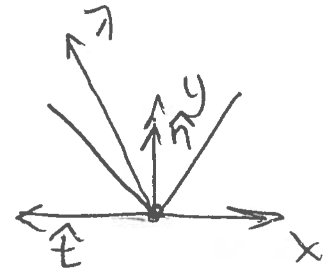
Add Coulomb Friction

Planar Case

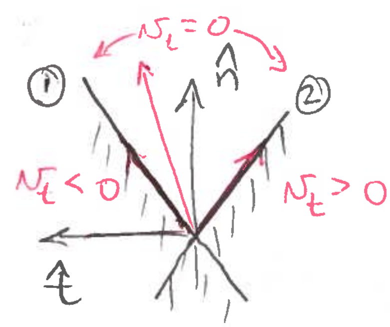
body vel $\quad \nu = \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} \qquad \lambda = \begin{bmatrix} \lambda_n \\ \lambda_t \end{bmatrix}$



contact vel $\quad v = \begin{bmatrix} v_n \\ v_t \end{bmatrix}$

Coulomb $\quad -\mu\lambda_n \leq \lambda_t \leq \mu\lambda_n \implies \lambda_n \geq 0$

Complementarity for friction:



(1) $\quad \mu\lambda_n - \lambda_t \geq 0$

(2) $\quad \mu\lambda_n + \lambda_t \geq 0$

if $v_t = 0$, (1) & (2) hold

if $v_t > 0$, (2) holds w/ equality

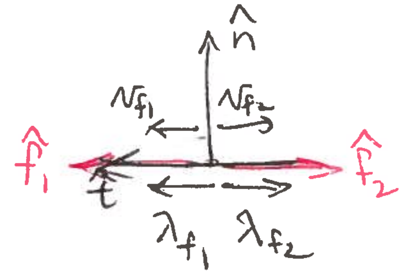
if $v_t < 0$, (1) holds w/ equality

$$0 \leq \mu\lambda_n + \lambda_t \perp v_t \geq 0$$

$$0 \leq \mu\lambda_n - \lambda_t \perp (-v_t) \geq 0$$

Convenient to introduce (+)ive & (-)ive parts of variables

$$\left.\begin{aligned} \lambda_t &= \lambda_{f_1} - \lambda_{f_2} \\ 0 \le \lambda_{f_1} &\perp \lambda_{f_2} \ge 0 \end{aligned}\right\} (1)$$



$$\boxed{\begin{aligned} \psi_n &= 0 \\ v_n &= 0 \end{aligned}}$$

$$\left.\begin{aligned} v_t &= v_{f_1} - v_{f_2} \\ 0 \le v_{f_1} &\perp v_{f_2} \ge 0 \end{aligned}\right\} (2)$$

Coupling across these conditions:

$$v_{f_1} > 0 \implies \lambda_{f_2} = \mu \lambda_n , \ \lambda_{f_1} = 0 , \ v_{f_2} = 0$$

$$v_{f_2} > 0 \implies \lambda_{f_1} = \mu \lambda_n , \ \lambda_{f_2} = 0 , \ v_{f_1} = 0$$

$$v_{f_1} = v_{f_2} = 0 \implies (\lambda_{f_1}, \lambda_{f_2}) \text{ in cone}$$

Introduce "slack" variable that is a sliding indicator

$s \ge 0 \longrightarrow$ $s > 0 \implies$ sliding $\implies$ cone bndry
$s = 0 \implies$ sticking $\implies$ inside cone.

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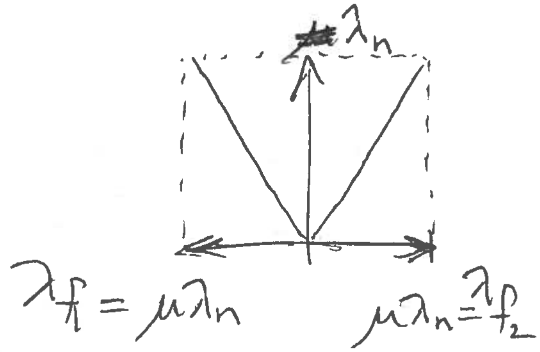
Cone constraint:

$$0 \le s \perp \mu\lambda_n - \lambda_{f_1} - \lambda_{f_2} \ge 0 \qquad (3)$$

Now, how do we make conditions to detect if v_{f_1} or $v_{f_2} \neq 0$?



Try using the coupling conditions:

$$0 \le \begin{bmatrix} \lambda_{f_1} \\ \lambda_{f_2} \end{bmatrix} \perp \begin{bmatrix} v_{f_1} \\ v_{f_2} \end{bmatrix} \ge 0$$

$\Longleftarrow$ Doesn't help. We don't involve s ~~or have~~ ~~v_{f_1} v_{f_2}~~

$$0 \le \begin{bmatrix} \lambda_{f_1} \\ \lambda_{f_2} \end{bmatrix} \perp \begin{bmatrix} \hat{f}_1^T v \\ \hat{f}_2^T v \end{bmatrix} \ge 0$$

if $\hat{f}_1^T v > 0$, then $\lambda_{f_1} = 0$. Good

But $\hat{f}_2^T v < 0$ which violates ≥ 0!

Put the slip indicator in!

$$0 \leq \begin{bmatrix} \lambda_{f_1} \\ \lambda_{f_2} \end{bmatrix} \perp \begin{bmatrix} \hat{f}_1^T \nu + s \\ \hat{f}_2^T \nu + s \end{bmatrix} \geq 0 \qquad (4)$$

Suppose $\overbrace{\hat{f}_1^T \nu}^{\nu_{f_1}} > 0 \Rightarrow \lambda_{f_1} = 0$

But we also want $\lambda_{f_2} = \mu \lambda_n > 0$ $\therefore$

$\therefore \hat{f}_2^T \nu + s = 0 \Rightarrow s = -\hat{f}_2^T \nu = \hat{f}_1^T \nu = \nu_{f_1} > 0$

But s could be bigger, then $\lambda_{f_1} = \lambda_{f_2} = 0$.

Condition (3) to the rescue

$$0 \leq s \perp \mu \lambda_n - \lambda_{f_1} - \lambda_{f_2} \geq 0 \qquad (3)$$

if $s > 0$, $\mu \lambda_n = \lambda_{f_1} + \lambda_{f_2} > 0$, so

λ_{f_1} and λ_{f_2} cannot be zero simultaneously

$\therefore$ s must be the smallest value possible to satisfy (4).

if $s > 0$, s = slip speed, and only one of $\lambda_{f_i} > 0$

$s = 0$, and both $\lambda_{f_i} > 0$.