Energetic Emus at the End of the Semester

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Original Goals

I sought to approach the problem of Turing Machine productivity as a function of the number of defined transitions of the machine, rather than the traditional criteria: number of states.

I had hoped to find the productivities of machine perhaps up to even 10 transitions, and have developed some ways for classifying machines by their observed behavior.

I also desired to explore the nature of machine behavior and examine what aspects of a machine had more impact on that machine's productivity: states or transitions?

Energetic Emus

The Busy Beaver problem attempts to solve the productivity function for machines using the number of states as the measure of machine. The current Rennselaer attack on the Busy Beaver problem is strong, and offers much insight into productivity, and efficient ways to explore it. Their approach has been to examine Turing Machines of the quadruple formation, implicitly halting, binary Turing Machines.

So as not to completely reinvent the wheel, I chose the same representation of the Turing Machine (quadruple formulation, implicit halt...) and define the *Energetic Emu* function $EE(n)$. A Machine is said to be an *Energetic Emu* if and only if the machine has n transitions defined, and no other machine with n states, when run with a blank input tape, halts with more 1's having been printed to the tape.

Implemented Approach

My approach to implementation has been to first enumerate Turing Machines strategically, and then simulate them and observe their behavior. Admittedly, this approach fails for larger numbers of transitions, but for the small values of n with which I was working, this method suffices.

Unfortunately, while this method works well to determine the machine's productivity, it does little to explain the underlying behavior of the machine.

Results

As of the beginning of May 2004, I've found $EE(n)$ values for $n < 5$. Results up to three may, however, be considered somewhat trivial. Due to some bugs in the Turing Machine Generation algorithm, I had to restart some exhaustive runs in order to guarantee that I had not missed some significant number of important machines.

It also is now clear that it is possible to have machines in which it is impossible for the addition of a transition to increase productivity. That is, $\exists m, n Productivity(EE(n)) = Productivity(EE(m))$. Currently this is only seen at $EE(1)$, and $EE(2)$, but the possibility raises some interesting issues.

Future Direction

Future Directions of this project must include the expansion to higher values of n, and the development of filters to detect non-halting machines, as well as other provably non-optimal machines.

The relationship between transitions and productivity as compared to that between states and productivity should also be explored. With the augmentation of each additional state, many possible transitions are considered, and some added. Growing by transitions however, each new transitions introduces at most one transition and one state. I would like to see this area explored as well.

I would also like to see (though this is probably a hard problem) whether there is any machine which is both an *Energetic Emu* and a *Busy Beaver* , aside from trivially small machines (though I'm not sure where to place the 'trivially small' cutoff yet).