16. Friction

Mechanics of Manipulation

Matt Mason
matt.mason@cs.cmu.edu

http://www.cs.cmu.edu/~mason

Carnegie Mellon

Outline.

Coulomb's Law.

Friction angle, friction cone.

Moment labeling of friction cone.

Static equilibrium problems.

How do you move things around?

Kinematics, kinematic constraint.

Force.

- Force of constraint;
- Gravity;
- Friction;

Momentum.

Coulomb's experiments

An experiment:

Clean surfaces, but not too clean. Dry. Unlubricated.

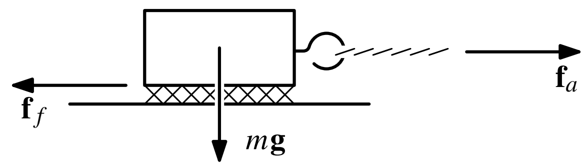
Pull on string with force $\mathbf{f}_a$, ramping up from 0.

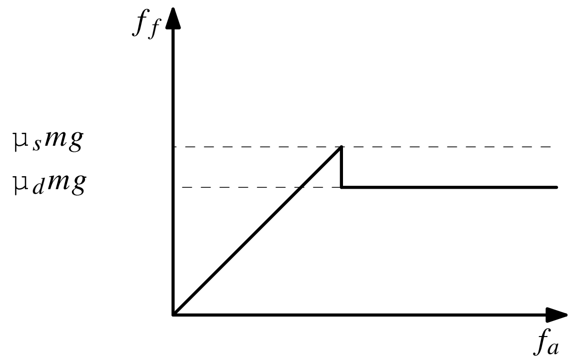
Friction force $\mathbf{f}_f$ will balance $\mathbf{f}_a$, up to a point.

Max $\mathbf{f}_f$ when not moving: $\mu_s mg$.

Max $\mathbf{f}_f$ when moving: $\mu_d mg$.

From now on we will assume $\mu_s = \mu_d = \mu$.





Coulomb's observations

Coulomb conducted hundreds of experiments, and over a *broad range* of conditions observed:

Frictional force is *approximately* independent of contact area.

Frictional force is *approximately* independent of velocity magnitude.

Coefficient of friction depends on pairs of materials.

Materials	μ
metal on metal	0.15–0.6
rubber on concrete	0.6–0.9
plastic wrap on lettuce	∞
Leonardo's number	0.25

Think when using Coulomb's law!

It holds over a *broad range*, but not nearly everywhere.

It is approximate.

Coefficients of friction tables are terrible.

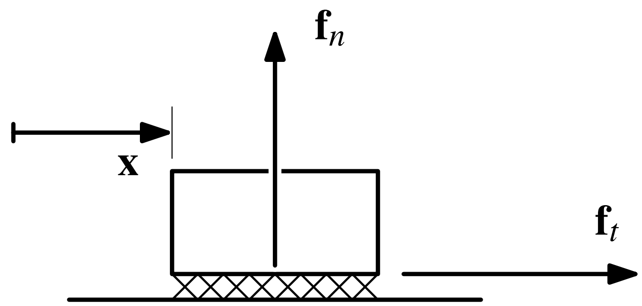
How can you use something so unreliable?

But, how can you *not* use it?

Contact modes

We can write Coulomb's law:

$\dot{x}$	$\ddot{x}$		
< 0		$f_t = \mu f_n$	left sliding
> 0		$f_t = -\mu f_n$	right sliding
$= 0$	< 0	$f_t = \mu f_n$	left sliding
$= 0$	> 0	$f_t = -\mu f_n$	right sliding
$= 0$	$= 0$	$\lvert f_t \rvert \leq \lvert \mu f_n \rvert$	rest



Friction angle

Block at rest on plane with angle α:

$$f_n = mg\cos\alpha$$

$$f_t = mg\sin\alpha$$

At rest $|f_t| \leq \mu f_n$. Maximum α:

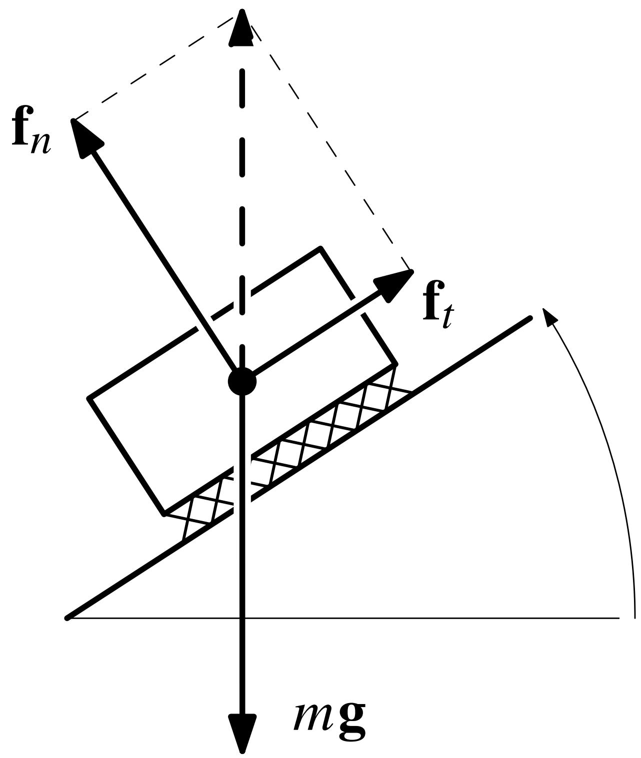
$$f_t = \mu f_n$$

Substituting,

$$mg\sin\alpha = \mu mg\cos\alpha$$

$$\alpha = \tan^{-1}\mu$$

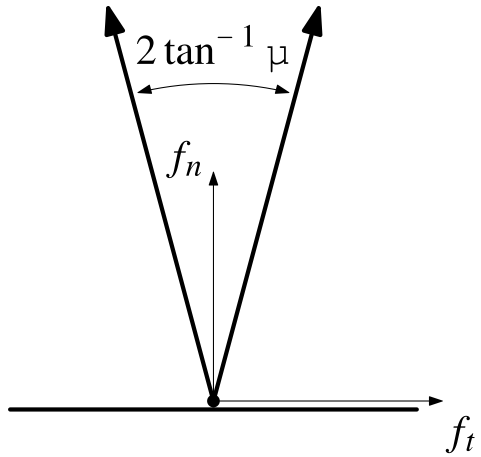
Sometimes called the *friction angle* or the *angle of repose*.



Friction cone

Define the **friction cone** to be the set of all wrenches satisfying Coulomb's law for an object at rest, i.e. all the wrenches satisfying:

$$|f_t| \leq \mu |f_n|$$



This set of forces describes a cone in wrench space.
Each wrench is applied at the contact point.
The dihedral angle is $2\tan^{-1}\mu$.
Then we can state Coulomb's law:

For left sliding	$\mathbf{f}_n + \mathbf{f}_t \in$ right edge of friction cone
For right sliding	$\mathbf{f}_n + \mathbf{f}_t \in$ left edge of friction cone
For rest	$\mathbf{f}_n + \mathbf{f}_t \in$ friction cone

Pipe clamp design problem

Why does pipe clamp work?

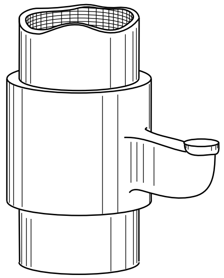
Let diameter be 2 cm.

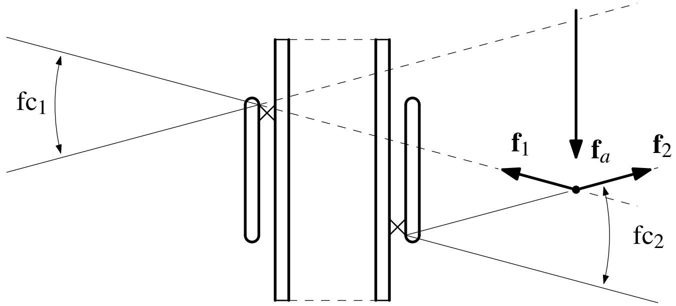
Let length be 2 cm.

Assume μ of 0.25.

Find min moment arm.

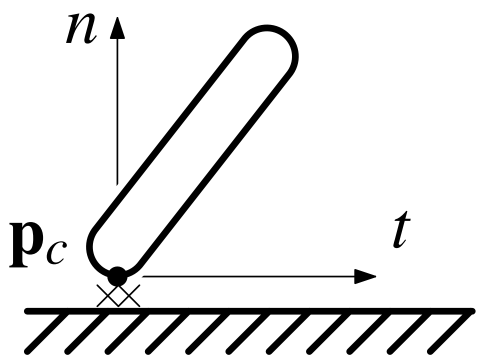
Extend to woodpecker toy?





Sliding rod

If we consider normal velocities. and accelerations:

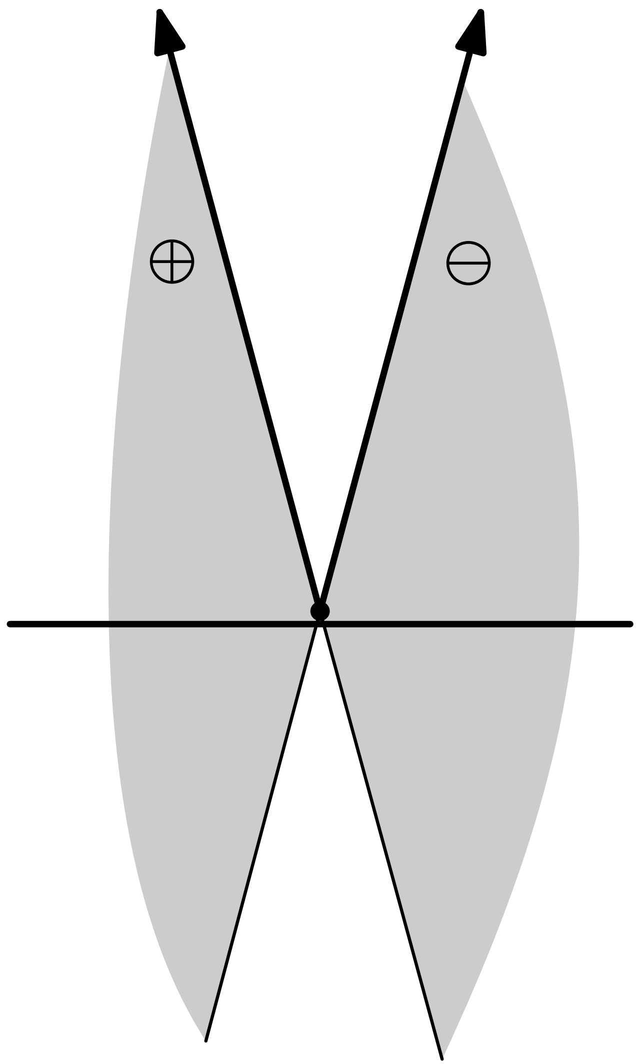


$\dot{p}_{cn}$	$\ddot{p}_{cn}$	$\dot{p}_{ct}$	$\ddot{p}_{ct}$		
< 0					impact
> 0					separation
$= 0$	< 0				impact
$= 0$	> 0				separation
$= 0$	$= 0$	< 0		$f_t = \mu f_n$	left sliding
$= 0$	$= 0$	> 0		$f_t = -\mu f_n$	right sliding
$= 0$	$= 0$	$= 0$	< 0	$f_t = \mu f_n$	left sliding
$= 0$	$= 0$	$= 0$	> 0	$f_t = -\mu f_n$	right sliding
$= 0$	$= 0$	$= 0$	$= 0$	$\lvert f_t \rvert \leq \lvert \mu f_n \rvert$	fixed

We're assuming pointy contact. Rolling is more complicated.

Moment labeling of friction cone

Friction cone is positive linear span of left edge unit vector and right edge unit vector.



Moment labeling

Examples

Block on table.

Wedged plank and piranha.

Triangle and three fingers.

What exactly does any of this prove?

Force closure versus stability.

Force closure versus first order form closure.