# 28. Quasidynamic parts orienting Mechanics of Manipulation 

Matt Mason<br>matt.mason@cs.cmu.edu<br>http://www.cs.cmu.edu/~mason

Carnegie Mellon

## Chapter 1 Manipulation 1

1.1 Case 1: Manipulation by a human 1
1.2 Case 2: An automated assembly system 3
1.3 Issues in manipulation 5
1.4 A taxonomy of manipulation techniques
1.5 Bibliographic notes 8

Exercises 8

## Chapter 2 Kinematics 11

2.1 Preliminaries 11
2.2 Planar kinematics 15
2.3 Spherical kinematics 20
2.4 Spatial kinematics 22
2.5 Kinematic constraint 25
2.6 Kinematic mechanisms 34
2.7 Bibliographic notes 36 Exercises 37

## Chapter 3 Kinematic Representation 41

3.1 Representation of spatial rotations 41
3.2 Representation of spatial displacements 58
3.3 Kinematic constraints 68
3.4 Bibliographic notes 72

Exercises 72

## Chapter 4 Kinematic Manipulation 7

4.1 Path planning 77
4.2 Path planning for nonholonomic systems 84
4.3 Kinematic models of contact 86
4.4 Bibliographic notes 88

Exercises 88

## Chapter 5 Rigid Body Statics 93

5.1 Forces acting on rigid bodies 93
5.2 Polyhedral convex cones 99
5.3 Contact wrenches and wrench cones 102
5.4 Cones in velocity twist space 104
5.5 The oriented plane 105
5.6 Instantaneous centers and Reuleaux's method 109
5.7 Line of force; moment labeling 110
5.8 Force dual 112
5.9 Summary 117
5.10 Bibliographic notes 117

Exercises 118

## Chapter 6 Friction 12

6.1 Coulomb's Law 121
6.2 Single degree-of-freedom problems 123
6.3 Planar single contact problems 126
6.4 Graphical representation of friction cones 127
6.5 Static equilibrium problems 128
6.6 Planar sliding 130
6.7 Bibliographic notes 139

Exercises 139
Chapter 7 Quasistatic Manipulation 143
7.1 Grasping and fixturing 143
7.2 Pushing 147
7.3 Stable pushing 153
7.4 Parts orienting 162
7.5 Assembly 168
7.6 Bibliographic notes 173

Exercises 175

## Chapter 8 Dynamics 18

8.1 Newton's laws 18
8.2 A particle in three dimensions 181
8.3 Moment of force; moment of momentum 183
8.4 Dynamics of a system of particles 184
8.5 Rigid body dynamics 186
8.6 The angular inertia matrix 189
8.7 Motion of a freely rotating body 195
8.8 Planar single contact problems 197
8.9 Graphical methods for the plane 203
8.10 Planar multiple-contact problems 205
8.11 Bibliographic notes 207

Exercises 208

Chapter 9 Impact 211
9.1 A particle 211
9.2 Rigid body impact 217
9.3 Bibliographic notes 223

$$
\text { Exercises } 223
$$

## Chapter 10 Dynamic Manipulation 225

10.1 Quasidynamic manipulation 225
10.2 Briefly dynamic manipulation 229
10.3 Continuously dynamic manipulation 23
10.4 Bibliographic notes 232

Exercises 235
Appendix A Infinity 237

## Outline.

- What is "quasidynamic"?
- Demonstrate tray tilting.
- Analysis of wrench in corner; of wrench at bottom.
- Analysis of wrench in uncertain pose.


## Quasidynamic

Remember the peg in hole problem?

Static analysis: identify jamming and wedging conditions, and avoid them.
Quasistatic analysis: not applicable. Non-jamming applied force is balanced only by dynamic load.
Dynamic analysis: too hard.
Doubles the state variables.
Quasidynamic
Assume velocity is negligible.
Solve for acceleration, assume object moves in direction of acceleration.

## Tray tilting

## Assumptions

1. Rigid body Newtonian mechanics with Coulomb friction.
2. Known shapes, mass distribution, coefficient of friction.
3. Friction with floor of tray is negligible.
4. Quasidynamic ...

- Centripetal acceleration is negligible.
- Impact: object just stops.


## Goal for example plan

Initially wrench is oriented, and anywhere along the bottom wall.
Goal is for wrench to be in same orientation, in lower left corner.


Find a single action, a single applied force, that accomplishes the goal.

## Plan of attack

Remember ambiguities. We cannot just look at the desired contact mode to find a good tilt angle. We have to exclude all the undesired contact modes.

For every possible initial configuration ...
For every undesired contact mode

## Exclude the tilt angles

If any tilt angles are not excluded by some contact mode for some initial configuration, then you have an action that produces the desired contact mode for every possible initial configuration.

## Recall contact modes for wrench in corner



## Recall analysis of mode "rsrs"

We used force dual:
Represent dynamic load by acceleration center

Transform negated contact forces by force dual
Find positive linear span of force dual and negated contact forces

Result is candidate applied
 forces

Intersect with line at infinity to get tilt directions

## Analysis of mode "'ssrr"

We'll use moment labeling
Transform acceleration center to dynamic load line of force
Find positive linear span of dynamic load and negated contact forces
Result is candidate applied forces
Intersect with wrenches through COM to get tray tilt directions.



## All possible actions from lower left corner

Analyze the other modes.
Record all modes corresponding to wrenches through COM
In this case, no ambiguities


## Analysis of wrench on bottom wall

Do the same thing for the wrench against the bottom wall, and intersect


## Parts orienting

Assume initial location of object is everywhere: any orientation, any place in the tray.

Goal is for it to be somewhere. Some particular somewhere.
For every pose and every contact mode, identify the corresponding tilt angles.

Define a graph.
Each node is a set of configurations.
Each edge is a range of tilt angles, the tilt angles that map the wrench from the source node configurations to the sink node configurations.
The root is the set of all configurations.
Do a breadth-first search of the tree. When you reach a node with just one configuration, you're done.

How big is the tree? In theory, it could be big. In practice, it's not.

## Chapter 1 Manipulation 1

1.1 Case 1: Manipulation by a human 1
1.2 Case 2: An automated assembly system 3
1.3 Issues in manipulation 5
1.4 A taxonomy of manipulation techniques 7
1.5 Bibliographic notes 8

Exercises 8

## Chapter 2 Kinematics 11

2.1 Preliminaries 11
2.2 Planar kinematics 15
2.3 Spherical kinematics 20
2.4 Spatial kinematics 22
2.5 Kinematic constraint 25
2.6 Kinematic mechanisms 34
2.7 Bibliographic notes 36 Exercises 37

## Chapter 3 Kinematic Representation 41

3.1 Representation of spatial rotations 41
3.2 Representation of spatial displacements 58
3.3 Kinematic constraints 68
3.4 Bibliographic notes 72

Exercises 72

## Chapter 4 Kinematic Manipulation 7

4.1 Path planning 77
4.2 Path planning for nonholonomic systems 84
4.3 Kinematic models of contact 86
4.4 Bibliographic notes 88

Exercises 88

## Chapter 5 Rigid Body Statics 93

5.1 Forces acting on rigid bodies 93
5.2 Polyhedral convex cones 99
5.3 Contact wrenches and wrench cones 102
5.4 Cones in velocity twist space 104
5.5 The oriented plane 105
5.6 Instantaneous centers and Reuleaux's method 109
5.7 Line of force; moment labeling 110
5.8 Force dual 112
5.9 Summary 117
5.10 Bibliographic notes 117

Exercises 118

## Chapter 6 Friction 12

6.1 Coulomb's Law 121
6.2 Single degree-of-freedom problems 123
6.3 Planar single contact problems 126
6.4 Graphical representation of friction cones 127
6.5 Static equilibrium problems 128
6.6 Planar sliding 130
6.7 Bibliographic notes 139

Exercises 139

Chapter 7 Quasistatic Manipulation 143
7.1 Grasping and fixturing 143
7.2 Pushing 147
7.3 Stable pushing 153
7.4 Parts orienting 162
7.5 Assembly 168
7.6 Bibliographic notes 173

Exercises 175

## Chapter 8 Dynamics 18

8.1 Newton's laws 181
8.2 A particle in three dimensions 181
8.3 Moment of force; moment of momentum 183
8.4 Dynamics of a system of particles 184
8.5 Rigid body dynamics 186
8.6 The angular inertia matrix 189
8.7 Motion of a freely rotating body 195
8.8 Planar single contact problems 197
8.9 Graphical methods for the plane 203
8.10 Planar multiple-contact problems 205
8.11 Bibliographic notes 207

Exercises 208

## Chapter 9 Impact 211

9.1 A particle 211
9.2 Rigid body impact 217
9.3 Bibliographic notes 223

Exercises 223

Chapter 10 Dynamic Manipulation 225
10.1 Quasidynamic manipulation 225
10.2 Briefly dynamic manipulation 229
10.3 Continuously dynamic manipulation 23
10.4 Bibliographic notes 232

Exercises 235

Appendix A Infinity 237

