Naming Games in Spatially-Embedded Random Networks [*]

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Abstract

We investigate a prototypical agent-based model, the Naming Game, on random geometric networks. The Naming Game is a minimal model, employing local communications that captures the emergence of shared communication schemes (languages) in a population of autonomous semiotic agents. Implementing the Naming Games on random geometric graphs, local communications being local broadcasts, serves as a model for agreement dynamics in large-scale, autonomously operating wireless sensor networks. Further, it captures essential features of the scaling properties of the agreement process for spatially-embedded autonomous agents. We also present results for the case when a small density of long-range communication links are added on top of the random geometric graph, resulting in a "small-world"-like network and yielding a significantly reduced time to reach global agreement.

Introduction

Reaching agreement without global coordination is of fundamental interest in large-scale autonomous multi-agent systems. In the context of social systems, the objective is to understand and predict the emergence of large-scale population-level patterns arising from empirically supported local interaction rules between individuals (e.g., humans). Examples for such phenomena driven by social dynamics include the emergence and the evolution of languages (Nowak, Plotkin, & Krakauer 1999; Nowak & Komarova 2001; Matsen & Nowak 2004) or opinion formation (Castellano *et al.* 2005; Ben-Naim 2005; Deffuant *et al.* 2000; Durlauf 1999; Krapivsky & Redner 2003; Sznajd-Weron & Sznajd 2000). From a system-design viewpoint in technological (e.g., sensor) networks (Lee *et al.* 2005; Collier & Taylor 2004), the objective can be somewhat reversed, in that it is to construct local rules giving rise to a fast and efficient convergence to a global consensus, when needed.

In this paper we consider and slightly modify a simple set of rules, referred to as Language or Naming Games

(NG), originally proposed in the context of semiotic dynamics (Steels 1997; Kirby 2002). Such problems have become of technological interest to study how artificial agents or robots can invent common classification or tagging schemes from scratch without human intervention (Steels 1997; Kirby 2002). The original model (Steels 1997; 1998; 1995; Steels & McIntyre 1997) was constructed to account for the emergence of shared vocabularies or conventions in a community of interacting agents. More recently, a simplified version of the NG was proposed and studied on various network topologies by Baronchelli et al. (2006b; 2005; 2006a), and by Dall'Asta et al. (2006) The advantage of studying a minimal model is that one can gain a deeper understanding of the spontaneous self-organization process of networked autonomous agents in the context of reaching global agreement, and can extract quantitative scaling properties for systems with a large number of agents. This simplified version of the NG was investigated on fully-connected (FC) (also referred to as mean-field or homogeneous mixing) (Baronchelli *et al.* 2006b; 2005), regular (Baronchelli *et al.* 2006a), and small-world (SW) networks (Dall'Asta *et al.* 2006).

In the FC network, each agent has a chance to meet with all others and compare their current local vocabularies before updating them. On regular networks, agents have only a limited and fixed number of neighbors with whom they can interact/communicate. The communication in both cases is "local", in that *pairs of agents* are selected to interact and to update their vocabularies. The basic algorithmic rules of the NG are as follows (Baronchelli *et al.* 2006b; 2006a). A pair of "neighboring" nodes (facilitated by the underlying communication topology), a "speaker" and a "listener", are chosen at random. The speaker will transmit a word from her list of "synonyms" to the listener. If the listener has this word, the communication is a success, and both players delete all other words, i.e., collapse their list of synonyms to this one word. If the listener does not have the word transmitted by the speaker, she adds it to her list of synonyms without any deletion. It was found that employing the above local rules (*pair-wise* interactions), after some time, the agents vocabularies converge to a unique vocabulary shared among all agents (Baronchelli *et al.* 2006b; 2005; 2006a; Dall'Asta *et al.* 2006). The major differences between the NG on FC and on regular (e.g., two-

dimensional) networks arise in the memory needed to develop the common language before convergence occurs, and in time t_c needed to reach global agreement. In the FC network, the convergence process to global agreement is fast [$t_c \sim \mathcal{O}(N^{1/2})$ for N agents], but large memory is needed per agent (Baronchelli *et al.* 2006b). For a regular two-dimensional network (or grid), spontaneous evolution toward a shared dictionary is slow [$t_c \sim \mathcal{O}(N)$], but the memory requirement is much less severe (Baronchelli *et al.* 2006a). When the NG is implemented on Watts-Strogatz (1998) SW networks, the agreement dynamics performs optimally in the sense that the memory needed is small, while the convergence is much faster than on the regular networks [$t_c \sim \mathcal{O}(N^{0.4})$, close to that of the FC network] (Dall'Asta *et al.* 2006).

The situation described above (i.e., the need for shared vocabularies) can also be quite realistic in the context of sensor networks (Lee *et al.* 2005; Collier & Taylor 2004). Envision a scenario where mobile or static sensor nodes are deployed in a large spatially-extended region and the environment is unknown, possibly hostile, the tasks are unforeseeable, and the sensor nodes have no prior classification scheme/language to communicate regarding detecting and sensing objects. Since subsequent efficient operation of the sensor network inherently relies on unique object identification, the autonomous development of a common "language" for all nodes is crucial at the exploration stage after network deployment (Collier & Taylor 2004; Lee *et al.* 2005). For this task, however, there are more efficient and faster schemes, guaranteeing to reach global agreement on the naming (tagging) of an object. In particular, basic leader-election (LE) algorithms (Angluin 1980; Hirschberg & Sinclair 1980; LeLann 1977; Malpani, Welch, & Vaidya 2000; Vasudevan *et al.* 2003) could be employed to arrive at a common word among a community of agents which observed the object to be named: Upon observation, each agent "coins" a random tag (identification number) for the object. Following the observation, the observing agents participate in the leader election algorithm (with not the purpose of electing a leader, but choosing a unique identifier for the object). For example, in a two-dimensional regular network of N agents, the convergence to a unique identifier takes time of $\mathcal{O}(\sqrt{N})$ on average. Convergence time in the NG in two dimensions is of $\mathcal{O}(N)$, significantly longer than the LE algorithm when N is large. Thus, for the purpose of constructing a shared classification or tagging scheme in a sensor network, launching the LE algorithm is the preferable choice. Unlike social networks, sensor networks, although operating autonomously, are intelligently designed (by humans), who can make the choice a priori which algorithm to employ based on their efficiency.

There are possible situations, however, when the NG algorithm, in addition to being interesting for its own merit in studying agreement dynamics on various networks, can also be beneficial from a system-design viewpoint. That can be the case when one does not intend the outcome of the agreement to be easily predictable. The actual process of electing a "leader" or coordinator among sensor nodes may actually be such a scenario. The leader must typically be a trusted node, with possible responsibilities ranging from routing coordination to key distribution (DeCleene *et al.* 2001). The basic LE algorithms are essentially based on finding global extremum (e.g., maximum) through local communications (Angluin 1980; Hirschberg & Sinclair 1980; LeLann 1977). Thus, the elections can be stolen by placing a node in the network with a sufficiently high ID (e.g., the largest number allowed by the number representation scheme of the sensor chips.) Along these lines, a possible application of the NG algorithm is autonomous key creation or selection for encrypted communication in a community of sensor nodes. Instead of having a centralized or hierarchial key management system with domain and area key distributors (DeCleene *et al.* 2001), group of sensor nodes can generate a shared "public" key (becoming visible to group members only).

Sensor networks are both spatial and random. As a large number of sensor nodes are deployed, e.g., from vehicles or aircrafts, they are essentially scattered randomly across large spatially-extended regions. In the corresponding abstract graph, two nodes are connected if they mutually fall within each others transmission range, depending on the emitting power, the attenuation function and the required minimum signal to noise ratio. Random geometric graphs (RGGs), also referred to as spatial Poisson/Boolean graphs, capturing the above scenario, are a common and well established starting point to study the structural properties of sensor network, directly related to coverage, connectivity, and interference. Further, most structural properties of these networks are discussed in the literature in the context of continuum percolation (Meester & Roy 1996; Penrose 2003; Dall & Christemsen 2002).

The common design challenge of these networks is to find the optimal connectivity for the nodes: If the connectivity of the nodes is too low, the coverage is poor and sporadic. If the node connectivity is too high, interference effects will dominate and result in degraded signal reception (Gupta & Kumar 2000; Xue & Kumar 2004; Krishnamachar, Wicker, & Béjar 2001; Krause *et al.* 2004; Branch, Chen, & Szymanski 2005). From a topological viewpoint, these networks are, hence, designed to "live" somewhere above the percolation threshold. This can be achieved by adjusting the density of sensor nodes and controlling the emitting power of the nodes; various power-control schemes have been studied along these lines (Gupta & Kumar 2000; Krause *et al.* 2004; Branch, Chen, & Szymanski 2005). In this paper we consider RGGs above the percolation threshold, as minimal models for the underlying network communication topology. Further, we consider RGGs with an added small density of "random" long-range links. The resulting structure resembles small-world (SW) networks (Watts & Strogatz 1998; Newman 2003), also well studied in the context of artificial (Korniss *et al.* 2003; Lu, Korniss, & Szymanski 2006) and social systems (Watts 1999; Newman 2003).The focus of this work is to study the NG algorithm *on* these well studied graphs.

Naming Games on Random Geometric Networks

As mentioned above in the Introduction, first we consider random geometric graphs in two dimensions (Meester & Roy 1996; Penrose 2003; Dall & Christemsen 2002) as the simplest topological structures capturing the essential features of ad hoc sensor networks. N nodes are uniformly random distributed in an $L \times L$ spatial area. For simplicity we consider identical radio range R for all nodes. Two nodes are connected if they fall within each other's range. An important parameter in the resulting random geometric graph is the average degree $\overline{k}$ (defined as the average number of neighbors per node), $\overline{k} = 2K/N$, where K is the total number of links and N is the number of nodes. In random geometrical networks, there is a critical value of the average degree, $\overline{k}_c$, above which the largest connected component of the network becomes proportional to the total number of nodes (the emergence of the giant component) (Meester & Roy 1996; Penrose 2003; Dall & Christemsen 2002). For two-dimensional RGGs $\overline{k}_c \approx 4.5$ (Dall & Christemsen 2002). There is a simple relationship between the average degree $\overline{k}$, the density of nodes $\rho = N/L^2$, and the radio range R of the nodes (Meester & Roy 1996; Penrose 2003; Dall & Christemsen 2002), $\overline{k} = \rho \pi R^2$, which can be used to control the connectivity of the network.

The Naming Game

We consider the Naming Game on random geometrical graphs. In the original context of the NG, agents try to reach agreement in finding a unique "word" for an object observed by them. In one of the above proposed potential applications, agents try to generate a shared public key for encrypted communication. For simplicity, we will use the term "word" for the latter as well when describing the algorithm.

We modify the communication rules to make them applicable for sensor networks. Instead of pairwise communications, nodes will initiate *broadcast* (to all neighbors) in a continuous-time asynchronous fashion. In this paper we consider the initial condition when the "vocabulary" of each node is empty. At every elementary time step, a node is chosen randomly out of N nodes (mimicking Poisson asynchrony for large N). This node (the "speaker") will broadcast a word from her list of "synonyms"; if her list of synonyms is empty, the speaker randomly invents a word; if she already has several synonyms, it randomly chooses one. Her neighbors (the "listeners") compare their vocabularies with the word transmitted by the speaker. If a listener has this word, she considers the communication a success, and she deletes all other words, collapsing her list of synonyms to this one word. If a listener does not have the word transmitted by the speaker, she adds it to her list of synonyms without any deletion. If at least one listener had the word transmitted, the speaker considers it (at least a partial) success, and (somewhat optimistically) collapses her list of synonyms to this one word. At every step, the "success" rate S is defined as the fraction of listeners who were successful (i.e., those that had the word transmitted by the speaker).

From the above it is clear that the listeners have to report the outcome of the "word matching" to the speaker, hence the elementary algorithmic step requires $(\overline{k}+1)$ broadcasts. In this paper time t is given in units of $(\overline{k}+1)$ broadcasts per node (during which, on average, N word matching have been attempted). The main difference between the above algorithm and the one in the works by Baronchelli et al. is the *broadcast* (instead of pairwise communications) and the underlying network (RGG in this paper) to capture the essential features of the NG in sensor networks.

Other initial conditions, e.g., applicable to public key generation for a community of networked agents, can also be employed; instead of starting from "scratch" (empty list of words for each node), each agent has a pre-generated (possibly long) list of words. The different initial conditions only have an effect on the early-time behavior of the system. For this scenario, results will be presented elsewhere.

When starting from empty vocabularies, agents invent words randomly. After time of $\mathcal{O}(1)$ [on average of order $(\overline{k}+1)$ broadcast per node], $\mathcal{O}(N/(\overline{k}+1))$ different words have been created. Following the early-time increase of the number of different words $N_d(t)$, through local broadcasts, agents slowly reconcile their "differences", and eventually will all share the same word. First, a large number of small spatial clusters sharing the same word develop. By virtue of the random *diffusive* motion of the interfaces separating the clusters, more and more of the small clusters are being eliminated, giving rise to the emergence of larger clusters, eventually leading to one cluster in which all nodes are sharing the same word. As suggested by Baronchelli et al. (2006a), this late-time process is analogous to coarsening, a well-known phenomenon from the theory of domain and phase ordering in physical and chemical systems (Bray 1994). Figure 1 shows snapshots of vocabularies of the nodes at different times. For later times, group of nodes which already share the same word, slowly coarsen, until eventually only one domain prevails. This behavior is also captured by Fig. 2(b), tracing the number of different words as a function of time $N_d(t)$, eventually reaching global agreement, $N_d = 1$.

Basic Scaling Considerations and the Analogy with Coarsening

Before turning to the detailed discussion of our simulation results, we first sketch the framework of coarsening theory (Bray 1994), applicable to the observed late-time dynamics of the NG on regular d-dimensional lattices (Baronchelli *et al.* 2006a). Coarsening has also been observed in other models relevant to opinion formation and social dynamics (Ben-Naim, Frachebourg, & Krapivsky 1996; Krapivsky & Redner 2003). Unlike other minimalist (typically two-state) models often employed to study opinion formation (Durlauf 1999), such as the one studied by Sznajd-Weron & Sznajd (2000), the Voter model (Liggett 1985; Ben-Naim, Frachebourg, & Krapivsky 1996), or the majority rule model (Krapivsky & Redner 2003), in the NG, each agent can be in an *unlimited* number of discrete states (corresponding to the chosen word). Further, at any instant before reaching global consensus, an agent can have
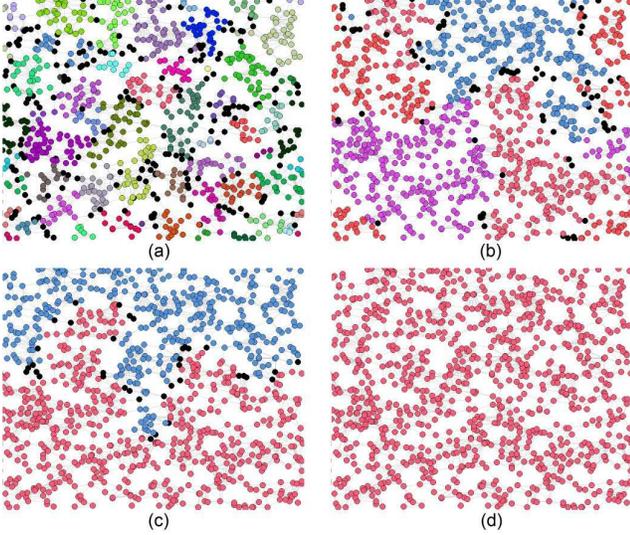
Figure 1: (Color figure) Snapshots of the time evolution of the contents of the agents' word lists during the process of reaching global agreement on RGG for $N = 1,000$ nodes at time (a) $t = 1$; (b) $t = 43$; (c) $t = 169$; (d) $t = 291$. The average degree is $\overline{k} \approx 12$. Initially, the word lists are empty for all agents. Time is measured in units of $(\overline{k} + 1)$ broadcasts per node. Different colors correspond to different words, with black indicating nodes with multiple words. After the early-time increase in the number of different words in the systems, small spatial clusters sharing the same word quickly form, then subsequently "coarsen" until eventually only one global cluster prevails.
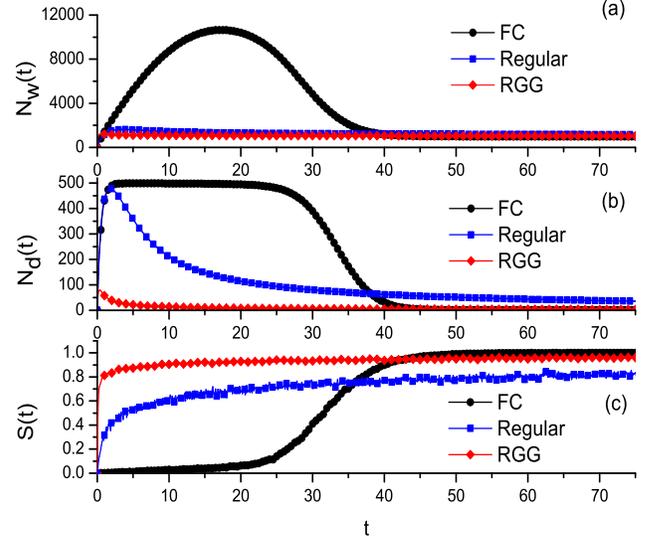

Figure 2: Time evolution of the relevant observables in the Naming Game in the fully-connected (FC), two-dimensional regular (with four nearest neighbors), and random geometric networks (RGG) for $N=1024$, averaged over $1,000$ independent network realizations; (a) the total number of words in the system $N_w(t)$; (b) the number of different words $N_d(t)$; (c) the average success rate $S(t)$. The average degree of the underlying RGG is $\overline{k} \approx 12$. Data for the FC and $2d$ regular networks are reproduced by our simulations, following Refs. (Baronchelli *et al.* 2006b; 2006a), for comparison.

different possible words for the object. Because of the potentially unlimited number of discrete states the agents can "reside in", we believe [this belief is also supported by preliminary data (Wykes *et al.* 2006)] that the full coarsening characteristics exhibited by the NG will match that of the infinite-state Potts model (Sire & Majumdar 1995b; 1995a; Masser & ben Avraham 2000). After the initial fast local ordering, essentially each cluster corresponds to a *different* state (word). Hence, through the coarsening process, domain walls (interfaces) separating different clusters will *coalesce* (as opposed to domain-wall *annihilation* in two-state models).

While RGG is a random structure, it is embedded in two dimensions, and we also attempt to employ elementary scaling arguments from coarsening theory. According to Ref. (Baronchelli *et al.* 2006a), on regular d-dimensional lattices, the typical size of domains (each with already agreed upon one word) is governed by a single length scale $\xi(t) \sim t^\gamma$ with $\gamma=1/2$, analogous to that of domain formation in systems with a non-conserved order parameter (Bray 1994). Thus, in d dimensions, the total number of *different* words N_d at time t scales as the typical number of domains

$$N_d(t) \sim \frac{N}{\xi^d(t)} \sim \frac{N}{t^{d\gamma}} . \qquad (1)$$

Further, the total number of words N_w (N_w/N being the average memory load per agent), at this late coarsening stage, can be written as the number of nodes N plus the number of nodes with more than one (on average, between one and two) words, separating the different domains. It is of order of typical number of domains times the typical length of the interface of one domain, yielding

$$N_w(t) - N \sim \frac{N}{\xi^d(t)} \xi^{d-1}(t) \sim \frac{N}{\xi(t)} \sim \frac{N}{t^\gamma} . \qquad (2)$$

Similarly, the "failure rate" for word matching, $1-S(t)$, (where $S(t)$ is the success rate) scales as the fraction of nodes at the interfaces separating domains with different words

$$1 - S(t) \sim \frac{1}{\xi(t)} \sim \frac{1}{t^\gamma} . \qquad (3)$$

The main feature of the above power-law decays (up to some system-size dependent cut-offs) is that the number of different words N_d, the total number of words N_w, and the success rate $S(t)$ only depend on t through the characteristic length scale $\xi(t)$. Further, for the typical time t_c to reach global agreement or consensus, one has $\xi^d(t_c) \sim N$, i.e.,
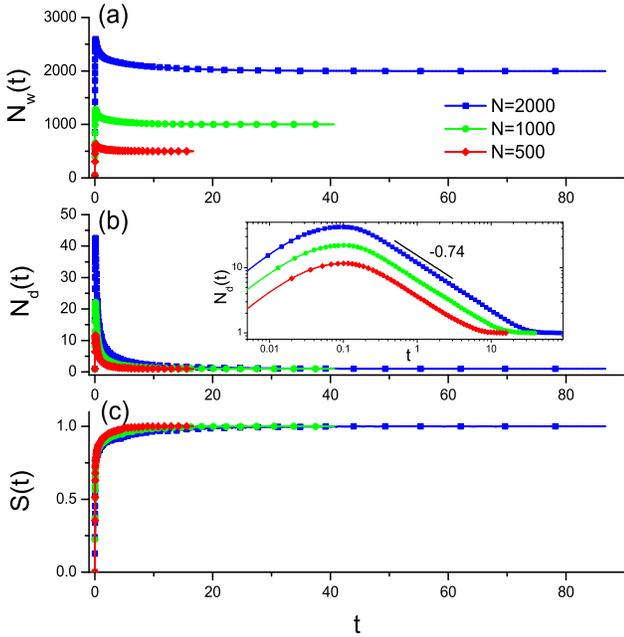
$$t_c \sim N^{1/(d\gamma)} . \qquad (4)$$

Figure 3: Time evolution of the relevant observables in the Naming Game in random geometric networks (RGG) for three system sizes, averaged over $1,000$ independent network realizations; (a) the total number of words in the system $N_w(t)$; (b) the number of different words $N_d(t)$; (c) the average success rate $S(t)$. The average degree of the underlying RGGs is $\overline{k}\approx 50$. The inset of (b) shows $N_d(t)$ on log-log scales, displaying the late-stage coarsening and the corresponding power-law decay, approximately $N_d(t) \sim t^{-0.74}$.

Simulation Results

Relevant quantities measured in the simulations are the total number of words in the system $N_w(t)$ (corresponding to the total memory used by the agents for word allocation at time t), the number of different words $N_d(t)$, and the average rate of success $S(t)$ of the word-matching attempts. Figure 2 displays the time evolution of these three quantities for the RGG, compared to the fully connected (FC) and to the $2d$ regular networks. Here, for the comparison, we reproduced the corresponding data of Refs. (Baronchelli *et al.* 2006b; 2006a). The behavior of the NG on RGG is qualitatively very similar to that of the NG on $2d$ regular graphs. After time of $\mathcal{O}(1)$, $\mathcal{O}(N/(\overline{k}+1))$ different words have been invented [Fig. 2(b) and 3(b)]. $N_w(t)$ also reaches its maximum in time of $\mathcal{O}(1)$ [Fig. 2(a) and 3(a)]].

Focusing on the late-time behavior of the systems, plotting $N_d(t)/N$, $N_w(t)/N-1$, and $1-S(t)$ vs t on log-log scales, confirms the power-law decays associated with the underlying coarsening dynamics, predicted by Eqs. (1), (2), and (3), respectively.

Further, in the scaling regime [Figs. 4(a) and (c)] we find two consistent estimates for the exponent of the typi-
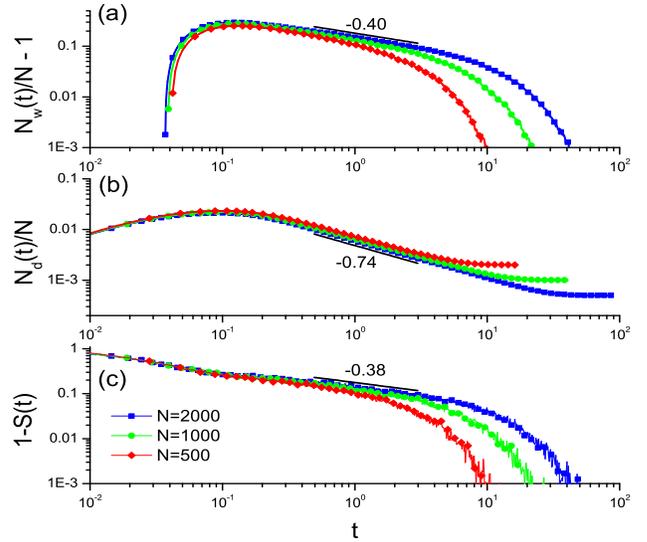

Figure 4: The scaled version of the same data shown in Fig. 3 on log-log scales; (a) the total number of words in the system $N_w(t)/N-1$; (b) the number of different words $N_d(t)/N$; (c) the average success rate $1-S(t)$. The straight line segments correspond to the best-fit power-law decays $N_w(t)/N-1 \sim t^{-0.40}$, $N_d(t)/N \sim t^{-0.74}$, $1-S(t) \sim t^{-0.38}$ for (a), (b), and (c), respectively.

cal length scale [see Eqs. (2) and (3)]: $\gamma \approx 0.40$ and $\gamma \approx 0.38$, respectively. The number of different words, according to Eq. (1), in turn, should scale as $N_d(t)/N \sim 1/t^{2\gamma}$, close to our measured exponent $2\gamma \approx 0.74$ [Figs. 3(b) and 4(b)]. The time to global agreement scales as $t_c \sim N^{1.07}$, shown in Fig. 5, somewhat deviating from the one predicted by Eq. (4) with the exponent $1/(2\gamma)$. This deviation is possibly due to the presence of multiple length and time scales in the late stage of the agreement dynamics and finite-size effects.

In addition to the average convergence time t_c, we also measured the standard deviation Δt_c [Fig. 5]. These results also indicate some weakness of the NG from a system-design viewpoint: up to the system sizes we could simulate, the standard deviation, within error, scales in the same fashion with the number of nodes as the average itself, $\Delta t_c \sim N^{1.07}$ [Fig. 5]. The lack of self-averaging for large systems (strong dependence on the individual runs) of the NG is inherently related to the coarsening dynamics, having a single interface wondering at the latest stage (and not to the underlying random structure). Suppressing large average convergence times and the corresponding large standard deviations will be addressed in the next section.

Naming Games in Small-World-Connected Random Geometric Networks

In light of recent results on NG on one-dimensional SW networks (Dall'Asta *et al.* 2006), we now consider accelerating the agreement process by adding random long-range communication links between a small fraction of nodes of the RGG. Such networks have long been known to speed up the spread of local information to global scales (Watts & Stro-
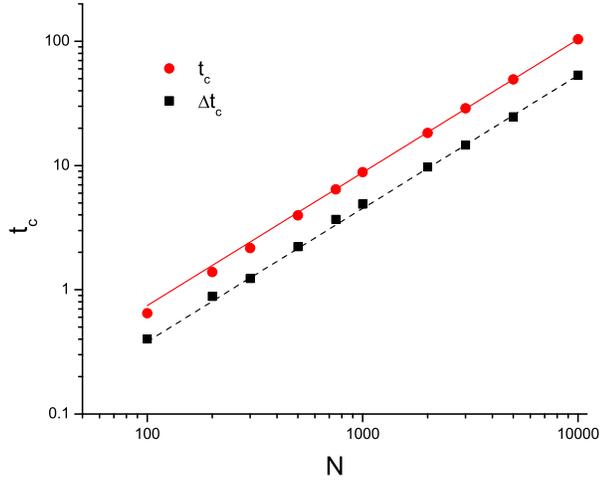
Figure 5: Average and the standard deviation of the convergence time t_c until global agreement is reached, as a function of the number of nodes on log-log scales, averaged over $1,000$ independent realizations of the RGG. The average degree of the underlying RGGs is $\overline{k} \approx 50$. The straight lines correspond to the best-fit power-laws with exponent 1.07 for both the average (solid line) and for the standard deviation (dashed line).

gatz 1998; Watts 1999; Newman 2003; Kozma, Hastings, & Korniss 2005), with applications ranging from synchronization problems in distributed computing (Korniss *et al.* 2003) to alarm-detection schemes in wireless sensor networks (Lu, Korniss, & Szymanski 2006). For sensor networks, this can be implemented either by *adding* a small fraction of sensors equipped with long-range unidirectional antennas ("physical" long-range connections) or by establishing designated multi-hop transmission patterns ("logical" long-range connections) between certain nodes (Helmy 2003).

We construct the small-world-like RGG (SW RGG) as follows. We start with the original RGG (embedded in d dimensions, where $d=2$ in this paper). Then we *add* "long-range" links (or "shortcuts") between randomly chosen nodes in such a way that the total number of long-range links per node (the density of random links) is p. This SW construction slightly differs from the original Watts-Strogatz one (Watts & Strogatz 1998) [also used by Dall'Asta et al. (2006)], where random links are introduced by "rewiring" some of the original connections. The resulting network, however, has the same universal properties in the small-p, large-N limit (Newman & Watts 1999), which is the center of our interest. Further, it is also motivated by actual implementations in sensor networks.

Basic Scaling Considerations

Before presenting simulation results, using scaling arguments, one can obtain an order of magnitude estimate for the crossover time $t_\times$ present in the SW RGG and for the time to reach global agreement t_c (Dall'Asta *et al.* 2006). In SW networks, embedded in d dimensions, the typical distance between two nodes with an added long-range link emanating

from them scales as $l_{SW} \sim p^{-1/d}$. Starting from empty initial word lists word, for early times (following the creation of $\mathcal{O}(N/(\overline{k}+1))$ different words in the system), the system will exhibit coarsening, until the typical size of the growing domains $\xi(t) \sim t^\gamma$ becomes comparable to l_{SW}. After that time, the agreement process is governed by the presence of random long-range connections, yielding mean-field-like behavior. Hence the crossover from d-dimensional coarsening to mean-field-like dynamics occurs when $t^\gamma \sim p^{-1/d}$, yielding

$$t_\times \sim p^{-1/(d\gamma)} . \tag{5}$$

In a system of N agents, the above crossover is only displayed if the convergence time of the original system with no random links would exceed the above crossover time $N^{1/d\gamma} \gg p^{-1/d\gamma}$, which is equivalent to the condition for the onset of the SW effect $N \gg p^{-1}$ (Newman & Watts 1999; Dall'Asta *et al.* 2006). Following the above system-size independent crossover time, the agreement dynamics is of mean-field like, and one can expect to observe a scaling behavior close to that of FC networks (Baronchelli *et al.* 2006b). In particular, the time to reach global agreement is expected to scale as

$$t_c \sim N^{1/2} , \tag{6}$$

a significant [and anticipated (Dall'Asta *et al.* 2006)] reduction compared to that of the "pure" RGG with no long-range links where $t_c \sim N^{1.07}$.

Simulation Results

Simulating the NG on SW RGGs confirms the above scaling scenario. Following the very early-time development of $\mathcal{O}(N/(\overline{k}+1))$ different words, the system of SW-networked agents, exhibits slow coarsening, with only small corrections to the behavior of the pure RGG. In fact, this early-time coarsening on SW RGGs is slightly slower compared to pure RGGs due to the effective pinning of interfaces near the shortcuts (Dall'Asta *et al.* 2006; Boyer & Miramontes 2003; Castellano, Vilone, & Vespignani 2003; Castellano *et al.* 2005). In the NG on SW networks, however, the agreement process only slows down (Dall'Asta *et al.* 2006), but is not halted by "frozen" (metastable) disordered configurations (Boyer & Miramontes 2003; Castellano *et al.* 2005). After a p-dependent crossover time [Eq. (5)], (when the typical size of the growing clusters becomes comparable to the SW length scale), an exponential convergence begins to govern the agreement process. This final-stage fast approach toward consensus sets in earlier for increasing values of the density of shortcuts p, yielding a significantly reduced convergence time compared to that of the NG on the "pure" RGG. Plotting the convergence time vs the density of long-range links, as shown in Fig. 6, suggests that (for sufficiently large but *fixed* N) the convergence time approaches an asymptotic power-law $t_c \sim p^{-s}$ with $s \approx 0.79$.

For fixed p and increasing N, however, the convergence time still increases with N [Fig. 7], $t_c \sim N^{0.31}$, closer to the anticipated mean-field-like behavior [Eq. (6)] (Dall'Asta *et al.* 2006).
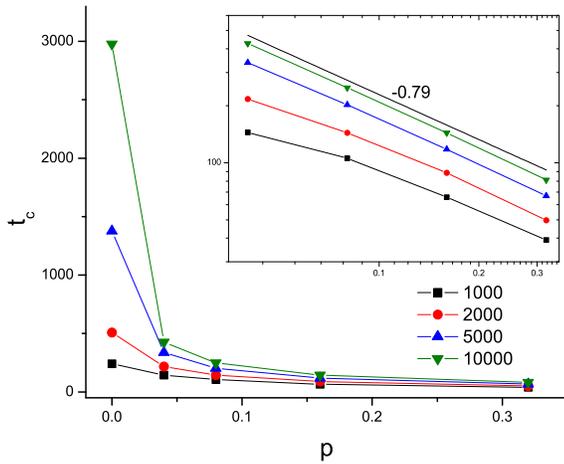
Figure 6: Average convergence time t_c for SW RGGs, as a function of the density of shortcuts for various system sizes, averaged over $1,000$ independent realizations of the network. The average degree of the underlying RGGs is $\overline{k} \approx 12$. The inset shows the same data on log-log scales. The straight lines corresponds to an estimate of the associated (asymptotic) power-law.
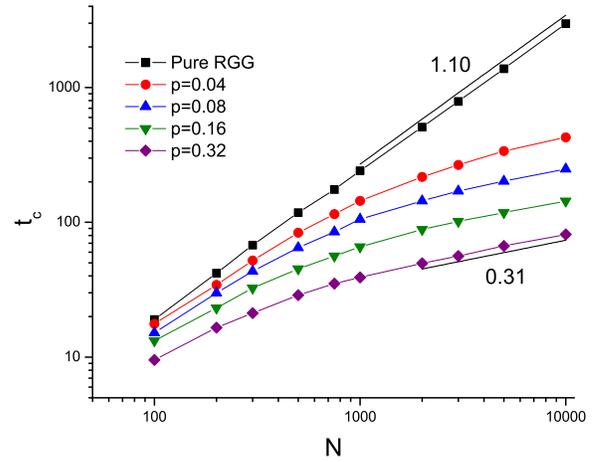

Figure 7: Average convergence time t_c for SW RGGs, as a function of the number of nodes on log-log scales for various density of long-range links p, averaged over $1,000$ independent realizations of network. The average degree of the underlying RGGs is $\overline{k} \approx 12$. The straight line segments correspond to the best-fit (asymptotic) power-laws with exponents 1.10 and 0.31, for the "pure" RGG (p=0) and for the SW RGG (p>0) cases, respectively.

Summary and Outlook

In this paper, we have explored the Naming Games on Random Geometric Graphs and SW-connected RGGs. While the underlying RGG communication topology is motivated by large-scale sensor networks, the NG on (SW) RGGs captures fundamental features of agreement dynamics of spatially-embedded networked agent-based systems. We have found that, qualitatively similar to two-dimensional regular networks (Baronchelli *et al.* 2006a), the NG on RGG can be reasonably well described by the physical theory of coarsening. In particular, local clusters of nodes sharing the same word quickly form, followed by slow coarsening of these clusters in the late stage of the dynamics. Our simulation results indicate that the average time to reach global agreement is of $\mathcal{O}(N^{1.07})$ (for *fixed* average degree). By adding a small density of shortcuts on "top" of the RGG, resulting in a SW-like network, the convergence time is strongly reduced and becomes of $\mathcal{O}(N^{0.31})$, closer to that of the FC network (Dall'Asta *et al.* 2006).

In future work we will investigate the NG on more realistic communication topologies, motivated by and relevant to wireless sensor networks, in particular, random spatial networks with heterogeneous range distribution, and also networks with dynamically changing connectivity.

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