

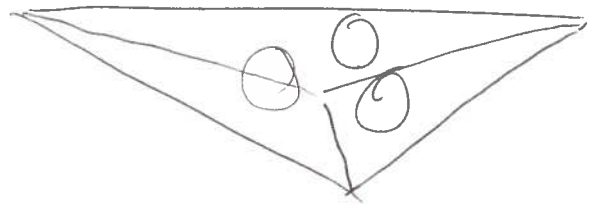
LCP Homework

2/23/12

①

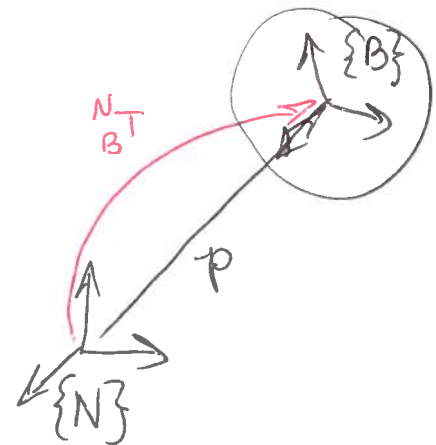
Simulate 3 spheres of different radii & mass properties in a 3-sided pyramidal depression / bowl.

Toss the balls into the bowl and plot forces, positions, velocities, distances, and sliding indicator variables over time.



Use ^{unit} \hat{q} quaternions
(a.k.a. Euler parameters)

Let \hat{k}, ϕ be axis-angle representation



$${}^N_B R = \begin{bmatrix} 1 - 2(e_2^2 + e_3^2) & 2(e_1 e_2 - e_3 e_0) & 2(e_1 e_3 + e_2 e_0) \\ 2(e_1 e_2 + e_3 e_0) & 1 - 2(e_1^2 + e_3^2) & 2(e_2 e_3 - e_1 e_0) \\ 2(e_1 e_3 - e_2 e_0) & 2(e_2 e_3 + e_0 e_1) & 1 - 2(e_2^2 + e_1^2) \end{bmatrix}$$

2/23/12

(2)

where

$$e_0 = \cos(\phi/2)$$

$$e_1 = k_1 \sin(\phi/2)$$

$$e_2 = k_2 \sin(\phi/2)$$

$$e_3 = k_3 \sin(\phi/2)$$

For each body:

You will also need the matrix V (in $\dot{u} = V \dot{u}$)

(7x1) (7x6) (6x1)

$$V_i = \begin{bmatrix} I_{3 \times 3} & 0 \\ 0 & {}^N B_i \end{bmatrix}$$

$$\text{where } {}^B B_i = \frac{1}{2} \begin{bmatrix} -e_1 & -e_2 & -e_3 \\ e_0 & -e_3 & e_2 \\ e_1 & e_0 & -e_1 \\ -e_2 & e_3 & e_0 \end{bmatrix} {}^B R_i$$

$$u_i = \begin{bmatrix} x \\ y \\ z \\ e_0 \\ e_1 \\ e_2 \\ e_3 \end{bmatrix}_i ; i=1,2,3$$

$$u = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$$

$$V = \text{blockdiag}(V_1, V_2, V_3)$$

Simulation Approach

2/23/12
(3)

