20. Quasistatic manipulation

Mechanics of Manipulation

Matt Mason
matt.mason@cs.cmu.edu

http://www.cs.cmu.edu/~mason

Carnegie Mellon

Outline.

Quasistatic manipulation.

Form closure and force closure.

Grasp and fixture planning.

Pushing.

Static and Quasistatic manipulation

Some tasks involve force balance with no motion.

Fixture planning.

Some tasks involve motion but with negligible inertial forces.

Grasp planning.

Pushing.

A cool application: parts orienting.

Grasping and fixturing

Fixture: immobilize something.

Grasp: immobilize something relative to the hand.

Form and force closure

Form closure: the object is at an isolated point in configuration space.

First order form closure: Every nonzero velocity twist is contrary to some contact screw.

Force closure: the contacts can apply an arbitrary wrench to the object.

Equilibrium: the contact forces can balance the object's weight and other external forces.

Stability: . . .

Flavors of closure

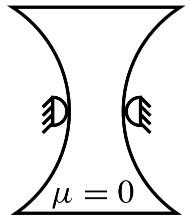
Frictionless force closure $\equiv$ first order form closure

First order form closure $\longrightarrow$ form closure

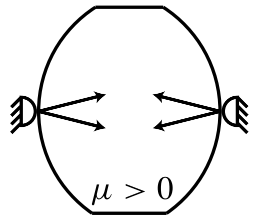
Frictionless force closure $\longrightarrow$ force closure

Form closure $\not\longrightarrow$ force closure

Force closure $\not\longrightarrow$ form closure



Form closure does not imply force closure



Force closure does not imply form closure

Issues in fixture and grasp design

Analysis. Given an object, a set of contacts, and possibly other information, determine whether closure applies.

Existence. Given an object, and possibly some constraints on the allowable contacts, does a set of contacts exist to produce closure?

Synthesis. Given an object, and possibly some constraints on the allowable contacts, find a suitable set of contacts.

Grasp and fixture analysis

Force closure: check positive linear span of friction cones.

Frictionless force closure or first order form closure: check positive linear span of contact normals.

Form closure: beyond the scope of the course! See Elon Rimon and Joel Burdick's work.

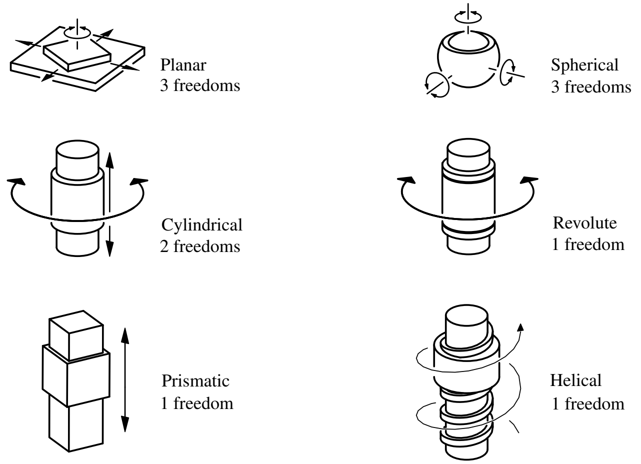
Existence

Given an object, does a force closure grasp exist?

Put fingers everywhere: the “zigzag locus”. Check whether positive linear span is all of wrench space.

Are there are any shapes that do *not* have force closure grasps.

Theorem (Mishra Schwartz and Sharir): For any bounded shape that is not a surface of revolution, a force closure (or first order form closure) grasp exists.



Synthesis

Consider a finger to be *redundant* if it can be deleted without reducing the positive linear span of all the fingers

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procedure GRASP
    put fingers "everywhere"
    while redundant finger exists
        delete any redundant finger
```

Everywhere means a dense sampling of the object boundary.

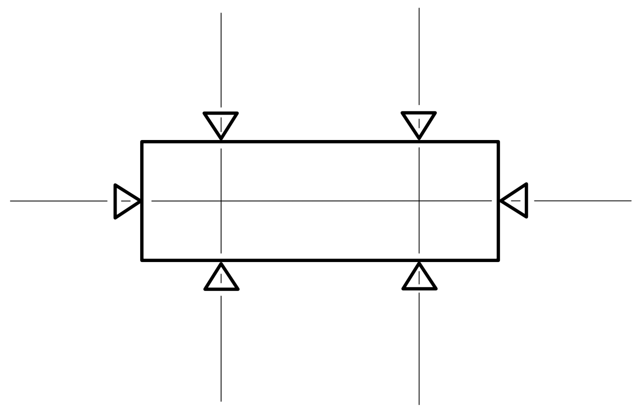
Clearly the algorithm generates a grasp for any object not a surface of revolution, if the sampling is dense enough. But how many fingers does it take?

How many fingers?

Theorem (Steinitz): Let X be a set of points in $\mathbf{R}^d$, with some point p in the interior of the convex hull of X. Then there is some subset Y of X, with $2d$ points or less, such that p is in the interior of the convex hull of Y.

Theorem (Mishra, Schwartz, and Sharir): For any surface not a surface of revolution, `GRASP` yields a grasp with at most 6 fingers in the plane, at most 12 fingers in three space.

In the absence of coincidences among the initial sampling of contact normals, how many fingers will `GRASP` terminate with?

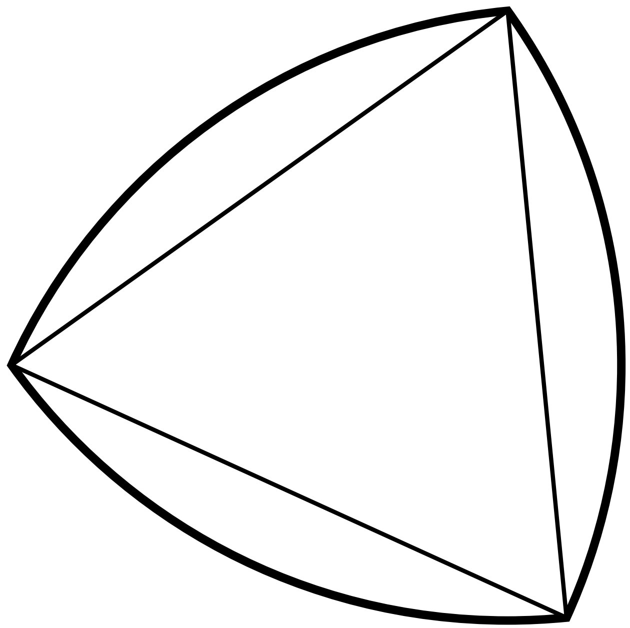


Problem

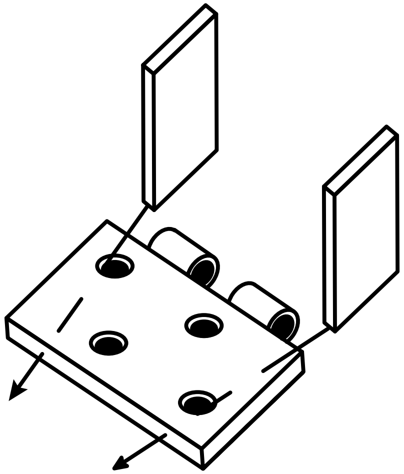
Reuleaux's triangle is a figure of constant diameter. Each edge is a circular arc centered on the opposite vertex.

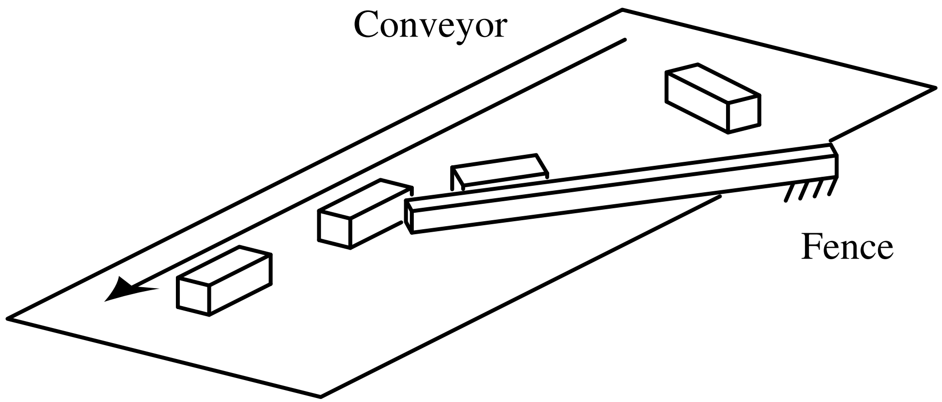
If only parallel jaw grippers are used, show that six fingers are required for frictionless form closure.

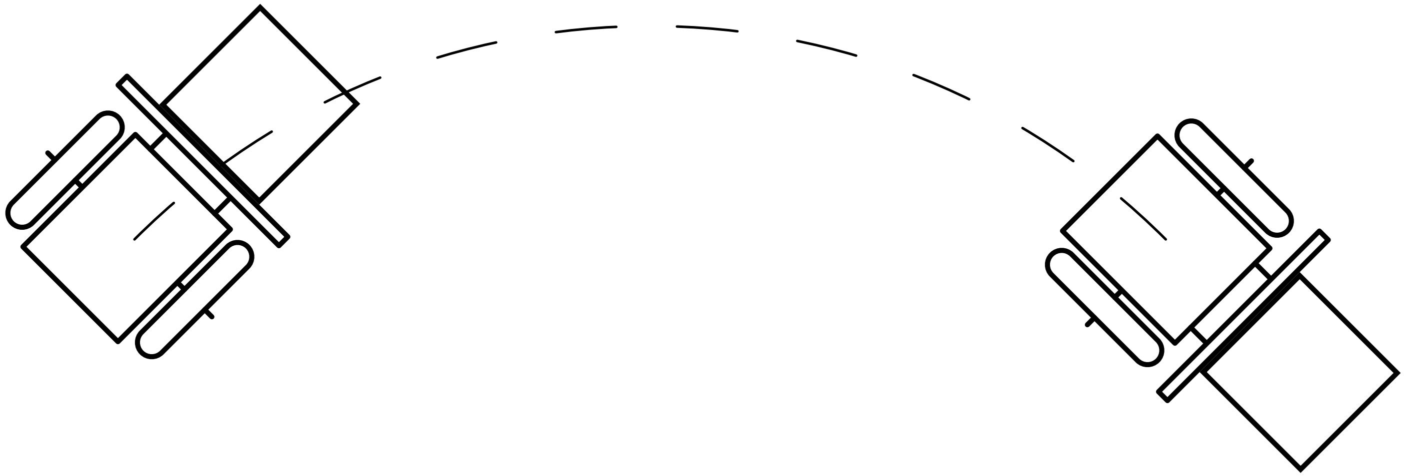
Construct a four-finger grasp. (Hint: don't use parallel jaw grippers!)



Examples of pushing







Pushing

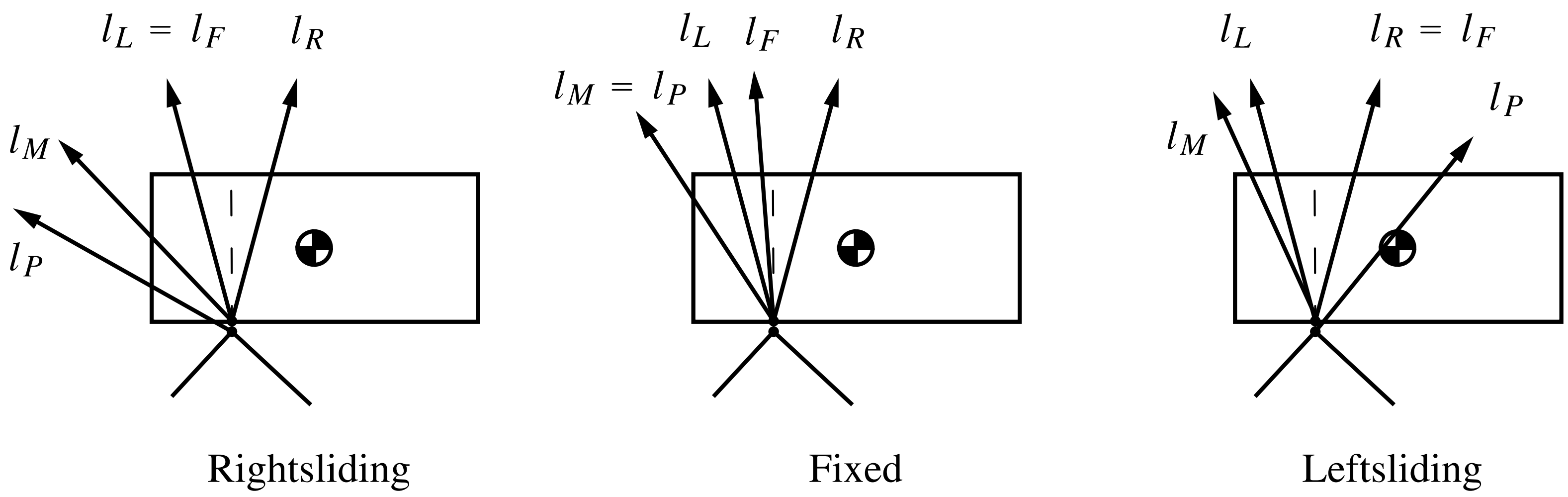
Can we predict direction of rotation?

Line of pushing l_P defined along vel of point in pusher.

Line of motion l_M defined along vel of point in slider.

Line of force l_F defined as usual.

Two edges of friction cone l_L and l_R.



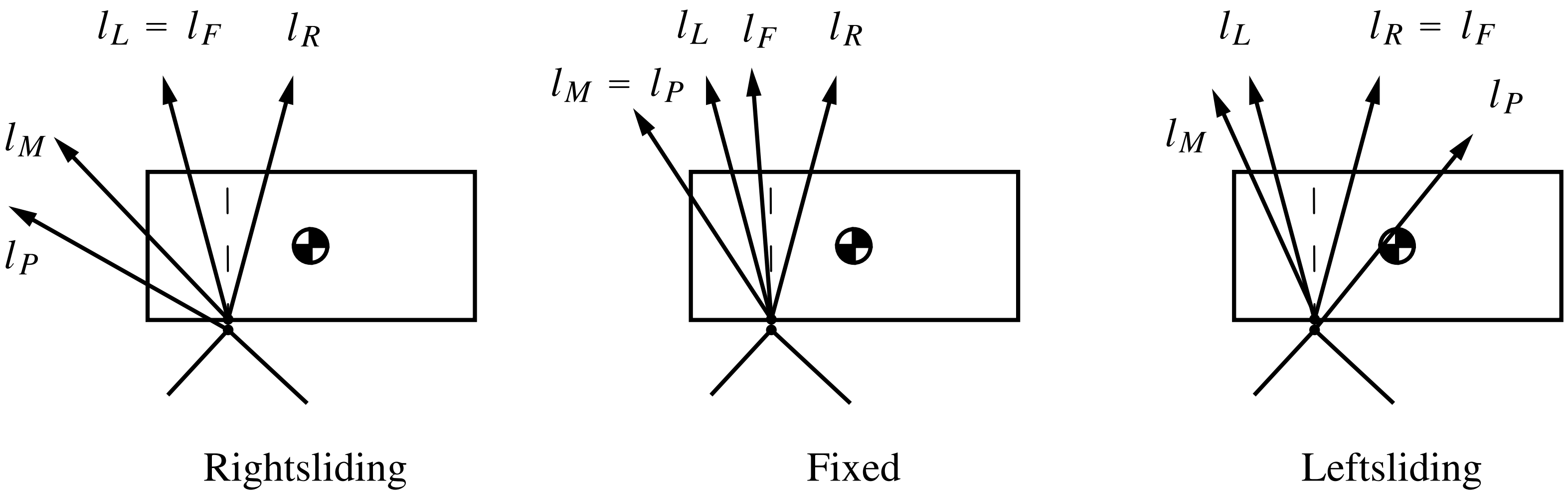
Which way will it turn?

Easy to predict from l_M or from l_F, but what you *know* is l_L, l_R, and l_P.

Main result: l_L, l_R, and l_P vote on rotation direction.

First: l_M dictates rotation direction.

Second: l_F dictates rotation direction.



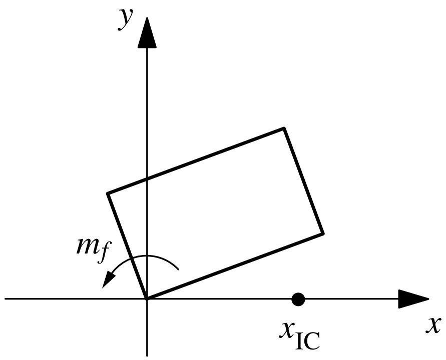
Line of motion dictates

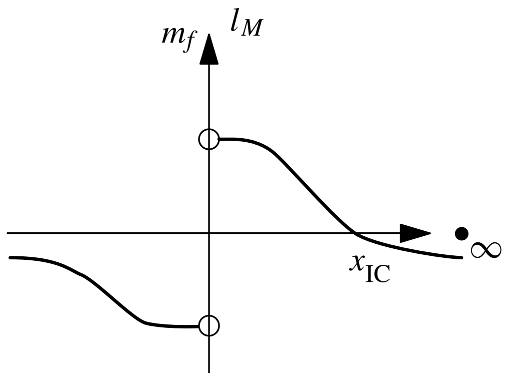
Theorem: For quasistatic pushing of a rigid body in the plane, with uniform coefficient of friction, the line of motion dictates the rotation direction.

Let y-axis be line of motion, let origin be contact point, let x_{IC} be IC coordinate, let $m_f(x_{\mathrm{IC}})$ be frictional moment as function of IC.

Show $m_f(x_{\mathrm{IC}})$ is monotone decreasing.

Look at values at 0^+, 0^-, ∞, apply intermediate value theorem.





Line of force dictates . . .

Theorem: For quasistatic pushing of a rigid body in the plane, with uniform coefficient of friction, the line of force dictates the rotation direction.
Proof:

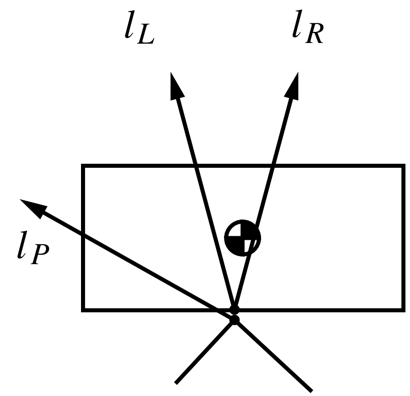
Choose origin at center of friction, construct limit surface.

Normals at f_x-f_y plane are horizontal.

By convexity, normals in upper half point up, in lower half point down.

Voting theorem

Theorem: For quasistatic pushing of a planar rigid body with uniform coefficient of friction, rotation direction is determined by a vote l_P, l_L, and l_R.



Construct voting tree.

If edges of friction agree, then so does line of force, and theorem follows.

Consider case where edges do not agree.

l_L votes $-$, l_R votes $+$, and l_P votes $-$. The majority is $-$.

Assume positive rotation. So l_F and l_M would vote $+$ by previous theorems. If l_M is right of $\mathbf{r}_0$ then it is right of l_P, so we have right sliding. So $l_F = l_L$: a contradiction.

The voting theorem really works.

Demo on overhead.

It tells you which way it turns but

not how fast, and

not about what IC.

Very useful when pushing with a translating edge.