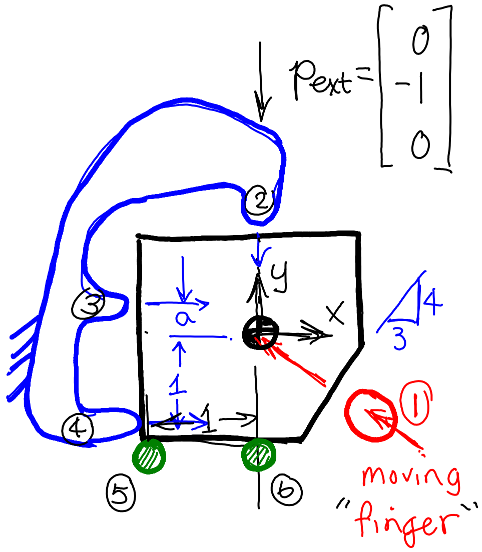
Soln LCP-3

Thursday, February 14, 2008
7:46 AM

3. It is desired to assemble the parts shown. The polygonal part is initially at rest on two shaded pegs. (contacts 5 & 6). Assembly is successful when contact between the E-shaped body at all three nubs and contact with the moveable "finger" have been achieved. We would like to complete the assembly in one time step.



Assume $\mu = 0$, $h = 1$, $m = 1$, $J = 1$, $a = 0.2$, and the gaps at contacts 1, 2, 3, & 4 are initially of size 0.1. As defined in the figure, $p_{ext} = [0\ -1\ 0]^T$.

Ⓐ Write the time-stepping LCP that models this situation and define the quantities, G_n, G_f, E, 1, ψ_n^l, $\frac{\partial \psi_n^l}{\partial t}$, M

No friction $\Rightarrow$ E, 1, G_f, $\frac{\partial \psi_f}{\partial t}$, U are vacuous.

$M = \text{diag}(m, m, J)$

$$G_n = \begin{bmatrix} -0.6 & 0 & 1 & 1 & 0 & 0 \\ 0.8 & -1 & 0 & 0 & 1 & 1 \\ 0 & 0 & -a & 1 & -1 & 0 \end{bmatrix}$$

$$\psi_n^T = \begin{bmatrix} 0.1 & 0.1 & 0.1 & 0.1 & 0 & 0 \end{bmatrix}$$

$$\left(\frac{\partial \psi_n}{\partial t}\right)^T = \begin{bmatrix} d & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \leftarrow$$ we don't know yet what d will be.

(B) Choose inequalities to be zero and positive such that, if satisfied, the parts will be properly assembled.

To ensure contacts form at (1), (2), (3), & (4), set $p_{1n}, p_{2n}, p_{3n}, p_{4n} > 0$

The complementary constraints are:

$$\rho_{1n} = \rho_{2n} = \rho_{3n} = \rho_{4n} = 0 \Longleftarrow$$ close gaps at the 4 desired contacts

We need 2 more equations. Allow contacts (5), (6) to break

set $p_{5n} = p_{6n} = 0$

(C) Solve for the nonzero element of $\frac{\partial \psi_n}{\partial t}$, ν^{l+1}, and p_n^{l+1}, such that the time-stepping equations are satisfied and p_n^{l+1} is small.

Hint: certain quantities must be zero at the end of the time step. This gives four equations that can be solved, $A p_n^{l+1} = b$, where $A_{(4\times4)}$ is of rank 3. Therefore solve by $p_n^{l+1} = A^+ b + N(A)\alpha$.
Use the scalar α to guarantee $p_n^{l+1} \geq 0$ ← scalar

$$\begin{bmatrix} \rho_{1n}^{l+1} \\ \rho_{2n}^{l+1} \\ \vdots \end{bmatrix} \quad \begin{bmatrix} -0.6 & 0.8 & 0 \\ 0 & -1 & 0 \\ \vdots \end{bmatrix} \begin{bmatrix} \nu_x \\ \vdots \end{bmatrix}^{l+1} \quad \begin{bmatrix} 0.1 \\ 0.1 \\ \vdots \end{bmatrix} \quad \begin{bmatrix} d \\ 0 \\ \vdots \end{bmatrix}$$

solve 3 linearly

$$\begin{bmatrix} \vdots \\ \rho_{2n}^{l+1} \\ \rho_{3n}^{l+1} \\ \rho_{4n}^{l+1} \\ \rho_{5n}^{l+1} \\ \rho_{6n}^{l+1} \end{bmatrix} = \begin{bmatrix} \vdots & & \\ 0 & -1 & 0 \\ 1 & 0 & -0.2 \\ 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} v_x \\ v_y \\ \omega_z \end{bmatrix}^{l+1} + \begin{bmatrix} \vdots \\ 0.1 \\ 0.1 \\ 0.1 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} \vdots \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

solve 3 linearly independent equations 1st

Solution of these eqs. gives $\underline{v^{l+1} = [-0.1 \quad 0.1 \quad 0]^T}$

Now solve for d to get the movement of finger 1.

$$d = -0.24$$

Substitute back into the 6 ρ equations giving:

$$(\rho_n^{l+1})^T = [0 \quad 0 \quad 0 \quad 0 \quad 0.1 \quad 0.1] \qquad \underline{\text{correct!}}$$

Now solve for p_n^{l+1}. There are only 4 that are nonzero.

Recall $\quad M v^{l+1} = G_n p_n^{l+1} + M \cancel{v^{l}}^{\,0} + p_{ext}$

Substitute known values

$$\begin{bmatrix} -0.1 \\ 0.1 \\ 0 \end{bmatrix} = \underbrace{\begin{bmatrix} -0.6 & 0 & 1 & 1 \\ 0.8 & -1 & 0 & 0 \\ 0 & 0 & -0.2 & 1 \end{bmatrix}}_{A} \begin{bmatrix} p_{1n}^{l+1} \\ p_{2n}^{l+1} \\ p_{3n}^{l+1} \\ p_{4n}^{l+1} \end{bmatrix} + \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}$$

has nontrivial null space

Rearrange & solve $\Longrightarrow$

$$p_n^{l+1} = \underbrace{\begin{bmatrix} 0.4860 \\ -0.7112 \\ 0.1596 \\ 0.0319 \end{bmatrix}}_{\text{pinv}(A)*\begin{bmatrix} -0.1 \\ 1.1 \\ 0 \end{bmatrix}} + \underbrace{\begin{bmatrix} -0.7255 \\ -0.5804 \\ -0.3627 \\ -0.0725 \end{bmatrix}}_{\text{null}(A)} \alpha$$

From Matlab

Choose $\alpha \ni p_n^{l+1} > 0$. $\alpha = -2.0 \Rightarrow$ smallest $p_{in}^{l+1} = 0.1770$

(D) Redo part (C) with contact 3 shifted downward by 2a. You will find that it is impossible to make $p_n^{l+1} \geq 0$. Relate this problem to grasping. What sort of grasps do the assemblies form in part (C) & (D) ?

Change -0.2 to 0.2 in G_n. and rework.

$$\nu^{l+1} = \begin{bmatrix} -0.1 \\ 0.1 \\ 0 \end{bmatrix} \Leftarrow \text{no change here.}$$

$$\text{Solve for } p_n^{l+1} = \begin{bmatrix} 0.4393 \\ -0.7485 \\ 0.2045 \\ -0.0409 \end{bmatrix} + \begin{bmatrix} -0.6704 \\ -0.5363 \\ -0.5028 \\ 0.1006 \end{bmatrix} \alpha$$

Note that $\alpha = 0 \Rightarrow p_{2n} < 0$ and $p_{4n} < 0$

If you increase $\alpha \ni p_{2n} > 0$ makes p_{4n} more negative.

In this case there is no solution with $p_n^{l+1} \geq 0$!

Look at the problem from the perspective of form closure and the corresponding ability to squeeze.

With the original numbers, the grasp had form cl. at t_{l+1}. The change for part D caused loss of form cl. at t_{l+1}.