

# 16. Friction

## *Mechanics of Manipulation*

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# Outline.

Coulomb's Law.

Friction angle, friction cone.

Moment labeling of friction cone.

Static equilibrium problems.

# How do you move things around?

Kinematics, kinematic constraint.

Force.

- Force of constraint;

- Gravity;

- Friction;

Momentum.

# Coulomb's experiments

An experiment:

Clean surfaces, but not too clean.  
Dry. Unlubricated.

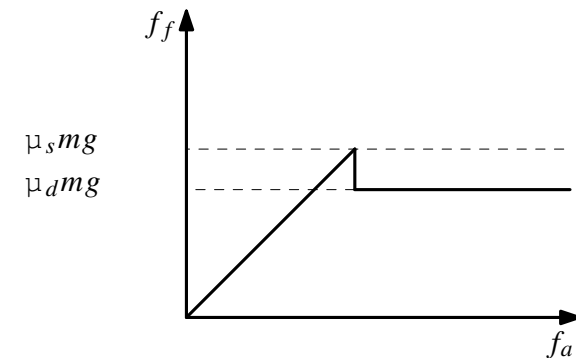
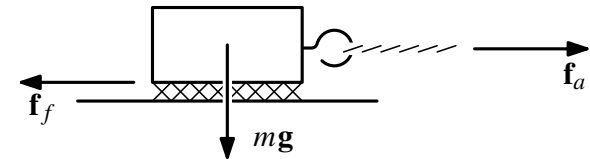
Pull on string with force  $\mathbf{f}_a$ , ramping up from 0.

Friction force  $\mathbf{f}_f$  will balance  $\mathbf{f}_a$ , up to a point.

Max  $\mathbf{f}_f$  when not moving:  $\mu_s mg$ .

Max  $\mathbf{f}_f$  when moving:  $\mu_d mg$ .

From now on we will assume  $\mu_s = \mu_d = \mu$ .



# Coulomb's observations

Coulomb conducted hundreds of experiments, and over a *broad range* of conditions observed:

Frictional force is *approximately* independent of contact area.

Frictional force is *approximately* independent of velocity magnitude.

Coefficient of friction depends on pairs of materials.

<b>Materials</b>	$\mu$
metal on metal	0.15–0.6
rubber on concrete	0.6–0.9
plastic wrap on lettuce	$\infty$
Leonardo's number	0.25

# Think when using Coulomb's law!

It holds over a *broad range*, but not nearly everywhere.

It is approximate.

Coefficients of friction tables are terrible.

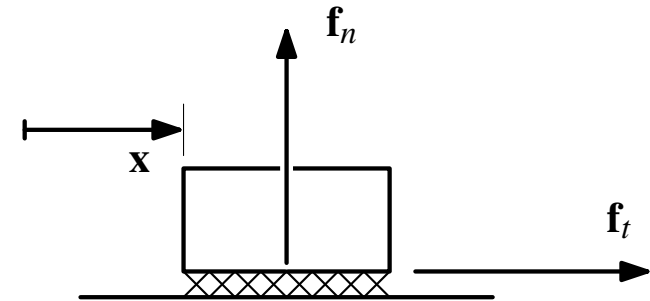
How can you use something so unreliable?

But, how can you *not* use it?

# Contact modes

We can write Coulomb's law:

$\dot{x}$	$\ddot{x}$		
$< 0$		$f_t = \mu f_n$	left sliding
$> 0$		$f_t = -\mu f_n$	right sliding
$= 0$	$< 0$	$f_t = \mu f_n$	left sliding
$= 0$	$> 0$	$f_t = -\mu f_n$	right sliding
$= 0$	$= 0$	$ f_t  \leq  \mu f_n $	rest





# Friction angle

Block at rest on plane with angle  $\alpha$ :

$$f_n = mg \cos \alpha$$

$$f_t = mg \sin \alpha$$

At rest  $|f_t| \leq \mu f_n$ . Maximum  $\alpha$ :

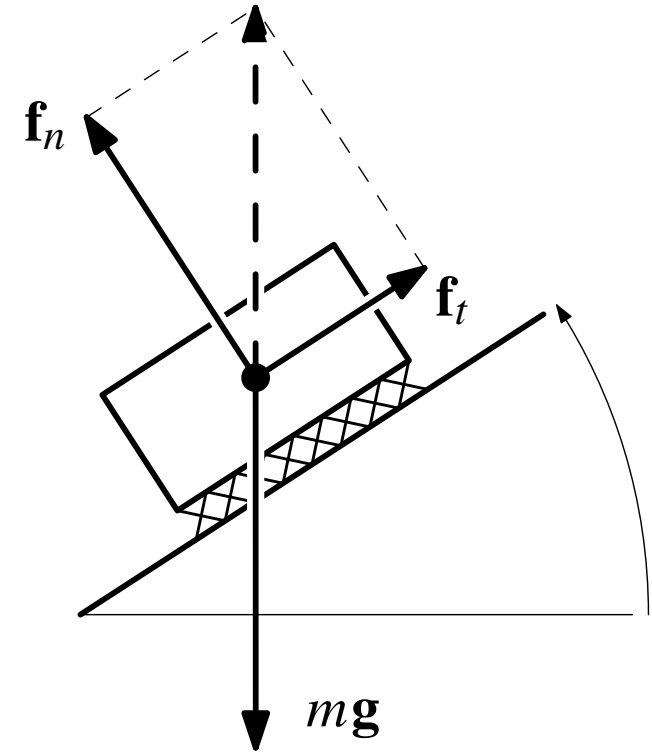
$$f_t = \mu f_n$$

Substituting,

$$mg \sin \alpha = \mu mg \cos \alpha$$

$$\alpha = \tan^{-1} \mu$$

Sometimes called the *friction angle* or the *angle of repose*.



# Friction cone

Define the **friction cone** to be the set of all wrenches satisfying Coulomb's law for an object at rest, i.e. all the wrenches satisfying:

$$|f_t| \leq \mu |f_n|$$

This set of forces describes a cone in wrench space.

Each wrench is applied at the contact point.

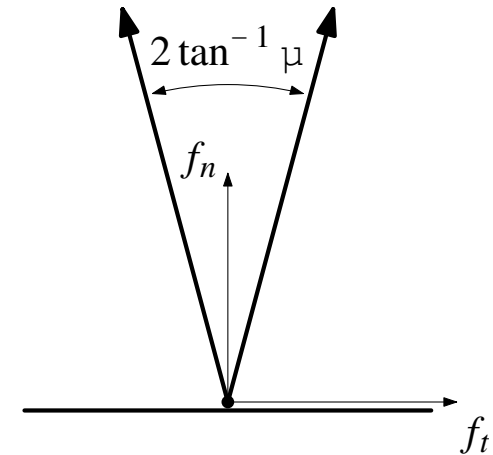
The dihedral angle is  $2 \tan^{-1} \mu$ .

Then we can state Coulomb's law:

For left sliding  $\mathbf{f}_n + \mathbf{f}_t \in$  right edge of friction cone

For right sliding  $\mathbf{f}_n + \mathbf{f}_t \in$  left edge of friction cone

For rest  $\mathbf{f}_n + \mathbf{f}_t \in$  friction cone



# Pipe clamp design problem

Why does pipe clamp work?

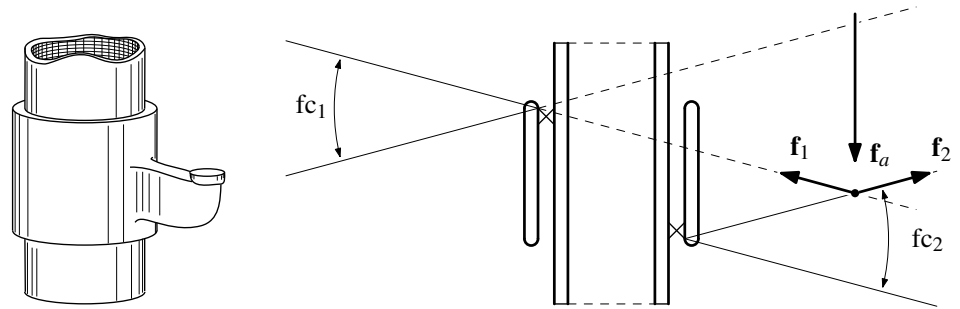
Let diameter be 2 cm.

Let length be 2 cm.

Assume  $\mu$  of 0.25.

Find min moment arm.

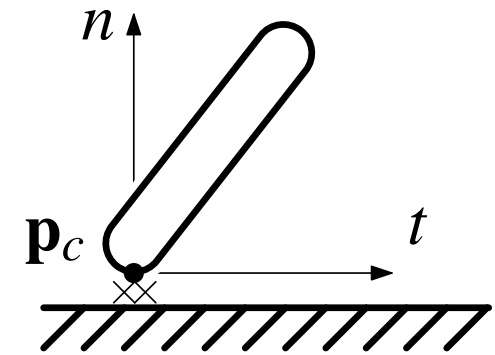
Extend to woodpecker toy?



# Sliding rod

If we consider normal velocities. and accelerations:

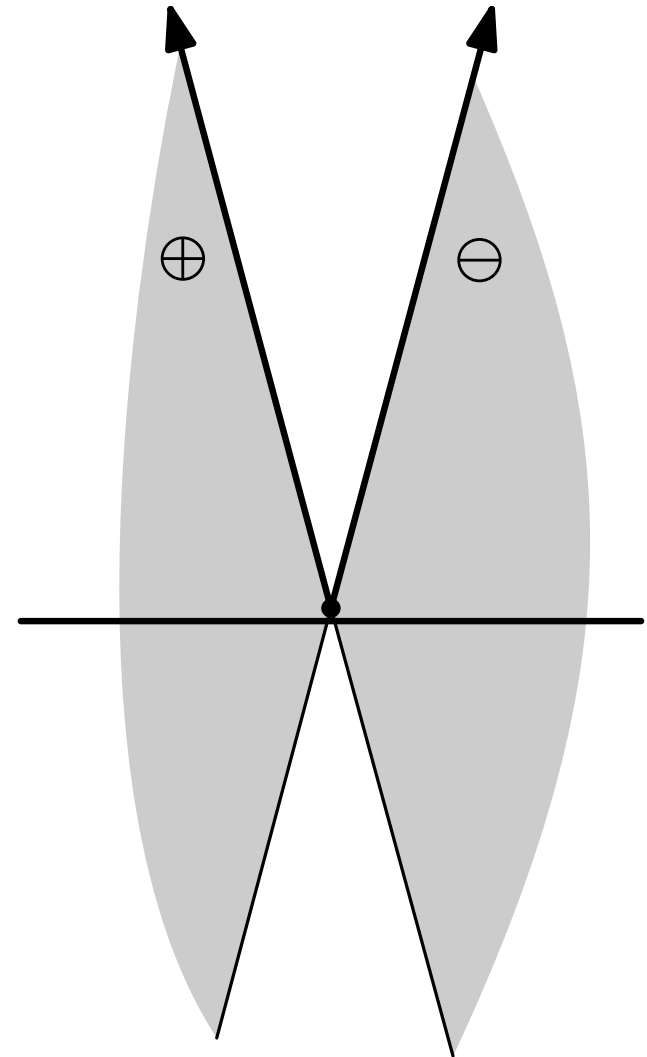
$\dot{p}_{cn}$	$\ddot{p}_{cn}$	$\dot{p}_{ct}$	$\ddot{p}_{ct}$		
$< 0$					impact
$> 0$					separation
$= 0$	$< 0$				impact
$= 0$	$> 0$				separation
$= 0$	$= 0$	$< 0$		$f_t = \mu f_n$	left sliding
$= 0$	$= 0$	$> 0$		$f_t = -\mu f_n$	right sliding
$= 0$	$= 0$	$= 0$	$< 0$	$f_t = \mu f_n$	left sliding
$= 0$	$= 0$	$= 0$	$> 0$	$f_t = -\mu f_n$	right sliding
$= 0$	$= 0$	$= 0$	$= 0$	$ f_t  \leq  \mu f_n $	fixed



We're assuming pointy contact. Rolling is more complicated.

# Moment labeling of friction cone

Friction cone is positive linear span of left edge unit vector and right edge unit vector.



# Examples

Block on table.

Wedged plank and piranha.

Triangle and three fingers.

# What exactly does any of this prove?

Force closure versus stability.

Force closure versus first order form closure.

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