Definition - Minkowski Sum (or Addition)

Let \( A \) and \( B \) be two sets.

\[
A + B = \{a + b \mid a \in A, b \in B\}
\]

\( A = \{(0,0), (0,1), (0,2)\} \)

\( B = \{(1,0), (-1,0)\} \)

\[
A + B = \{(1,0), (-1,0), (1,1), (-1,1), (1,2), (-1,2)\}
\]

Suppose \( A = \{a \mid a = (0, \frac{x}{2}) \text{ for } 0 \leq \frac{x}{2} \leq 2\} \)

\[
A + B = \{(1,0), (-1,0), (1,1), (-1,1), (1,2), (-1,2)\}
\]

Suppose \( B = \{b \mid -1 \leq b_x \leq 1, b_y = 0\} \)

\[
A + B = \{a + b \mid -1 \leq a_x + b_x \leq 1, 0 \leq b_y + a_y \leq 2\}
\]

= square.
Suppose \( A = \{(x,y) \mid x = 0, \ 1 \leq y \leq 3\} \)

\[ B = \{(u,v) \mid u^2 + v^2 \leq 1\} = \text{unit disc} \]

Minkowski Sum \( A \oplus B \)

Minkowski Difference of \( A \setminus B \equiv A \ominus B \)

\[ A = \{(0,0), (1,1), (1,0)\} \]

\[ B = \{(0,0), (1,0)\} \]

\[ A \ominus B \Rightarrow \quad \text{Not commutative in general.} \]

\[ B \ominus A \Rightarrow \]
What about change of CO due to ref. pt. choice?

If we add \((5,1)\) to every point in \(A\),

then

\(b-a\) shifts \((-1,-1)\)!
PRM – Probabilistic Road Map
Approach

Plaster C-space w/ points. Hope to construct C-space structure. Attempt to connect w/ “local planner”. Choose points “close enough.” Use C-space hints to refine sampling

Not complete?
How many points to sample?
Exponential complexity in dimension to cover C-space well & return C-space structure
Metric Space - a set for which a notion of distance is defined between set elements!

Possible potential functions - See Kaditschek's paper from 90's

\[ C_1 \| q - q_{\text{goal}} \|^2 \iff \text{quadratic surface with } q_{\text{goal}} \text{ the lowest point.} \]

\[ C_1 \in \mathbb{R}^+ \]

Let \( d_i(q) \) = distance of robot to obstacle \( i \)

\[ \frac{C_{2i}}{d_i(q)} \iff \text{hyperbolic function that grows as distance} \]

\[ C_{2i} \in \mathbb{R}^+ \]

Potential Function, \( F \)

\[ F(q) = C_1 \| q - q_{\text{goal}} \|^2 + \sum_{i=1}^{N_{\text{obst}}} \frac{C_{2i}}{d_i(q)} \]

Ideally, \( F(q) \) has a unique global minimum.

Then just follow the gradient.
Main data structure as a priority queue.

A priority queue is a container for which you can access only the highest priority item.

Need an objective function which defines "best".

E.g., Potential field plus distance.

How do you choose Grid Size?

Depends on whether you use collision check or swept volumes.
Feature of BFP:

- No need to compute C-space obstacle, which is exponential in dimension of C-space.
- Just need to do collision check when visiting a node.

Distance computation:

- You get this as a by-product of collision checking.
- Could also use total distance between pairs of points.

Behavior:

- If lucky, alg walks down slope to goal.
- If unlucky, alg reaches potential well and visits many points in the well before escaping.

List of visited nodes becomes very large.
Applicability

Holonomic systems?

Yes, in principle, but should at least be able to eliminate constrained variables, i.e., need a lower dimensional representation of C-space. Suppose we have a point robot constrained to lie on a circle:

\[ x^2 + y^2 - r^2 = 0 \]

We need to grid on the variable \( \theta \), not \( x \) & \( y \).

Tangent space is not enough necessarily. It could work for the circle, could lead to non-uniform coverage of C-space. Want points not "too close". Use geodesics in n-dimensional...
What about nonholonomic systems?

Massin says "no!"

Why? Can't get to arbitrary nearest neighbor easily

We can!

If constraints are Pfaffian, then we can plan a "free-flying" path and then do Lie bracket maneuvers to reach various sub-goals along the way.
How could we modify the alg to produce plans with fewer Lie bracket motions.

Integrate system forward over time $\delta t$ with input $a$.

$$\text{node} = n = \text{int}(q, a, \delta t)$$

Must discretize the space of actions. What's a suitable $\delta t$?

Running time is exponential in # of actions. Need function to determine closest node to a config, $q$.

What is the cost function for "best" node?

How do we choose discretization to ensure coverage of reachable set.
Nonholonomic Planner, NHP

<table>
<thead>
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<th>open</th>
<th>best</th>
<th>visited</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q_{init}$</td>
<td>$q_{init}$</td>
<td>$V_1$</td>
</tr>
<tr>
<td>$n_1, n_2$</td>
<td>$n_3$</td>
<td>$V_2, V_3, V_4$</td>
</tr>
<tr>
<td>$n_5, n_6$</td>
<td>$n_6$</td>
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</tbody>
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PROBLEMS!? Planer is at best Resolution Complete.

Can get stuck.

What if $a, St, n$

have been chosen

such that you can't

make progress toward goal?

What if $St$ is too small

for some portion of grid.

Planner requires space & time exponential in dimension of Spa
4 actions.

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Note that $v_4$ and $v_5$ are in the same (x, y) position as $v_1$, but above and below on x, y, z grid.