Probs_Chap4_Lavalle

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11:36 AM

4.5 C-space of cylindrical rod is $\boxed{\mathbb{R}^3 \times S^2}$

Imagine pinning rod end to center of sphere with a ball-in-socket joint.

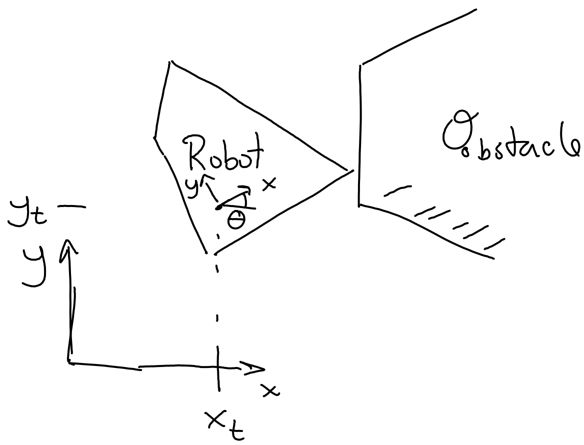
Rod can point in any direction, hence S^2

Base of rod can translate, hence $\mathbb{R}^3$

Rotation of rod about its axis makes no difference

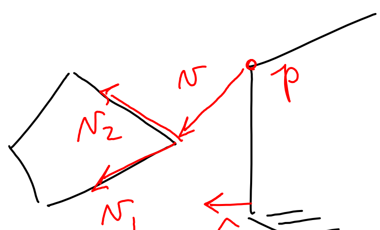
4.9 Derive H_A for type VE contact

vertex of robot contacts edge of obstacle

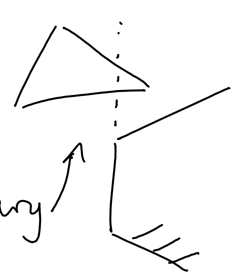


Let $q = (x_t, y_t, \theta)$

Sufficient conditions for $q \in C_{free}$



$$\boxed{\begin{aligned} N(q) \cdot \hat{n} &> 0 \\ N_1(\theta) \cdot \hat{n} &\geq 0 \\ N_2(\theta) \cdot \hat{n} &\geq 0 \end{aligned}}$$



Note: conditions are not necessary

Let $H_f = \{ q \mid v(q) \cdot \hat{n} > 0,\ v_1(\theta) \cdot \hat{n} \geq 0,\ v_2(\theta) \cdot \hat{n} \geq 0 \}$

$H_f \subset C_{free}$

Let $H_A = C \setminus H_f \quad \Rightarrow \quad H_A = H_1 \cup H_2 \cup H_3$

where

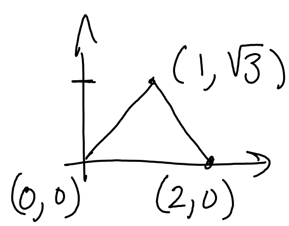
$H_1 = \{ q \mid \hat{n} \cdot v(q) \leq 0 \}$

$H_2 = \{ q \mid \hat{n} \cdot v_1(\theta) \leq 0 \}$

$H_3 = \{ q \mid \hat{n} \cdot v_2(\theta) \leq 0 \}$

The main difference between EV & VE contacts are the details of the formulas in H_i, $i=1,2,3$, when expanded as functions of x_t, y_t, θ.

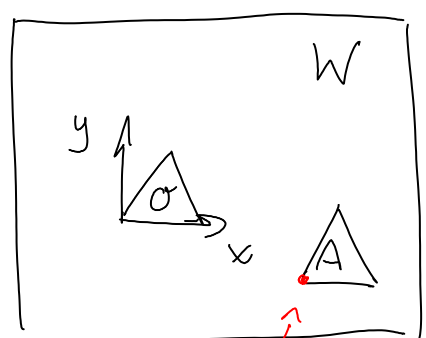
(4.11) This problem does not specify C-space. Solve w/o rotation first.



$\mathcal{O}, A$ = equilateral triangle

Let the world frame coincide with the frame of $\mathcal{O}$

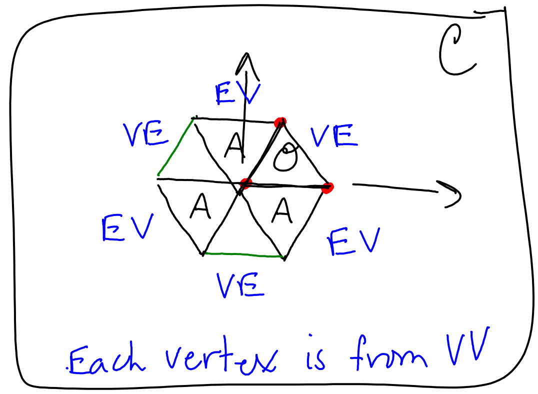
Choose the reference point of A at its lower left corner.



The $C_{obs} = \mathcal{O} \ominus A$.

The corners of the vertices are (by inspection):

$$\{ (2,0), (1,\sqrt{3}), (-1,\sqrt{3}), (-2,0), (-1,-\sqrt{3}), (1,-\sqrt{3}) \}$$



Indicate $EV_{A\theta}$, $VE_{A\theta}$ & $VV_{A\theta}$ combinations on C_{obs}

(4.16) 5 bodies free to move in $\mathbb{R}^3$ gives

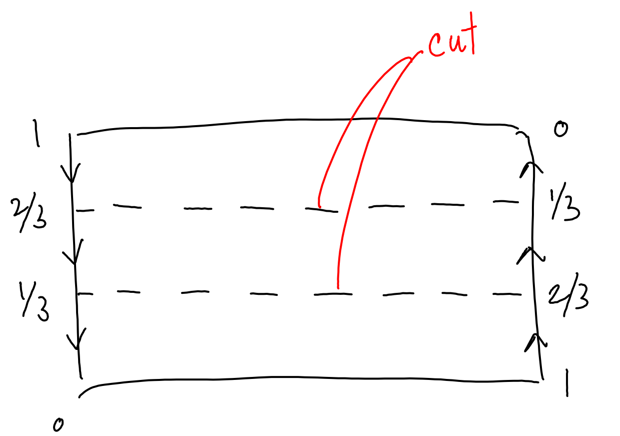
$$C = SE(3) \times \cdots \times SE(3) = (SE(3))^5$$

C is 30-dimensional

(4.18)

Form mobius band then cut along dashed line.

$\boxed{\text{What results ?}}$



One gets a mobius band of length 1 from the center of the strip.

The outer ⅓-wide strips stay connected to each other & form a hoop like a mobius band with two twists.

Also the mobius band and double twist band are connected like links in a chain.

Question: Is the double twist band homeomorphic to a cylindrical surface, i.e., $S \times I$?

<u>Is the result a manifold?</u>

Yes, it is a manifold w/ boundary.

Every point not on the edge has a nbhd. that is Euclidean ($\mathbb{R}^2$). Every point on the edge has a nbhd that is a half-plane.

(4.21) Tetrahedron & Cube. How many facets on the C_{obs} if they are free to move in $\mathbb{R}^3$?

$$\left.\begin{aligned} VF &= 8 \times 4 = 32 \\ FV &= 6 \times 4 = 24 \\ EE &= 12 \times 6 = 72 \end{aligned}\right\} \Rightarrow 128 \quad \text{2D facets.}$$