LabelRankT: Incremental Community Detection in Dynamic Networks via Label Propagation

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ABSTRACT
An increasingly important challenge in network analysis is efficient detection and tracking of communities in dynamic networks for which changes arrive as a stream. There is a need for algorithms that can incrementally update and monitor communities whose evolution generates huge real-time data streams, such as the Internet or on-line social networks. In this paper, we propose LabelRankT, an online distributed algorithm for detection of communities in large-scale dynamic networks through stabilized label propagation. Results of tests on real-world networks demonstrate that LabelRankT has much lower computational costs than other algorithms. It also improves the quality of the detected communities compared to dynamic detection methods and matches the quality achieved by static detection approaches. Unlike most of other algorithms which apply only to binary networks, LabelRankT works on weighted and directed networks, which provides a flexible and promising solution for real-world applications.

Keywords
social network, community detection, clustering, network evolution, dynamic network, temporal

1. INTRODUCTION
Communities are the basic structures in sociology in general and in social networks in particular. They have been intensively researched for more than a half of the century [17]. In sociology, community usually refers to a social unit whose members share common values and the identity of the members as well as their degree of cohesiveness depend on individuals' social and cognitive factors such as beliefs, preferences, or needs. The ubiquity of the Internet and social media eliminated spatial limitations on community range, enabling on-line communities to link people regardless of their physical location. The newly arising computational sociology relies on computationally intensive methods to analyze and model social phenomena [3], including communities and their detection.

Analysis of social networks became one of the basic tools of sociology [24] and has been used for linking micro and macro levels of sociological theory. The classical example of the approach is presented in [10] that elaborated the macro implications of one aspect of small-scale interaction, the strength of dyadic ties. Moreover, a lot of commercial applications, such as digital marketing, behavioral targeting and user preference mining, rely heavily on community analysis.

With the rapid growth of large-scale on-line social networks, e.g., Facebook connected a billion users in 2012, there is a high demand for efficient community detection algorithms that will be able to handle their evolution growth. Communities in on-line social networks are discovered by analyzing the observed and often recorded on-line interactions between people.

Numerous techniques have been developed for community detection. However, most of them require a global and often static view of the network and ignore temporal correlations between different snapshots over time. Such algorithms are not scalable enough to cope with dynamically evolving networks, especially when new data about them are generated continuously. Another limitation shared by most of the existing algorithms is that they are applicable only to networks with binary adjacency matrix, that is with undirected and unweighted edges.

Label propagation based community detection algorithms such as LPA [18, 26], COPRA [11] and SLPA\(^1\) [27] have been shown to perform well in static networks. However, due to random tie breaking strategy, they produce different partitions in different runs. Such instability is highly undesirable when tracking the evolution of communities in a dynamic network.

The contributions of this paper are two-fold. First, we generalized the LabelRank algorithm introduced in [28] to incorporate important network features such as edge weights and directions. Second, built upon LabelRank, we introduce LabelRankT algorithm that incrementally detects evolving communities in dynamic networks. The new algorithm presented here delivers significant improvements over the existing solutions in both the quality of detected evolving communities and the speed of program execution.

\(^1\)Source codes: https://sites.google.com/site/communitydetectionslpa/
Algorithm 1 Generalized LabelRank

1: add self-loop to adjacency matrix $W$
2: initialize label distribution $P$ using Eq. 3
3: repeat
4: $P' = W \times P$
5: $P' = \Gamma_{in} P'$
6: $P' = \Phi, P'$
7: $P = \Theta_q(P', P)$
8: until stop criterion satisfied
9: output communities

2. GENERALIZATION OF LABELRANK

To make the paper self-contained, this section first summarizes and generalizes the LabelRank algorithm introduced in [28] which is the basis for LabelRankT. Both algorithms are based on the idea of simulating the propagation of labels in the network.

LabelRank relies on four operators applied to the labels: (i) propagation, (ii) inflation, (iii) cutoff, and (iv) conditional update. The data structure that lies in the core of this algorithm is the sparse matrix of label distribution. Each node maintains a label distribution locally during the propagation. At the end of the algorithm, LabelRank ranks labels in each node. Nodes with the same highest probability label form a community.

LabelRank, a stabilized LPA, was initially introduced in [28] for binary networks. Here, we generalize the propagation operation over edges in order to take both edge direction and weight into account. Given a network $G = (E, V)$, where $E$ is the set of edges and $V$ is the set of nodes, this operator can be expressed in matrix form as:

$$W \times P,$$  \hspace{0.5cm} (1)

where $W$ is the $n \times n$ weight matrix and $n$ is the number of nodes. For each $w_{ij}$, if there exist a directed edge $e_{ij} \in E$ from node $j$ to $i$, $w_{ij}$ takes on a positive value; otherwise it is 0. $w_{ij} > 0$ is the weight placed on the directed edge, which bears important and application specific information. $P$ is the $n \times n$ label distribution matrix. Label is just a unique identifier. For simplicity, it usually takes on the same value as the node ID. $P$ is composed of $n \times n$ row vectors $P_i$, one for each node $i$. Each element $P_{ic}$ or $P_i(c)$ holds the current estimation of probability of node $i$ observing label $c \in C$ taken from a finite set of alphabet (here $|C| = n$).

Each node broadcasts the distribution to its neighbors at each iteration step and computes the new distribution $P'_i$ simultaneously using the following equation:

$$P_{ic} = \sum_{c' \in \text{Nb}(i)} \frac{w_{ij}P_{ic'}}{\sum_{k \in \text{Nb}(i)} w_{ik}}, \forall C \in C, \hspace{0.5cm} (2)$$

where $\text{Nb}(i)$ is a set of neighbors of node $i$ and the numerator sums up weights of all edges incoming node $i$. That is, a node sends out its information along outgoing edges to its neighbors and at the same time receives information along incoming edges from the neighborhood. Note that, $P'_i$ is normalized to make proper probability distribution. One can show that the new distribution vector $P'_i$ is the distribution that minimizes the KL divergence between any possible $P'_i$ and $P_i$ [22].

To initialize $P$, each node is assigned a distribution of probabilities of all incoming edges by assigning to each incoming edge the initial probability of seeing this neighbor’s label proportional to the weight of this edge:

$$P_{ij} = \frac{w_{ij}}{\sum_{k \in \text{Nb}(i)} w_{ik}}. \forall j \in C \text{ s.t. } w_{ij} > 0. \hspace{0.5cm} (3)$$

We briefly characterize the three remaining operations of LabelRank as follows.

(1) The inflation operator $\Gamma_{in}$ on $P$ [23] is used to contract the propagation, where $in$ is the parameter taking on real values. It operates on the label distribution matrix $P$ (rather than to a stochastic matrix or adjacency matrix) to decouple it from the network structure. After applying $\Gamma_{in} P$, each $P_{ic}$ is proportional to $P_{ic}^n$, i.e., $P_{ic}$ rises to the $in$th power.

(2) The full label propagation distribution induces a cost for memory. To alleviate this problem, the cutoff operator $\Phi_r$ on $P$ is introduced to remove labels that are below threshold $r \in [0, 1]$. More importantly, $\Phi_r$ is shown empirically to reduce the space complexity efficiently, from quadratic to linear. The average number of labels in each node is typically less than 3.0 for $r = 0.1$.

(3) The conditional update $\Theta_q$ operator is used to trap the process in the quality space (e.g., modularity [14]) to avoid trivial network states where each node holds the same distribution. The algorithm updates a node only when it is significantly different from its neighbors in terms of labels. This allows us to preserve detected communities and detect termination based on scarcity of changes to the network. At each iteration, the change is accepted only by nodes that satisfy the following update condition:

$$\sum_{j \in \text{Nb}(i)} \text{isSubset}(C^*_i, C^*_j) \leq qk_i, \hspace{0.5cm} (4)$$

where $C^*_i$ is the set of maximum labels which includes labels with the maximum probability at node $i$ at the previous time step. Function $\text{isSubset}(s_1, s_2)$ returns 1 if $s_1 \subseteq s_2$, and 0 otherwise. $k_i$ is the degree of node $i$, and $q$ is a real number parameter chosen from the interval $[0, 1]$. Intuitively, $\text{isSubset}$ can be viewed as a measure of similarity between two nodes.

These four operators together with a post-processing that groups nodes whose highest probability labels are the same into a community form our algorithm (see Alg. 1).

The running time of the generalized LabelRank is the same as the original algorithm, as it is $O(n)$, linear with the number of edges $m$. The space complexity is $O(n)$ in practice because the number of labels in each node monotonically decreases and drops to a small constant in a few steps due to both cutoff and inflation operators. The $P$ matrix is replaced by sparse matrix representation of $n$ variable-length list of pairs (usually short) carried by each node; each pair contains label and its probability (with labels whose probability reduced to 0 not listed).
Figure 1: The example network G(0) with \( n = 15 \). Colors represent communities discovered by LabelRankT.

Figure 2: The example network G(1) with \( n = 15 \) that change from G(0) by splitting and merging. Nodes 10 and 14 moved from blue group into the red group. Three edges were deleted from and three were added. Colors represent communities discovered by LabelRankT after these events.

Figure 3: The example network G(2) with \( n = 15 \) in which node 6 was removed and node 16 was born. The green group dissolved and its members merged into blue and red groups. Colors represent communities discovered by LabelRankT.

Algorithm 2 LabelRankT
1: input: snapshots \( G([0, 1, \ldots, T]) \)
2: for \( t = 1:T \) do
3:   (a) Tracking the changed nodes in \( G(t) \) due to the changes in edges they attach to since \( t \rightarrow t - 1 \).
4:   (b) Initialize \( P^t \). For node \( i \) that does not change since \( t \rightarrow t - 1 \), we copy its label distributions, i.e., \( P^t_i = P^{t-1}_i \). For changed nodes, we reinitialize their label distributions as in LabelRank.
5:   (c) Iteratively update only changed nodes’ label distribution and assign them to the corresponding communities as in LabelRank.
6: end for

Unlike in binary networks, there are various ways of adding a self-loop to each node to stabilize the results. It is interesting to see that the algorithm might perform slightly differently with different ways of defining self-loop. The most common ways include setting \( w_{ii} = 1 \), \( w_{ii} = \max(w_{ik}) \) or \( w_{ii} = \sum_k w_{ik} \). In the experiments run for this paper, we use \( w_{ii} = 1 \). Yet, it is still an open question how to optimally select the selfloop weight for each node.

3. LABELRANKT: AN EXTENSION FOR DYNAMIC NETWORKS

The extended algorithm called LabelRankT is based on the generalized LabelRank introduced in Alg. 1. The description of LabelRankT is contained in Alg. 2. The main idea is to adjust our detection as the network structure changes. We take advantage of what we already obtain in previous snapshot for inferring the dynamics in the current time step. Since local structure information is encoded in the node label distributions, the evolving of communities is expected to be caught and reflected in these distributions.

LabelRankT can be viewed as a LabelRank with one extra conditional update rule by which only nodes involved any change accept the new distribution. Moreover, we only need to update nodes that are changed between consecutive snapshots, including cases where an existing node adds or deletes links, or a node is removed from or newly joins the network. An example that shows different evolution events in three consecutive snapshots, G(0), G(1) and G(2) is shown in Figs. 1, 2 and 3. During the evolution, nodes (edges) are added or removed, and communities split, merge and dissolve, all of which is captured by LabelRankT. To discover communities in these snapshots, we ran our algorithm with the same parameters for all three snapshots.

In our algorithm, all these cases are handled by simply comparing neighbors of a node \( i \) at two consecutive steps, \( t \rightarrow t - 1 \) and \( t \), i.e., \( \mathcal{NB}^t(i) \) and \( \mathcal{NB}^{t-1}(i) \). If \( \mathcal{NB}^t(i) \) and \( \mathcal{NB}^{t-1}(i) \) are not equal, then node \( i \) is called a changed node\(^2\). For changed nodes, we reinitialize their label distributions (i.e., \( P^t \)) and update until the simulation stops as in LabelRank. However, since only changed nodes and their neighbors are involved (some neighbors only propagate labels but not update), LabelRankT is more efficient than LabelRank.

\(^2\)Since it is often a case in practice, we assume here that all nodes in all steps are uniquely and consistently named.
The time complexity for LabelRankT can be derived as follows. It is easy to see for step (a) that we can track the number of changed nodes in $O(1)$. For steps (b) and (c), since at most $n$ can be changed, we need to communicate across each edge at most twice in each iteration. Since the number of iterations required is a constant $T$ (usually less than 50 iterations), the overall complexity for detecting evolving communities between two consecutive snapshots is $O(Tm)$, implying $O(m)$ in general.

We first compared the performance of LabelRankT with static algorithms MCL, using for all snapshots parameters that optimized performance for the first snapshot, and Infomap. Both run through each snapshot independently. Since a dynamic (especially incremental) algorithm like LabelRankT does not recompute the entire network, static algorithms might perform better. In fact, on AS Graph, see Fig. 8, three algorithms actually have close performance. Infomap slightly outperforms LabelRankT by about 5.03% in modularity on average. LabelRankT and MCL performance differs just by 0.43%. On arXiv HEP-TH (which is of much larger size than AS), as seen in Fig. 9, Infomap and LabelRankT perform within 0.88% of each other. However, LabelRankT outperforms MCL significantly by 15.37% (Note that the behavior of MCL is partially caused by its sensitivity to parameters). On the other hand, LabelRankT has benefit of efficiency. It runs 4 and 12 times faster than Infomap on AS Graph and arXiv HEP-TH respectively. And it is faster than MCL by a factor of 27 to 52 on AS Graph and arXiv HEP-TH respectively.

We also compared LabelRankT with two publicly available dynamic algorithms that employ incremental detection methods: facetNet $^3$ [13] and iLCD $^4$ [7]. On AS Graph, see Fig. 10, facetNet and LabelRankT achieve performance within 3. Since facetNet requires the number of communities as input, we assign it the value produced by LabelRankT.

$^4$After detection, if a node belongs to more than one community, we assign it to the the one with maximum size to be able to output only unique and disjoint partitions.
Figure 6: arXiv HEP-TH. Degree distributions at the beginning, middle and the end of evolution (main). Average degree over time (inset).

Figure 7: The structure changes in arXiv HEP-TH, including the number of edges added ($E_+$) and deleted ($E_-$), as well as the number of nodes involved in changes ($N_{+/-}$).

Figure 8: Comparison of modularity over time $Q(t)$ with static detection algorithms on AS-Internet Routers Graph.

Figure 9: Comparison of modularity over time $Q(t)$ with static detection algorithms on arXiv HEP-TH.

Figure 10: Comparison of modularity over time $Q(t)$ with dynamic detection algorithms on AS-Internet Routers Graph.

Figure 11: Comparison of modularity over time $Q(t)$ with dynamic detection algorithms on arXiv HEP-TH. We ran iLCD on only the first 130 snapshots due to the time complexity.
just 0.07% of each other, while iLCD fails to find strong community structure at all. As shown in Fig. 11, on arXiv HEP-TH, facetNet does not work due to the overflow in memory, while LabelRankT performs at least twice better than iLCD. Moreover, LabelRankT is more than 100 times faster than both facetNet and iLCD on the two datasets used here.

We also analyzed the number of communities and the distribution of community sizes relative to time. As shown in Fig. 11, on arXiv HEP-TH, LabelRankT does not work due to the overflow in memory, while LabelRankT performs at least twice better than iLCD. Moreover, LabelRankT is more than 100 times faster than both facetNet and iLCD on the two datasets used here.

We also analyzed the number of communities and the distribution of community sizes relative to time. The abrupt drop in the number of communities at time 410 signals a dramatic change in structure. As shown in Fig. 12, AS Graph does not evolve smoothly all the time. The abrupt drop in the number of communities at time 410 signals a dramatic change in structure, which is verified by a completely different distribution of community sizes in comparison with the beginning one. Although this violates our assumption, LabelRankT still worked well as evidenced by consistency of its results with the results of static algorithm Infomap. In contrast, arXiv HEP-TH exhibits a fairly smooth pattern shown in Fig. 13. The distributions of community sizes at time 1 and 350 (near the end of evolution) obey power laws with essentially identical exponents. Small size communities grow faster as more and more papers are published as indicated by the downward shift in these distributions. Some communities grow relatively faster than the others and the largest communities expand as indicated by the shift to the right (see the inset).

5. EFFECT OF EDGE WEIGHT AND DIRECTION

In this section, we demonstrate that incorporating the edge weight and direction into the network description allows LabelRankT to identify communities better. The experiments were conducted on a dataset including weighted and directed networks.

Reality Mining Bluetooth Scan Data [8]. This dataset was created from the records of Bluetooth Scans generated among the 94 subjects in Reality Mining study conducted from 2004-2005 at the MIT Media Laboratory. In the network, nodes represent the subjects and the directed edges correspond to the Bluetooth Scan records and the weight of each edge represent the number of directed Bluetooth scans between the two subjects. In the comparison described below, we only adopted the records from August 02, 2004 (Monday) to May 29, 2005 (Sunday) and we divided them weekly snapshots, so each snapshot represents scans collected during the corresponding week. There are total of 43 snapshots.

We compared the community detection results produced by LabelRankT on the Reality Mining Bluetooth Scan network with and without edge weight and direction. By varying the parameter $q$ (from 0.05 to 0.95) of the conditional update $\Theta_q$ operator, we calculated the average modularity differences, shown in Table 1, between the weighted and directed version and the unweighted and undirected version of LabelRankT. All the average modularity differences in Table 1 are positive. This demonstrates that including the edge weight and direction improves the performance of our algorithm. Further, Fig. 14 presents the modularity of LabelRankT with and without edge weight and direction on all the 43 snapshots with the conditional update parameter $q = 0.6$ when the average modularity difference is the largest. In conclusion, these results demonstrate that inclusion of edge weight and direction of the network improves the quality of communities detected by LabelRankT.

6. RELATED WORK

Label propagation and random walk based algorithms are most relevant to our work. LPA [18, 26] identifies disjoint groups as nodes with the same label. COPRA [11] and SLPA [27] extend LPA to detection of overlapping communities by allowing multiple labels. However, none of these algorithms resolves the LPA randomness issue, where different communities may be detected in different runs over the same network. Markov Cluster Algorithm (MCL) proposed in [23] is based on simulations of flow (random walk). MCL ex-
cutes repeatedly matrix multiplication followed by inflation operator.

LabelRankT, like its predecessor LabelRank (see [28]) differs from MCL in at least two aspects. First, LabelRankT applies the inflation to the label distributions and not to the matrix \( M \). Second, the update of label distributions on each node in LabelRankT requires only local information. Thus it can be computed in a decentralized way. Regularized-MCL [19] also employs a local update rule of label propagation operator. Despite that, the authors observed that it still suffers from the scalability issue of the original MCL. To remedy, they introduced Multi-level Regularized MCL, making it complex. In contrast, we address the scalability by introducing new operator, conditional update, and the novel stopping criterion, preserving the speed and simplicity of the LPA based algorithms. Moreover, neither MCL nor Regularized-MCL is suitable for dynamic networks.

For dynamic networks, there has been work that focus on exploring the properties of evolving communities that could be used to guide the detection algorithms. Palla et al. [16] developed an algorithm based on the clique percolation method and investigated the time dependence of overlapping communities to uncover basic relationships characterizing community evolution. Tantipathananandh and Berger-Wolf [21] extended their previous social cost model to arbitrary dynamic networks and approximately solved the optimization problem using semidefinite programming relaxation and a rounding heuristic. Bassett et al. [5] proposed an approach to construct representative partitions. This approach adopts a null model to correct for statistical noise in sets of partitions to improve robustness of community detection result in time-dependent networks.
8. REFERENCES


