LCP4_soln

Monday, February 21, 2011
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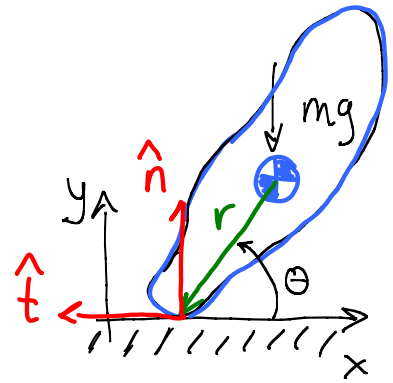
This is a planar problem

At the current time a rigid body is in contact with a horizontal support surface ($y=0$) while translating to the left.



$d = \|r\|$

μ = friction coeff

(x,y) = center of grav. location

m = mass of body

J = moment of inertia

Write the instantaneous equations of motion in terms of λ_n and $\ddot{\psi}_n$. The result should be an LCP of size 1 in λ_n: $\quad 0 \le \lambda_n \perp A\lambda_n + b \ge 0$

Getting started: $\quad u = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix} \quad v = \begin{bmatrix} v_x \\ v_y \\ \omega_z \end{bmatrix}$

Translating left and in contact $\Rightarrow$ $\omega_z = v_y = 0$

$v_x < 0$.

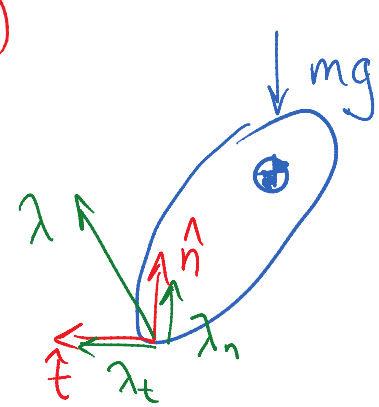
Use these initial conditions to eliminate λ_t!

1. Derive the scalars A and b in terms of g, m, J, d, μ, and θ.

Kinematics:

$$\psi_n = y - d\sin(\theta) \qquad \dot{\psi}_n = \dot{y} - \dot{\theta} d\cos(\theta)$$

$$\ddot{\psi} = \ddot{y} + \dot{\theta}^2 d\sin(\theta) - \ddot{\theta} d\cos(\theta)$$



Sliding contact:

$$\lambda_t = \mu \lambda_n \qquad \lambda = \begin{bmatrix} \lambda_n \\ \lambda_t \end{bmatrix}$$

Newton Euler Equation:

$$m\ddot{x} = -\lambda_t$$

$$m\ddot{y} = -mg + \lambda_n$$

$$J\ddot{\theta} = -\lambda_n d\cos(\theta) - \lambda_t d\sin(\theta)$$

Complementarity:

$$0 \le \lambda_n \perp \ddot{\psi}_n \ge 0$$

$$\ddot{\psi} = \ddot{y} + \cancelto{0}{\dot{\theta}^2} d\sin(\theta) - \ddot{\theta} d\cos(\theta)$$

$\omega_z = 0$

Use $\lambda_t = \mu \lambda_n$ to eliminate λ_t

" $J\ddot{\theta}$ equation to elim $\ddot{\theta}$

" $m\ddot{y}$ equation to elim $\ddot{y}$

Then rearrange to get

$$A = \frac{1}{m} + \frac{d^2}{J}\cos(\theta)\left[\cos(\theta) - \mu \sin(\theta)\right]$$

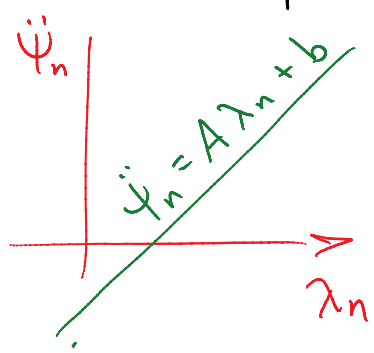
$$b = -g$$

2. Let $m = d = \mu = 1$ and $J = 0.1$.

a. Find values of g & θ such that λ_n is unique

We just need $A > 0$ for unique soln.

$\pi/2 < \theta < \pi$ works $\forall g \in \mathbb{R}^1$



b. Find values of g & θ such that no solution exists.

First, make $A < 0$:

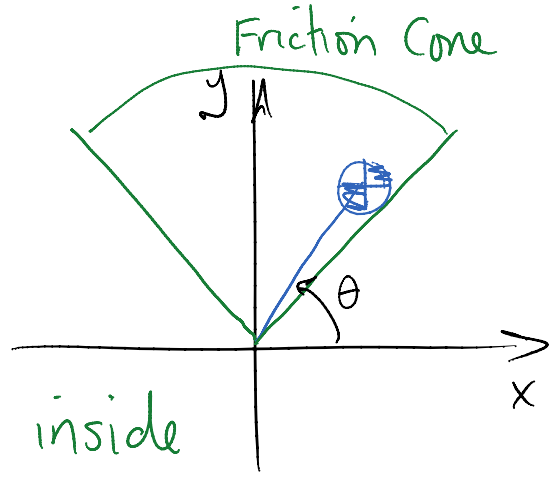
Look for θ such that $\cos(\theta) > 0$

AND $\cos(\theta) - \mu \sin(\theta) < 0$

If $\mu = 1$, $\boxed{\theta = 1.1}$ does the trick.

Now make $b < 0$:

$\boxed{g > 0}$ does it.



More generally, if the c.g. is inside the right half of the friction cone, and the body is translating to the left, there is no finite solution for λ_n! The acceleration-level model breaks down. A velocity-level model solves this problem.