# The Naming Game in Social Networks: Community Formation and Consensus Engineering

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Abstract We study the dynamics of the Naming Game [Baronchelli et al., (2006) J. Stat. Mech.: Theory Exp. P06014] in empirical social networks. This stylized agent-based model captures essential features of agreement dynamics in a network of autonomous agents, corresponding to the development of shared classification schemes in a network of artificial agents or opinion spreading and social dynamics in social networks. Our study focuses on the impact that communities in the underlying social graphs have on the outcome of the agreement process. We find that networks with strong community structure hinder the system from reaching global agreement; the evolution of the Naming Game in these networks maintains clusters of coexisting opinions indefinitely. Further, we investigate agent-based network strategies to facilitate convergence to global consensus.

# 1 Introduction

Agent-based models and simulations provide invaluable frameworks and tools to gain insight into the collective behavior of social systems (Epstein and Axtell, 1996; Challet et al, 2005; Anghel et al, 2004). Opinion spreading and social dynamics (Durlauf, 1999; Castellano et al, 2008) on regular and random networks are examples of the latter. A large number of studies have investigated models of opinion dynamics (Castellano et al, 2005; Ben-Naim, 2005; Deffuant et al, 2000; Hegselmann and Krause, 2002; Lorenz, 2007, 2008; Krapivsky and Redner, 2003; Sood and Redner, 2005; Sznajd-Weron and Sznajd, 2000; Kozma and Barrat, 2008a; Benczik et al, 2008; Antal et al, 2005; Jung et al, 2008) and the dissemination of culture (Axelrod, 1997; San Miguel et al, 2005; Mazzitello et al, 2007), while fundamental models for residential and ethnic segregation have also attracted strong interest (Schelling, 1971; Zhang, 2004; Vinkovic and Kirman, 2006; Lim et al, 2007). Most recently, researchers have

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also turned their focus to models where both the network topology and opinions change over time (Kozma and Barrat, 2008a,b). With the availability of empirical data sets and cheap and efficient computing resources, one can implement stylized socio-economic models on empirical social networks, and evolve "artificial societies" (Epstein and Axtell, 1996) to study the collective properties of these systems.

Here, we focus on one such stylized model, the Naming Game (NG) (Baronchelli et al, 2006b). The NG is a minimal model, employing local communications, that can capture generic and essential features of an agreement process in networked agent-based systems. For example, in the context of a group of robots (the original application), the NG dynamics mimics the emergence of shared communication schemes (synthetic languages), while in the context of sensor networks, such an agreement process can describe to the emergence of a shared key for encrypted communications. In a system of human agents, the NG can be considered as a minimal model to describe the recent phenomenon of collaborative tagging or social bookmarking (Cattuto et al, 2006, 2007; Golder and Huberman, 2006) on popular web portals like like Del.icio.us (http://del.icio.us), Flickr (www.flickr.com), CiteULike (www.citeulike.org), and Connotea (www.connotea.org). Another common example is the evolution and spread of coexisting dialects in everyday use (see, e.g., the geographical distribution of "Pop" vs "Soda" for soft drinks in the US (McConchie, 2002)). In a broader context, the NG can be employed to investigate the emergence of large-scale population-level patterns arising from empirically supported local interaction rules between individuals.

The common feature in the above examples and applications is that global agreement can emerge spontaneously (without global enforcement) purely as a result of local (e.g., pairwise agent-to agent) communications. The NG has been studied intensively on regular and random complex network models (see next Section). Here we investigate the evolution of the agreement process in the NG on empirical social graphs. It is well known that empirical social graphs exhibit strong community structure (Girvan and Newman, 2002; Newman and Girvan, 2004; Onnela et al, 2007; Palla et al, 2005, 2007). It is also known that in networks with community structure, reaching global agreement can be hindered (Lambiotte and Ausloos, 2007; Candia and Mazzitello, 2008). Here, we investigate the NG precisely from this viewpoint. Further, we analyze strategies to destabilize otherwise indefinitely coexisting clusters of opinions, to reach global consensus of a selected opinion. The later can also be considered as an abstract agent-based marketing approach.

The paper is organized as follows. In Sec. 2 we briefly review recent results on the NG on various regular and complex networks models. In Sec. 3, we present results for the NG on empirical social networks. In particular, we investigate the effect of communities in the underlying static social graphs on the agreement process (typically leading to indefinitely coexisting clusters of opinions). In Sec. 4, we study and analyze node-selection strategies to facilitate the convergence to a global opinion. In Sec. 5, we conclude our paper with a brief summary.

### 2 Background, Model, and Prior Results on the Naming Game on Regular and Complex Network Models

In the simplified version of the NG, agents perform *pairwise* games in order to reach agreement on the name to assign to a *single* object. This version of the NG was investigated on the complete graph (CG) (corresponding to mean-field or homogeneous mixing) (Baronchelli et al, 2006b, 2005), on regular (Baronchelli et al, 2006a), on small-world (SW) (Dall'Asta et al, 2006a; Lin et al, 2006), and on scale-free (SF) networks (Dall'Asta et al, 2006b; Baronchelli et al, 2006c). On a CG, each agent has a chance to meet with all others and compare their current local vocabularies (list of "synonyms") before updating them. On regular networks, agents have only a small number of nearest neighbors with whom they can interact/communicate (e.g., four nearest neighbors in two dimensions). The communication in both cases is "local", in that *pairs of agents* are selected to interact and to update their vocabularies. The basic algorithmic rules of the NG are as follows (Baronchelli et al, 2006b,a). A pair of neighboring nodes (as defined by the underlying communication topology), a "speaker" and a "listener", are chosen at random.<sup>1</sup> The speaker will transmit a word from her list of synonyms to the listener. (If the speaker has more than one word on her list, she randomly chooses one; if it has none, it generates one randomly.) If the listener has this word, the communication is termed "successful", and both players delete all other words, i.e., collapse their list of synonyms to this one word. If the listener does not have the word transmitted by the speaker (termed "unsuccessful" communication), she adds it to her list of synonyms without any deletion. In this paper, we measure time in units during which N agents are selected at random as speakers, where N is the number of agents in the network. The above rules are summarized in Fig. 1.

Here, we considered initial conditions when all agents have an empty vocabulary. Then such an agent, when chosen to be a speaker, invents a *random* word to be transmitted to the listener. (For initial conditions with a single random word per agent, see the Electronic Supplementary Material.) In terms of the number of different words, the evolution of the game will go through stages of growth (due to unsuccessful communications) and stages of elimination (due to successful ones). In all of the above mentioned networks, starting from empty vocabularies, an early time explosion of words is followed by a slow elimination of all synonyms, except one; that is agents come to global agreement on the naming of the object in question.

Also recently, the NG was studied on homogeneous random geometric graphs (RGGs) (Lu et al, 2006, 2008). RGG is both spatial and random

<sup>&</sup>lt;sup>1</sup> Note that on strongly heterogeneous (scale-free) graphs, the order whether the listener or the speaker is chosen first, strongly impacts the efficiency toward global agreement. Choosing the listener first at random will increase the chance for selecting a node (as a neighbor) with larger degree for speaker. In turn, hubs will be the most frequent speakers, giving rise to faster convergence to global agreement at a mildly elevated memory cost (Dall'Asta et al, 2006b; Baronchelli et al, 2006c).



Fig. 1 Schematic rules of the Naming Game (Baronchelli et al, 2006b) as described in the text. If the speaker has more than one word on her list, she randomly chooses one; if it has none, it generates one randomly.

(Meester and Roy, 1996; Penrose, 2003; Dall and Christemsen, 2002). Motivated by the deployment of sensor networks, nodes are randomly scattered in a two-dimensional area, and two nodes are connected if they fall within each others transmission range. Further, pairwise communications were replaced by local broadcasts to capture the essential features of communication protocols in sensor networks. Similar to earlier findings on regular, SW, and SF networks, we found that the NG on RGG with homogeneous node density also leads to global consensus, facilitating an application to autonomous key creation for encrypted communication in a community of sensor nodes (Lu et al, 2006, 2008).

It was found that the NG dynamics on the above networks will lead to global consensus among all agents, i.e., after some time, agents' vocabularies eventually converge to a unique word, the same for all agents (Baronchelli et al, 2006b, 2005, 2006a; Dall'Asta et al, 2006a). The major differences between the NG on CGs (homogeneous mixing) and on low-dimensional networks (such as regular one- or two-dimensional grids, and RGGs) arise in the scaling of the memory need and in the scaling of the time  $t_c$  with the number of agents N to reach global agreement. The memory need in the present context is the typical value of the largest number of words an agent may posses throughout the evolution of the game (Baronchelli et al, 2006b,a).] In CGs, the convergence process to global agreement is fast,  $t_c \sim \mathcal{O}(N^{1/2})$  (measured in units of communications per agent), but large memory,  $\mathcal{O}(N^{1/2})$ , is needed per agent (Baronchelli et al, 2006b). For a regular two-dimensional network or RGG, spontaneous evolution toward a shared dictionary is slow,  $t_c \sim \mathcal{O}(N)$ , but the memory requirement per agent is much less severe,  $\mathcal{O}(1)$  (Baronchelli et al, 2006a). When the NG is implemented on Watts-Strogatz (Watts and Strogatz, 1998) SW networks (Dall'Asta et al, 2006a), or when long-range random links are added to the RGG (Lu et al, 2008), the agreement dynamics performs optimally in the sense that the memory needed is small, while the convergence process is much faster than on the respective low-dimensional network,  $t_c \sim \mathcal{O}(N^{0.4})$ , closer to that of CGs or homogeneous mixing.

The above results, on spatial graphs, can be understood within the framework of coarsening, a well know phenomenon from the theory of domain and phase ordering in physical and chemical systems (Brav, 1994). Starting from empty vocabularies, agents invent words randomly. After time of  $\mathcal{O}(1)$  [on average one communication per node],  $\mathcal{O}(N)$  different words have been created. Following the early-time increase of the number of different words (essentially corresponding to the number of different clusters of agents)  $N_d(t)$ , through pairwise or local communications, agents slowly reconcile their "differences", and eventually will all share the same word. First, a large number of small spatial clusters sharing the same word develop. By virtue of the slow coalescence of the interfaces separating the clusters, more and more of the small clusters are being eliminated, giving rise to the emergence of larger clusters, eventually leading to one cluster in which all nodes are sharing the same word, i.e.,  $N_d=1$ (Baronchelli et al. 2006a; Lu et al. 2008). In domain coarsening, the typical size of domains (each with already agreed upon one word) is governed by a single length scale  $\xi(t) \sim t^{\gamma}$  with  $\gamma = 1/2$ . Thus, in d dimensions the average domain size (inside which all agent share the same word) scales as  $\xi^d(t) \sim t^{d\gamma}$ and the total number of *different* words  $N_d$  at time t scales as the typical number of domains  $N_d(t) \sim N/\xi^d(t) \sim Nt^{-d\gamma}$ . Global consensus is reached when  $\xi^d(t_c) \sim N$  (or equivalently  $N_d(t) \sim 1$ ), hence the typical time to global agreement scales as  $t_c \sim N^{1/(d\gamma)}$ .

On SW and SF random network models with no community structure, the long-time behavior of the NG is essentially governed by the mean-field fixed point,<sup>2</sup> and global consensus time scales as  $t_c \sim N^{1/2}$  (although with noticeable finite-size corrections) (Dall'Asta et al, 2006a,b; Lu et al, 2008). On the other hand, Dall'Asta et al (2006b) found that on stylized network models with community structure (composed of fully connected cliques with a single link between cliques) the evolution of the NG runs into long-living meta-stable configurations, corresponding to different co-existing words (different for each clique). Here we study precisely this later scenario by implementing the NG on static empirical social graphs.

In this work we employ the basic NG where agents have infinite memories (i.e., the number of words they can store on their individual lists is unlimited). For a finite-memory version (Wang et al, 2007) of the model on social networks see the Electronic Supplementary Material.

In passing we note that the issue of the emergence of meta-stable or frozen opinion clusters and fostering consensus have been discussed for models of opinion formation under bounded confidence (Deffuant et al, 2000; Hegselmann and Krause, 2002; Lorenz, 2007, 2008). In those models, however, community formation or opinion segmentation is the result of the agents' inter-

 $<sup>^2</sup>$  Here, by the mean-field fixed point, we refer to the characteristic scaling behaviors of the NG on the complete graph (also referred to as homogenous mixing) where each agent can potentially interact with all others (Baronchelli et al, 2006b; Dall'Asta et al, 2006b).

action being limited by bounded confidence: an agent can gradually adjust her opinion toward another one's only if their opinions were already sufficiently close to one another to begin with. As a result, opinion segmentation can emerge in networks with no community structure with low-confidence agents. In contrast, the NG dynamics does not require that agents' opinions are sufficiently close in order to potentially interact (i.e., their confidence is unbounded), and as mentioned earlier, the NG dynamics always lead to global consensus on networks *without* community structure. Our motivation here, by studying the NG on empirical social graphs, is to directly study how the community structure of the underlying graphs affects the emergence of meta-stable or long-living opinion clusters.

#### 3 The Naming Game on Empirical Social Networks

One of the most important feature of social graphs is their modularity: these networks typically consist of a number of communities; nodes within communities are more densely connected, while links bridging communities are sparse. Since the community structure of empirical networks is often not known a priori, detecting communities in large networks itself is a difficult problem (Palla et al, 2005). A number of current methods for finding community structures utilize various forms of hierarchical clustering, spectral bisection methods (Scott, 2000; Newman, 2006; Wu and Huberman, 2004), and iterative high-betweenness edge removal (Newman and Girvan, 2004; Newman, 2004; Girvan and Newman, 2002). A different approach involves searching for the ground-states of generalized multi-state spin models (corresponding to different opinions) on these networks, such as the q-state Potts model (Blatt et al, 1996; Reichardt and Bornholdt, 2004; Kumpula et al, 2007; Fortunato and Barthelemy, 2007). Also, recently a novel method has been developed to detect overlapping communities in complex networks (Palla et al, 2005).

The NG, as summarized in the Sec. 2, in low-dimensional networks exhibits slow coarsening, while networks with small-world characteristic (small shortest path, such as in SW and SF networks) facilitate faster (and guaranteed) convergence to a global consensus among nodes. But in all cases, global consensus is reached, provided the network has no heterogeneous clustering or modularity (i.e., community structure).

Here, we study the NG on networks which do exhibit strong community structure. The set of social networks (high-school friendship networks), on which we implemented the NG, were provided by the National Longitudinal Study of Adolescent Health (Add Health)<sup>3</sup>. The high-school friendship

<sup>&</sup>lt;sup>3</sup> This research uses the network-structure data sets from Add Health, a program project designed by J. Richard Udry, Peter S. Bearman, and Kathleen Mullan Harris, and funded by a grant P01-HD31921 from the National Institute of Child Health and Human Development, with cooperative funding from 17 other agencies. For data files contact Add Health, Carolina Population Center, 123 W. Franklin Street, Chapel Hill, NC 27516-2524, (addhealth@unc.edu, http://www.cpc.unc.edu/projects/addhealth/.



Fig. 2 (a) Number of different words  $N_d$  vs time for a friendship network (thin lines) and for the Watts-Strogatz network (bold line). N = 1, 127,  $\overline{k} = 8.8$ , and  $\overline{C} = 0.067$  for both systems. Results for the WS network are averaged over 1000 independent realizations. For the high-school friendship network we show three individual realizations (thin lines), reaching different final states with  $N_d = 1$ ,  $N_d = 2$ , and  $N_d = 3$  (indicated with horizontal dashed lines). Note the log scales on both axes. (b) The probability (relative frequency) of final configurations with  $N_d$  different words (opinions) for the same high school friendship network as in (a) based on 10,000 independent runs. Statistically, in this particular network, the most likely final configuration exhibits three opinions.

networks investigated here, were constructed from the results of a paper-andpencil questionnaire in the AddHealth project (Moody, 2001). Here, nodes represent students while the edges are for their mutual relations or friendships. Two students are considered to be friends (thus have a link between them) when one nominates the other as her/his friend and both of them participated in some activities, e.g., talked over the phone, spent the weekend together, etc., in the last seven days. (for this study, we considered the relationships reciprocal, and associated them with undirected links in the NG). These networks exhibit exponential degree distributions (no hubs), with an average degree of the order of 10. For a baseline comparison we also constructed a Watts-Strogatz (WS) network (Watts and Strogatz, 1998) network with the same number of nodes N, average degree  $\overline{k}$ , and clustering coefficient  $\overline{C}$  as the friendship network. The WS network has homogeneous clustering, hence, no community structure.

We selected a few networks with a large number of students (on the order of 1,000) from the available data set. Starting from an empty word list for all agents, both the friendship network and WS network show nearly identical early-time development of the number of different words  $N_d$ . However, the friendship-network simulations exhibit a long-time behavior very different from the ones discussed in Section 2, and also from the baseline reference, the NG on the WS network [Fig. 2(a)]. In the late stage of the NG, networked



Fig. 3 Snapshots of the time-evolution of the Naming Game on a high-school friendship network. Initially agents have an empty word list (no opinions). In the snapshots, different colors correspond to different words. In the very early stage of the game (a), "speakers" with no words has to create one randomly. After a slow but steady coarsening of opinions, in the final stages of the game, the system exhibit relatively long plateaus in the number of different opinions. The corresponding clusters, i.e., agents with the same opinions, can be regarded as communities. For the particular network shown here, in the next to final stage (b), the network exhibits four communities. Eventually, two of these communities coalesce, leading to a final configuration (c) with three communities.

agents without community structure (including the WS network) always exhibit a spontaneous evolution toward a shared "dictionary" (or opinion), i.e., a global consensus is reached. In contrast, in the empirical high-school networks, consensus is rarely reached (for long but finite simulation times) [Fig. 2(a)]. For this particular high-school friendship network, performing 10,000 independent runs of the NG with a fixed simulation time of  $t = 10^4$  steps, 10%, 35%, and 55% of these runs, ended up with one, two, and three different words, respectively, in their final configurations [Fig. 2(b)]. Thus, in this network, the most likely (or typical) outcome of the NG is one with three different clusters of opinions. Snapshots taken from the typical evolution of the NG on this network are shown in Fig. 3. In analogy with domain formation in physical systems, we can regard these long-living configurations with coexisting multiple opinions as "meta-stable" ones.

The emergence of different long-living clustered opinions is not unexpected. In fact, the same high-school networks have been analyzed for community structures in a study of friendship segregation along racial lines among highschool students (Moody, 2001; Gonzáles et al, 2007). For example, close to the final stage, the time-evolution of the NG on the particular network shown in Fig. 3(b) exhibits four communities. These four clusters of opinions correspond to segregation along the two-schools involved in the particular network, highschool (HS) – middle-school (MS) pair, and along racial lines, whites students - black students in each. Checking the race and school-grade attribute of the node information in the raw data, we confirmed that the four communities exhibited by the NG in Fig. 3(b) correspond to black HS (upper left), white HS (upper right), black MS (lower left), and white MS (lower right) students. Then, in the final state [Fig. 3(c)], only three communities remain; opinions, segregated along the racial line coalesce in the middle-school portion of the students, simply indicating that racial segregation in friendships is weaker in this group, in this particular network set.

Admittedly, the objective of our paper is not to draw over-ambitious conclusions from a social science viewpoint. Instead, we are interested in how the evolution of the NG (a stylized model for opinion formation) is affected by the community structure of the underlying graphs, such as the high-school friendship networks which are well-known to exhibit strong community structure (Moody, 2001; Gonzáles et al, 2007). We demonstrated that the outcome of the NG, is strongly affected by the existence of communities in the underlying network. Conversely, at some coarse level, the long-living late-stage meta-stable clusters of words (opinions) reveal important aspects of the community structure of the underlying network. Thus, the NG, together with other stylized models for opinion formation, can not only be used as a tool to understand generic features of spontaneous agreement processes in a network of artificial or human agents, but can also be employed to extract relevant information on the community structure of complex networks (Blatt et al, 1996; Reichardt and Bornholdt, 2004; Kumpula et al, 2007; Fortunato and Barthelemy, 2007).

#### 4 Engineering Consensus in Social Networks

There are several ways to influence the outcome of social dynamics, e.g., to facilitate the outcome of a specific global opinion that one would prefer the system to achieve (preferred opinion for short). All methods essentially rely on "breaking the symmetry" of the otherwise equivalent coexisting opinions. One possibility is to expose and couple many or all agents to an "external" global signal (analogous to mass media effects) (Mazzitello et al, 2007; Candia and Mazzitello, 2008). Alternatively, one can break the symmetry by choosing a small number of well-positioned "committed" agents who will stick to a preferred opinion without deviation. In the next subsection, we investigate this latter scenario first.

#### 4.1 Committing Agents

In the simulations, by committed agents we mean an agent who has a fixed opinion which cannot be changed. In the context of the NG, a committed agent has a single word. As a listener, she does not accept any new word from their neighbors, but as a speaker, always transmits her word. Of the three co-existing communities at the end-stage of the NG [Fig. 3(c)], we choose one community as the one representing the "preferred" opinion, and we "indoc-trinate" selected committed agents with this opinion. Fig. 4 shows snapshots of the evolution of the NG with committed agents. Initiating the simulations from the final configuration of the original NG (exhibiting three meta-stable opinion clusters), introducing a small number of committed agents yields a relatively fast convergence to the global consensus of the selected opinion.

To quantify this phenomena we investigated the temporal behavior of this agreement process, in particular, its dependence on the method of selecting



**Fig. 4** Snapshots of the Naming Game on a high-school friendship network with committed agents. The system is initialized from a state with three coexisting meta-stable communities [see (a)] with a small number of well-positioned committed agents (indicated with yellow core around the nodes as indicated in (b). Global consensus (i.e., a single opinion) is reached exponentially fast. Here we employed 50 committed agents, selected according to their degree ranking.

committed agents and on the number of these selected agents. Among the methods to select committed agents are selecting nodes with the highest degrees (nodes with the highest number of neighbors), with the highest betweenness (likely to bridge different communities), with hop-distance proximity to the core cluster (nodes outside, but no farther than two hops from the core cluster of "preferred" opinion), and for comparison, also selecting committed agents at random.

Our main observation is that once the number of committed agents is sufficient to induce global consensus, it happens exponentially fast, independently of the selection method. More precisely, we ran 10,000 realizations of the NG with committed agents. The initial configuration here is the final multi-opinion meta-stable configuration of the original NG with no committed agents (with  $N_d = 3$  [Fig. 4]. We kept track of the fraction of surviving runs,  $n_s(t)$ , defined as the fraction of runs that have not reached global consensus by time t, i.e., runs that have more than one opinion at time t. (This quantity then can also be interpreted as the probability that a single run has not reached consensus by time t.) We choose committed agents, to maximize their influence in reaching global consensus, according to their ranking in a number of graph theoretical measures. We selected the top M agents according to their degree, shortest-path betweenness centrality, (Newman and Girvan, 2004; Newman, 2004), hop-distance proximity to the preferred core opinion cluster, or at random, for reference. Figure 5 displays the fraction of surviving runs  $n_s(t)$  for the degree and for the betweenness ranking for a number of different committed agents.

A common feature of all methods is that a very small fraction (f = M/N)of committed nodes is sufficient to induce global consensus. I.e., there seems to be a very low threshold in  $f_{c1}$ , such that for  $f > f_{c1}$  the dynamics with committed nodes leads to global agreement. Further, in this case, the fraction of surviving runs (fraction of runs with more than one opinion),  $n_s(t)$ , in the long-time regime, decays exponentially

$$n_s(t) \propto e^{-t/\tau} . \tag{1}$$



Fig. 5 Fraction of surviving runs as a function of time for varying number of committed agents M when agents are selected according to their (a) degree ranking and (b) (shortest-path) betweenness ranking. The total number of agents is N = 1, 127. For the degree-based ranking selection method different symbols represent the fraction of surviving runs for 12, 13, 15, 25, 40, 60, and 80 committed agents, from top to bottom. In betweenness selection method the number of committed agents M ranges from 22, 25, 30, 35, 40, 50, 60, to 70, from top to bottom.

The time scale of the exponential decay  $\tau$ , of course, depends on the selection method and the fraction of committed nodes. The inverse time scale  $1/\tau$ , i.e., the rate at which global consensus is approached is, initially, an increasing function of the number of committed nodes, but it quickly *saturates* and essentially remains constant. This can be seen in Fig. 5, as the slopes of the exponential decays are becoming progressively steeper, up to a certain M, then they remain constant. Thus, there is second characteristic fraction of committed agents, such that for  $f > f_{c2}$  the rate of reaching global consensus becomes essentially a constant (saturates).

These three features, (i) small threshold  $f_{c1}$  required for global consensus, (ii) exponential decay of  $n_s(t)$  if  $f > f_{c1}$  [Figs. 5 and 6], and (iii) saturation of the rate to reach consensus for  $f > f_{c2}$  [Fig. 7], are exhibited by all selection method we considered here. Further, both characteristic values and the gap between them are very small,  $f_{c1}$ ,  $f_{c2}$ ,  $f_{c2}-f_{c1} \ll 1$ . These results are essentially summarized in Fig. 7. The convergence rate for the randomly selected committed nodes is also shown for comparison. On this particular social network, selecting a small number of the nodes with the highest degree works best, followed by the hop-distance proximity (to the core cluster) ranking. (We refer to a strategy as more efficient if the convergence rate  $1/\tau$  is larger for the same fraction of committed agents.) For example, selecting the committed agents according to their degree ranking,  $f_{c1} \approx 0.01$  and  $f_{c2} \approx 0.03$  [Fig. 7]. Selecting committed agents just above this latter fraction is optimal, since the rate of convergence does not improve beyond this value.



Fig. 6 Fraction of surviving runs as a function of time for different strategies with the same number of committed agents on the same network (M = 35, N = 1, 127,  $f \simeq 0.031$ .). The three strategies (selection of committed agents) shown here are based on degree ranking (squares), hop-distance proximity to the core cluster (diamonds), and shortest-path betweenness (circles). For comparison, the result of selecting committed agents randomly is also shown (triangles).

In general, the optimal selection method will vary, depending on the community structure of the particular underlying network. However, because we changed the dynamics of the NG by breaking the symmetry of otherwise equivalent opinions, the exponential decay and the saturation of the convergence rate is expected to be generic for a large class of opinion formation models on networks with community structure.

### 4.2 Global External Influence

As mentioned in the introductory paragraph of this Section, another natural way of influencing the outcome of the competition among otherwise neutral and meta-stable opinions, is to couple all or a fraction of agents to a global external "signal" [mimicking a mass media effect (Mazzitello et al, 2007; Candia and Mazzitello, 2008)]. For comparison, we implemented the NG with an external field (affecting all agents) corresponding to the selected opinion among the three meta-stable ones in the final stage of the NG. Then, similar to the committed-agent approach, we initialize the system with that final meta-stable state with co-existing opinions of the original NG. In the presence of mass media, an agent, when randomly chosen, with probability p will adopt the externally promoted opinion. Otherwise, the usual rules of the game are invoked (i.e., the node, as a speaker, initiates communication with a listener). Our findings indicate that even for extremely small values of p, the fraction of



Fig. 7 Convergence rate as a function of the fraction of committed agents f=M/N, for different selection methods of committed agents, including the degree ranking (squares), hop-distance proximity to core cluster (diamonds), and shortest-path betweenness (circles). For comparison, the result of selecting committed agents randomly is also shown (triangles).

surviving runs (the fraction of runs that have not reached global consensus) decays exponentially, ultimately leading to global order Fig. 8(a)]. The rate of convergence  $1/\tau$  increases monotonically and smoothly with p [Fig. 8(b)]. For application oriented studies, one should associate a cost with the mass-media coupling, and a cost with committing an agent (e.g., finding these nodes and giving them incentives impossible to resists), then perform a relevant costbenefit comparative analysis for the selection or optimal combination of two approaches.

#### 5 Summary

We studied the Naming Game on social networks. Earlier works have shown that this simple model for agreement dynamics and opinion formation always leads to global consensus on graphs with no community structure. On the other hand, social networks are known to have rich community structure. The Naming Game on such networks exhibits, in the late-stage of the dynamics, several meta-stable coexisting communities; these configurations, in effect, are the computationally observed final configurations.

In the context of models for social dynamics, communities manifest themselves in the context in which distinct stylized opinions (e.g., religions, cultures, languages) have evolved and emerged over time. Clusters of nodes having reached consensus are part of a community, reflecting the inherent community structure of the underlying social networks. Thus, if at the late stages of the social dynamics on the networks, several communities persist (different opin-



**Fig. 8** (a) The fraction of surviving runs as a function of time for several values of the strength of external influence p (p is the probability that in a time step an agent will adopt the fixed externally and globally promoted opinion). (b) Convergence rate to global consensus as a function of the strength of external field p.

ions survive), they are the authentic signatures of the underlying community structure. The Naming Game, together with other similar models for opinion formation, can be employed to probe these properties of complex networks.

We then considered the task of destabilizing the coexisting meta-stable opinions (in order to reach consensus) by selecting the optimal number of committed agents with a preferred opinion, as an alternative to a global external signal (mass media effect). The results implied that a small number of committed agent is sufficient to facilitate an exponential decay toward global consensus of the selected opinion. Further, selecting more agent than a systemspecific upper cut-off, yields no improvement in the convergence rate. Hence, there seems to be an optimal number of agents for this task, beyond which it does not pay off to invest in committing more agents. Selecting the committed agents according to their degree, betweenness, or hop-distance proximity to the core cluster of the preferred opinion, all displayed the above qualitative features. Further, they all significantly outperformed committing the same number of agents at random.

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## **Electronic Supplementary Material**

### The Naming Game with one-word-per-agent initial conditions

Here, we show the behavior of the Naming Game (NG) with initial configurations where each agent has exactly one word (or opinion), different for each agent  $(N_d(0) = N)$ . The rest of the rules of the NG are the same as described in Sec. 2 of the main article. Since each agent has a word initially, no new words will be invented, and there is no growth phase ( $N_d$  is monotonically decreasing as a function of time); pairwise communications will lead to the gradual elimination of existing opinions. At the end, only a few opinion clusters remain, again, reflecting the community structure of the underlying graphs. There is no significant difference in the *late-stage scaling behavior* between the empty-dictionary and the one-word-per-agent initial conditions: In the former case, it takes of  $\mathcal{O}(1)$  time steps to reach the maximum of  $N_d$ ,  $N_d^{\max} \sim N/2$  [Fig. 2(a) of the main article], after which slow "opinion coarsening" begins. In the latter case, the number of different words initially starts from N and slowly begins to decay [Fig. S1(a).] The relative frequencies of the final configurations with one, two, and three words are very similar [Fig. 2(b) of the main article vs Fig. S1(b)], and the underlying opinion clusters (communities) exhibited this way are the same. Note that for the one-word-per-agent initial conditions, out of 10,000 independent runs, we also recorded 3 runs with four surviving opinion clusters ( $N_d$ =4) after 10<sup>4</sup> time steps, corresponding to a 3 × 10<sup>-4</sup> relative frequency. Since it is three orders of magnitudes smaller than the probability of other possible final configurations, it is not visible on the same scales and is not shown in Fig. S1(b).



Fig. S1: The Naming Game on a friendship network where the simulations are initialized from configurations with a single word per agent (different for each agent). (a) Number of different words  $N_d$  vs time for a friendship network with N = 1, 127,  $\overline{k} = 8.8$ , and  $\overline{C} = 0.067$  (same friendship network as in Fig. 2 of the main article). Results are shown for three individual realizations, reaching different final states with  $N_d = 1$ ,  $N_d = 2$ , and  $N_d = 3$  (indicated with horizontal dashed lines). (b) The probability (relative frequency) of final configurations with  $N_d$  different words (opinions) for the same friendship network as in (a) based on 10,000 independent runs.

# The Naming Game with finite-memory agents

As was discussed by Dall'Asta et al. (2006), the typical memory need (the maximum number of words in its list at any given time) of an agent of degree k is of  $\sqrt{k}$ . Limiting the agents' memory to a finite value L, in general, can slow down the consensus process (Wang et al., 2007). For the particular friendship network we used here,  $k_{\min} = 1$ ,  $k_{\max} = 33$ , and  $\overline{k} = 8.8$ . Here we considered a "first in - first out" finite-memory version of the NG: in case of an unsuccessful communication, if the memory of the listening agents is full, it drops the word from its list which has been there the longest, and adds the one just heard. All the other rules of the NG, as described in Sec. 2 of the main article, remain the same.

For comparison, here we show the agreement process in terms of number of different words (communities) vs time for both the Watts-Strogatz network and for the friendship network with the same number of nodes, average degree, and clustering coefficient [Fig. S2]. Since the average degree of these networks is rather small, there is very little



Fig. S2: Naming Game with finite-memory agents. The number of different words vs time on (a) the Watts-Strogatz network; (b) a friendship network with the same number of nodes, average degree, and clustering coefficients ( $N = 1, 127, \overline{k} = 8.8$ , and  $\overline{C} = 0.067$ ). All curves represent averages over 10,000 independent runs.

variation in the late-stage agreement process and convergence times (the infinite-memory behavior is asymptotically approached as the memory length is of order  $\sqrt{k}$ ). The only exception is the L=1 case: here the listener simply replaces its current opinion with the one of the speaker's, hence the model becomes equivalent to the q-state voter model (with  $q = \infty$ ) (Howard, 1998). Unlike the NG with  $L \ge 2$ , the voter model on random graphs has no tendency to form compact domains; the 'interfaces'' or boundaries separating domains stochastically disintegrate. While the voter model on finite networks eventually orders, it is the result of a large spontaneous fluctuation of ordered regions spanning the full system (Castellano, 2003; Castellano 2005). Global consensus for the NG with L=1 (equivalent to the voter model) on networks with no community structure takes much longer than for  $L\ge 2$  [Fig. S2(a)]. On networks with community structure (such as the friendship network), however, the tendency of the NG with  $L\ge 2$ to form domains leads to the formation of compact long-living or meta-stable opinion clusters, hence global order is rarely reached. On the other hand, when L=1, there are no stable domain boundaries and the underlying community structure has little or no effect on reaching global consensus [Fig. S2(b)]. The NG with L=1 (voter model) orders on any finite network, essentially independently of the underlying community structure.

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