2. It is desired to assemble the parts shown. The polygonal part is initially at rest on two shaded pegs. (contacts 5 & 6). Assembly is successful when contact between the E-shaped body at all three nubs and contact with the moveable "finger" have been achieved. We would like to complete the assembly in one time step.

Assume $\mu = 0$, $h = 1$, $m = 1$, $J = 1$, $a = 0.2$, and the gaps at contacts 1, 2, 3, & 4 are initially of size 0.1. As defined in the figure, $p_{ext} = [0 \ -1 \ 0]^T$.
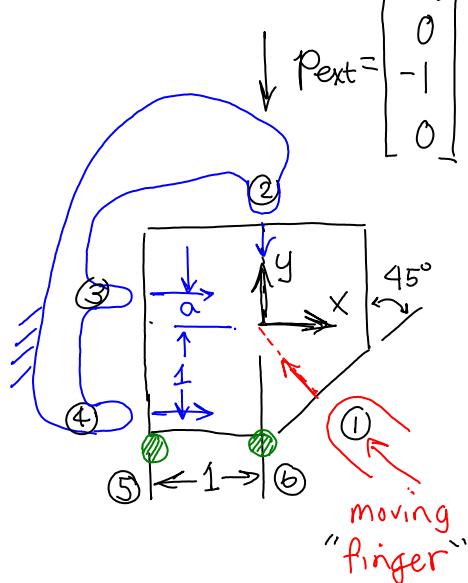
$$p_{ext} = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}$$

(figure labels: ②, ③, ④, ⑤, ⑥, ①, y, x, a, 1, $45°$, moving "finger", $\leftarrow 1 \rightarrow$)

Ⓐ Write the time-stepping LCP that models this situation and define the quantities, G_n, G_f, E, 1, ψ_n^ℓ, $\frac{\partial \psi_n^\ell}{\partial t}$, M

No friction $\Rightarrow$ E, 1, G_f, $\frac{\partial \psi_f}{\partial t}$, U are vacuous.

$M = \text{diag}(m, m, J)$

$$G_n = \begin{bmatrix} -\sqrt{2}/2 & 0 & 1 & 1 & 0 & 0 \\ \sqrt{2}/2 & -1 & 0 & 0 & 1 & 1 \\ 0 & 0 & -a & 1 & -1 & 0 \end{bmatrix}$$

$$\psi_n^T = \begin{bmatrix} 0.1 & 0.1 & 0.1 & 0.1 & 0 & 0 \end{bmatrix}$$

$$\left(\frac{\partial \psi}{\partial t}_n\right)^T = \begin{bmatrix} d & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad \leftarrow \text{we don't} \\ \text{know yet what} \\ d \text{ will be.}$$

Ⓑ Choose inequalities to be zero and positive such that, if satisfied, the parts will be properly assembled.

To ensure contacts form at ①, ②, ③, & ④,
  set $p_{1n}, p_{2n}, p_{3n}, p_{4n} > 0$

The complementary constraints are:

  $p_{1n} = p_{2n} = p_{3n} = p_{4n} = 0$ $\Longleftarrow$ close gaps at
      the 4 desired contacts

We need 2 more equations. Allow contacts ⑤, ⑥
                to break

  set $p_{5n} = p_{6n} = 0$

Ⓒ Solve for the nonzero element of $\frac{\partial \psi_n}{\partial t}$, $\nu^{\ell+1}$, and $p_n^{\ell+1}$, such that the time stepping equations are satisfied and $p_n^{\ell+1}$ is small.

Hint: certain quantities must be zero at the end of the time step. This gives four equations that can be solved, $A p_n^{\ell+1} = b$, where $A_{(4\times4)}$ is of rank 3. Therefore solve by $p_n^{\ell+1} = A^+ b + N(A)\alpha$.
Use the scalar α to guarantee $p_n^{\ell+1} \geq 0$ $\uparrow$ scalar

$$\begin{bmatrix} p_{1n}^{\ell+1} \\ 0.^{\ell+1} \end{bmatrix} \begin{bmatrix} -\sqrt{2}/2 & \sqrt{2}/2 & 0 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} \nu^{\ell+1} \end{bmatrix} \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix} \begin{bmatrix} d \\ 0 \end{bmatrix} \quad \text{solve} \\ \text{3 linearly}$$

$$\begin{bmatrix} \rho_{2n}^{\ell+1} \\ \rho_{3n}^{\ell+1} \\ \rho_{4n}^{\ell+1} \\ \rho_{5n}^{\ell+1} \\ \rho_{6n}^{\ell+1} \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & -0.2 \\ 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} N_x \\ N_y \\ \omega_z \end{bmatrix} + \begin{bmatrix} 0.1 \\ 0.1 \\ 0.1 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad \left.\begin{array}{l} \text{solve} \\ 3 \text{ linearly} \\ \text{independent} \\ \text{equations 1st} \end{array}\right.$$

Solution of these eqs. gives $\quad \underline{\nu^{\ell+1} = \begin{bmatrix} -0.1 & 0.1 & 0 \end{bmatrix}^T}$

Now solve for d to get the movement of finger 1.

$$d = -0.2414$$

Substitute back into the 6 ρ equations giving:

$$(\rho_n^{\ell+1})^T = \begin{bmatrix} 0 & 0 & 0 & 0 & 0.1 & 0.1 \end{bmatrix} \quad \text{correct!}$$

Now solve for $p_n^{\ell+1}$. There are only 4 that are nonzero.

Recall $\quad M\nu^{\ell+1} = G_n\, p_n^{\ell+1} + M\nu^{\ell\,0} + p_{ext}$

Substitute known values

$$\begin{bmatrix} -0.1 \\ 0.1 \\ 0 \end{bmatrix} = \underbrace{\begin{bmatrix} -\sqrt{2}/2 & 0 & 1 & 1 \\ \sqrt{2}/2 & -1 & 0 & 0 \\ 0 & 0 & -0.2 & 1 \end{bmatrix}}_{A} \begin{bmatrix} p_{1n}^{\ell+1} \\ p_{2n}^{\ell+1} \\ p_{3n}^{\ell+1} \\ p_{4n}^{\ell+1} \end{bmatrix} + \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}$$

has nontrivial null space

Rearrange & solve $\Longrightarrow$

$$p_n^{\ell+1} = \underbrace{\begin{bmatrix} 0.4454 \\ -0.7851 \\ 0.1791 \\ 0.0358 \end{bmatrix}}_{\substack{\text{From Matlab} \\ \text{pinv}(A)* \begin{bmatrix} -0.1 \\ 1.1 \\ 0 \end{bmatrix}}} + \underbrace{\begin{bmatrix} 0.7330 \\ 0.5183 \\ 0.4319 \\ 0.0864 \end{bmatrix} \alpha}_{\text{null}(A)}$$

Choose $\alpha \ni p_n^{\ell+1} > 0 \quad \alpha = 2.0 \Rightarrow p \simeq 0.22$

Choose $\alpha \ni p_n^{\ell+1} > 0$. $\alpha = 2.0 \Rightarrow p_{2n} \cong 0.22$

(D) Redo part (C) with contact 3 shifted downward by 2a.
You will find that it is impossible to make $p_n^{\ell+1} \geq 0$.
Relate this problem to grasping. What sort of grasps
do the assemblies form in part (C) & (D) ?

Change -0.2 to 0.2 in G_n and rework.

$$v^{\ell+1} = \begin{bmatrix} -0.1 \\ 0.1 \\ 0 \end{bmatrix} \Leftarrow \text{no change here.}$$

Solve for $p_n^{\ell+1} = \begin{bmatrix} 0.3860 \\ -0.8270 \\ 0.2162 \\ 0.0432 \end{bmatrix} + \begin{bmatrix} 0.6576 \\ 0.4650 \\ 0.5812 \\ -0.1162 \end{bmatrix} \alpha$

Note that $\alpha = 0 \Rightarrow p_{2n} < 0$.

   Increase α to about 1.8 to make $p_{2n} > 0$.

   But this cause $p_{4n} < 0$

In this case there is no solution with $p_n^{\ell+1} \geq 0$!

Look at the problem from the perspective of form
closure and the corresponding ability to squeeze.

With the original numbers, the grasp had form cl.
at $t_{\ell+1}$. The change for part D caused loss

of form d. at $t_{\ell+1}$.