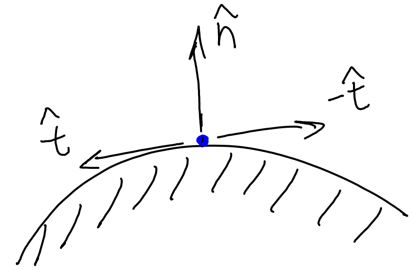
Soln LCP-1

Thursday, February 14, 2008
7:46 AM

1. Consider a particle in the plane moving in contact with a fixed obstacle.



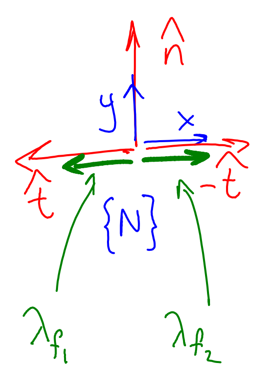
Determine the physical interpretations (sliding left, sliding right, sticking, degenerate sliding, degenerate sticking) of the eight generic solutions of the friction model given by:

$$0 \le \lambda_f \perp G_f^T \nu + 1 s \ge 0$$

$$0 \le s \perp \mu \lambda_n - 1^T \lambda_f \ge 0$$

where λ_{f_1} is in the $\hat{t}$ direction
λ_{f_2} is in the $-\hat{t}$ direction

$$G_f = \begin{bmatrix} -1 & 1 \\ 0 & 0 \end{bmatrix} \qquad \nu = \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix}$$



Expand comple. conditions

$$0 \le \lambda_{f_1} \perp -\dot{x} + s \ge 0$$

$$0 \le \lambda_{f_2} \perp \dot{x} + s \ge 0$$

$$0 \le s \perp \mu \lambda_n - \lambda_{f_1} - \lambda_{f_2} \ge 0$$

$\boxed{\text{8 cases}}$

$$0 \le s \perp \mu\lambda_n - \lambda_{f_1} - \lambda_{f_2} \ge 0$$

(Assume $\mu\lambda_n$ given & positive)

$\boxed{\text{8 cases}}$

Case#	Left	Right
1	+ + +	0 0 0
2	+ + 0	0 0 +
3	+ 0 +	0 + 0
4	+ 0 0	0 + +
5	0 + +	+ 0 0
6	0 + 0	+ 0 +
7	0 0 +	+ + 0
8	0 0 0	+ + +

Case 1:

$$\left.\begin{aligned} \dot{x} &= s \\ \dot{x} &= -s \end{aligned}\right\} \Rightarrow s = \dot{x} = 0 \Rightarrow \text{sticking}$$

$$\left.\begin{aligned} &\mu\lambda_n - \lambda_{f_1} - \lambda_{f_2} = 0 \\ &\lambda_{f_1}, \lambda_{f_2} \ge 0 \end{aligned}\right\} \Rightarrow \text{contact force in cone.}$$

$s > 0$

Case 1 is inconsistent, but degenerate case

$\begin{bmatrix} + & 0 \\ + & 0 \\ 0 & 0 \end{bmatrix}$ is NOT. $\therefore$

$\boxed{\text{DEGENERATE STICKING}}$

notice different form, but effectively same constraint

Case 2:

$$\left.\begin{aligned} \dot{x} &= s \\ \dot{x} &= -s \end{aligned}\right\} \Rightarrow \dot{x} = s = 0 \Rightarrow \text{sticking}$$

$$\left.\begin{aligned} &\mu\lambda_n - \lambda_{f_1} - \lambda_{f_2} \ge 0 \\ &\lambda_{f_1}, \lambda_{f_2} \ge 0 \end{aligned}\right\} \Rightarrow \text{contact force in cone}$$

$\boxed{\text{STICKING}}$

Case 3:

$\dot{x} = s$

$\lambda_{f_2} = 0$

$\mu\lambda_n - \lambda_{f_1} = 0$

$\lambda_{f_1} \ge 0$ ← valid

$$\left.\begin{aligned} \dot{x} &\ge -s \\ s &\ge 0 \end{aligned}\right\} \Rightarrow \dot{x} \ge 0$$

$\mu\lambda_n - \lambda_{f_1} = 0$ | $s \geq 0$

$\boxed{\text{SLIDING RIGHT}}$

Case 4:

$\left.\begin{array}{l} \dot{x} = s \\ \lambda_{f_2} = 0 \\ s = 0 \end{array}\right\} \Rightarrow$ sticking with $\lambda_{f_2} = 0$

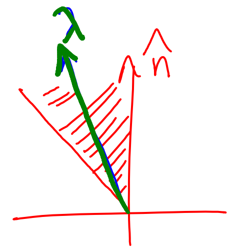
The inequalities

$\lambda_{f_1} \geq 0$

$\dot{x} \geq s$

$\mu\lambda_n - \lambda_{f_1} \geq 0$

$\boxed{\text{STICKING}}$ but with contact force limited to half of friction cone



Case 5:

$\lambda_{f_1} = 0$

$\dot{x} = -s$

$\mu\lambda_n - \lambda_{f_2} = 0$

Ineqs.

$-\dot{x} \geq -s$

$\lambda_{f_2} \geq 0$

$s \geq 0$

$\boxed{\text{SLIDING LEFT}}$

Case 6:

$\lambda_{f_1} = 0$

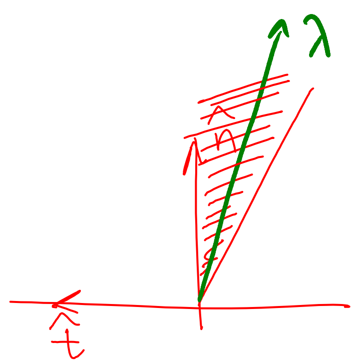
$\left.\begin{array}{l} \dot{x} = -s \\ s = 0 \end{array}\right\} \Rightarrow \dot{x} = 0$

$-\dot{x} \geq -s$

$\lambda_{f_2} \geq 0$

$\mu\lambda_n - \lambda_{f_2} \geq 0$

$\boxed{\text{STICKING}}$



Case 7:

$$\left.\begin{aligned} \lambda_{f_1} &= 0 \\ \lambda_{f_2} &= 0 \\ \mu\lambda_n - \lambda_{f_1} - \lambda_{f_2} &= 0 \end{aligned}\right\} \Rightarrow \lambda = 0$$

Ineqs.

$$\left.\begin{aligned} -\dot{x} &\geq -s \\ \dot{x} &\geq -s \end{aligned}\right\} -s \leq \dot{x} \leq s$$

$$s \geq 0$$

$\boxed{\text{DEGENERATE SLIDING}}$ sliding is maintained without compressive force!

Case 8:

$$\lambda_{f_1} = 0$$

$$\lambda_{f_2} = 0$$

$$s = 0$$

ineqs

$$\left.\begin{aligned} -\dot{x} &\geq -s \\ \dot{x} &\geq -s \end{aligned}\right\} -\cancel{s}^{\,0} \leq \dot{x} \leq \cancel{s}^{\,0} \Rightarrow \dot{x} = 0$$

$$\mu\lambda_n - \cancel{\lambda_{f_1}}_{\,0} - \cancel{\lambda_{f_2}}_{\,0} \geq 0 \Rightarrow \lambda_n \geq 0$$

$\boxed{\text{DEGENERATE STICKING}}$ friction is not used to prevent sliding!