

LCP1_soln

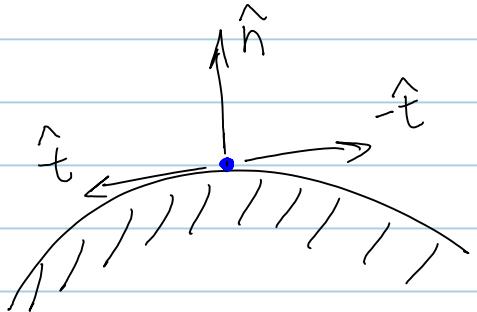
Thursday, February 14, 2008
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1. Consider a particle in the plane moving in contact with a fixed obstacle.

Determine the physical interpretations (sliding left, sliding right, sticking, degenerate sliding, degenerate sticking) of the eight generic solutions of the friction model given by:

$$0 \leq \lambda_f \perp G_f^\top v + 1s \geq 0$$

$$0 \leq s \perp \mu \lambda_n - 1^\top \lambda_f \geq 0$$

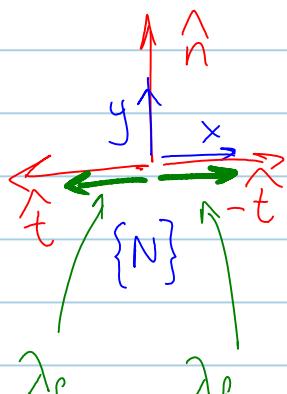


where λ_{f_1} is in the t direction

λ_{f_2} is in the $-t$ direction

$$G_f = \begin{bmatrix} -1 & 1 \\ 0 & 0 \end{bmatrix} \quad v = \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix}$$

Find compl. conditions



Expand comple. conditions

$$\begin{array}{c|c} & | \\ \lambda_{f_1} & \lambda_{f_2} \end{array}$$

$$0 \leq \lambda_{f_1} \perp \dot{x} + s \geq 0$$

$$0 \leq \lambda_{f_2} \perp \dot{x} + s \geq 0$$

$$0 \leq s \perp \mu \lambda_n - \lambda_{f_1} - \lambda_{f_2} \geq 0$$

(Assume $\mu \lambda_n$ given & positive)

Approach. For each case :

- 1.) Choose 3 equations and solve to the extent possible
- 2.) Substitute solution into complementary inequalities and check for consistency (i.e. feasibility).

If solution implies one or more inequalities must be satisfied as equalities, then that case is mathematically degenerate.

Case 1:

$$\begin{cases} \dot{x} = s \\ \dot{x} = -s \end{cases} \Rightarrow s = \dot{x} = 0 \Rightarrow \text{sticking}$$

3 cases

$$\begin{aligned} \mu \lambda_n - \lambda_{f_1} - \lambda_{f_2} &= 0 \\ \lambda_{f_1}, \lambda_{f_2} &\geq 0 \end{aligned} \quad \left. \right\} \Rightarrow \begin{array}{l} \text{contact} \\ \text{force in} \\ \text{cone.} \end{array}$$

$s \geq 0$

CASE 1 is inconsistent, notice $\lambda_{f_1} + \lambda_{f_2} = \mu \lambda_n$

Case #	Left	Right
1	0	0
2	0	+

$S \geq 0$ CASE 1 is inconsistent,
 but degenerate case
 $\begin{bmatrix} + & 0 \\ + & 0 \\ 0 & 0 \end{bmatrix}$ is NOT. \therefore
DEGENERATE STICKING

notice different form
 but effectively same constraint

\angle	+	+
0	0	+
3	0	0
4	0	0
5	0	0
6	0	+
7	0	+
8	0	+

Case 2:

$$\begin{cases} \dot{x} = s \\ \dot{x} = -s \end{cases} \Rightarrow \dot{x} = s = 0 \Rightarrow \text{sticking}$$

$$\begin{cases} \mu \lambda_n - \lambda_{f_1} - \lambda_{f_2} \geq 0 \\ \lambda_{f_1}, \lambda_{f_2} \geq 0 \end{cases} \Rightarrow \text{contact force in cone}$$

STICKING

Case 3:

$$\begin{array}{l|l}
\begin{array}{l}
\dot{x} = s \\
\lambda_{f_2} = 0 \\
\mu \lambda_n - \lambda_{f_1} = 0
\end{array} &
\begin{array}{l}
\lambda_{f_1} \geq 0 \quad \leftarrow \text{valid} \\
\dot{x} \geq -s \\
s \geq 0
\end{array} \Rightarrow \dot{x} \geq 0
\end{array}$$

SLIDING RIGHT

Case 4:

$$\begin{cases} \dot{x} = s \\ \lambda_{f_1} = 0 \end{cases} \Rightarrow \text{sticking} \dots$$

The inequalities

$$\begin{array}{l}
\lambda_{f_1} \geq 0 \\
\dot{x} \geq s
\end{array}$$

$$\begin{array}{l} \dot{x} = 0 \\ \lambda_{f_2} = 0 \\ s = 0 \end{array} \left\{ \begin{array}{l} \Rightarrow \text{sticking} \\ \text{with } \lambda_{f_1} = 0 \end{array} \right.$$

$$\begin{array}{l} \lambda_{f_1} = 0 \\ \dot{x} \geq s \\ \mu \lambda_n - \lambda_{f_1} \geq 0 \end{array}$$

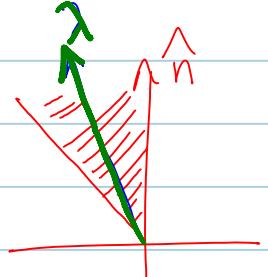
DEGENERATE STICKING

Equations

force $\dot{x} \geq s$ to be satisfied strictly, i.e. $\dot{x} = s = 0$

$$\therefore \begin{vmatrix} + & 0 \\ 0 & + \end{vmatrix} \Rightarrow \begin{vmatrix} + & 0 \\ 0 & + \end{vmatrix}$$

but with contact force limited to half of friction cone



Case 5:

$$\begin{array}{l} \lambda_{f_1} = 0 \\ \dot{x} = -s \\ \mu \lambda_n - \lambda_{f_2} = 0 \end{array}$$

$$\begin{array}{l} \text{Ineqs.} \\ -\dot{x} \geq -s \\ \lambda_{f_2} \geq 0 \\ s \geq 0 \end{array}$$

SLIDING LEFT

Case 6:

$$\lambda_{f_1} = 0$$

$$\begin{array}{l} \dot{x} = -s \\ s = 0 \end{array} \Rightarrow \dot{x} = 0$$

$$-\dot{x} \geq -s$$

$$\lambda_{f_2} \geq 0$$

$$\mu \lambda_n - \lambda_{f_2} \geq 0$$

DEGENERATE STICKING



STICKING

Degeneracy is analogous to

that of Case 4:

$$s = \dot{x} = 0 \quad \begin{vmatrix} 0 & + \\ + & 0 \\ 0 & + \end{vmatrix} \Rightarrow \begin{vmatrix} 0 & 0 \\ + & 0 \\ 0 & + \end{vmatrix}$$

$$\Rightarrow \ddot{x} = -s. \quad -\dot{x} > -s \text{ is not possible}$$

Case 7:

$$\left. \begin{array}{l} \lambda_{f_1} = 0 \\ \lambda_{f_2} = 0 \\ \mu \lambda_n - \lambda_{f_1} - \lambda_{f_2} = 0 \end{array} \right\} \Rightarrow \lambda = 0 \quad \left. \begin{array}{l} \text{Ineqs.} \\ -\dot{x} \geq -s \\ \dot{x} \geq -s \\ s \geq 0 \end{array} \right\} -s \leq \dot{x} \leq s$$

DEGENERATE SLIDING

sliding is maintained
without compressive force!

Not mathematically degenerate, since $\begin{vmatrix} 0 & + \\ 0 & + \\ + & 0 \end{vmatrix}$ is possible.

The degeneracy is physical.

Case 8:

$$\left. \begin{array}{l} \lambda_{f_1} = 0 \\ \lambda_{f_2} = 0 \\ s = 0 \end{array} \right| \quad \begin{array}{l} \text{ineqs} \\ -\dot{x} \geq -s \\ \dot{x} \geq -s \\ \mu \lambda_n - \lambda_{f_1} - \lambda_{f_2} \geq 0 \end{array} \Rightarrow \begin{array}{c} \overset{0}{-\dot{x}} \leq \overset{0}{\dot{x}} \leq \overset{0}{s} \\ \Rightarrow \dot{x} = 0 \end{array}$$

$$\Rightarrow \lambda_n \geq 0$$

DEGENERATE

friction is not used

DEGENERATE STICKING

friction is not used
to prevent sliding!

Mathematically degenerate because choosing $\lambda_{f_1} = \lambda_{f_2} = s = 0$

$$\Rightarrow \dot{x} = 0$$

\therefore case

solution is

$$\begin{vmatrix} 0 & 0 \\ 0 & 0 \\ 0 & + \end{vmatrix}$$