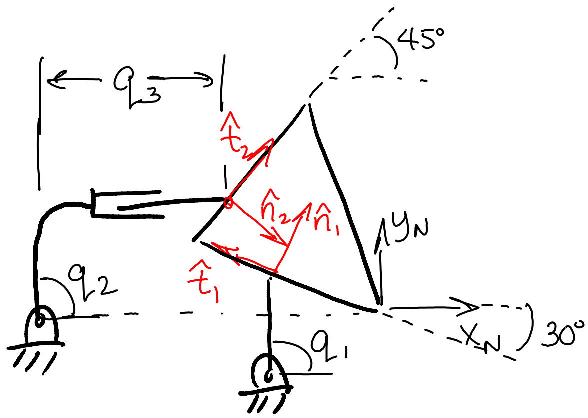
Soln. in-class

Monday, January 26, 2009

10:12 PM

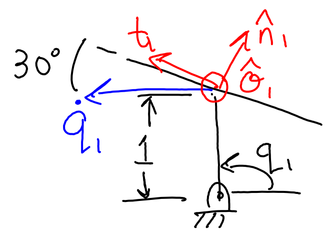
Assume 2 HF contacts

Construct G & J directly in 2D.



$$\underline{L\tilde{J}_1}\dot{q} = \begin{bmatrix} \nu_{1n} \\ \nu_{1t} \\ \omega_{1o} \end{bmatrix} = \begin{bmatrix} -\tfrac{1}{2}\dot{q}_1 \\ +\tfrac{\sqrt{3}}{2}\dot{q}_1 \\ \dot{q}_1 \end{bmatrix}$$

2D Jacobian



$$\Longrightarrow \quad \underline{L\tilde{J}_1} = \begin{bmatrix} -\tfrac{1}{2} & 0 & 0 \\ \tfrac{\sqrt{3}}{2} & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

2D Jacobian

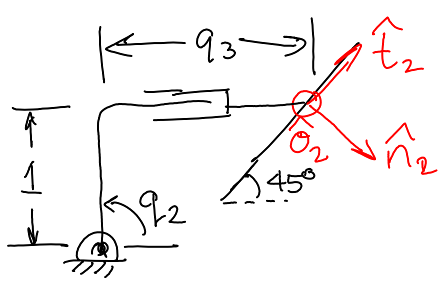
$$H_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

Use 2D selection matrix for HF contact

$$J_1 = H_1 L\tilde{J}_1 = \boxed{\begin{bmatrix} -\tfrac{1}{2} & 0 & 0 \\ \tfrac{\sqrt{3}}{2} & 0 & 0 \end{bmatrix} = J_1}$$

$$\underline{L\tilde{J}_2} = \begin{bmatrix} \nu_{2n} \\ \nu_{2t} \\ \omega_{2o} \end{bmatrix}$$

2D Jacobian



$$\begin{bmatrix} -\tfrac{\sqrt{2}}{2}\dot{q}_2 - q_3\tfrac{\sqrt{2}}{2}\dot{q}_2 \qquad \tfrac{\sqrt{2}}{2}\dot{q}_3 \end{bmatrix}$$

$$\underline{\underline{L}}\tilde{J}_2\dot{q} = \begin{bmatrix} \cdots & \cdots \\ -\sqrt{2}/2\ \dot{q}_2 + q_3\sqrt{2}/2\ \dot{q}_2 & \sqrt{2}/2\ \dot{q}_3 \\ \dot{q}_2 & 0 \end{bmatrix}$$

$$\Rightarrow \underline{\underline{L}}\tilde{J}_2 = \begin{bmatrix} 0 & -\frac{\sqrt{2}}{2}(1+q_3) & \sqrt{2}/2 \\ 0 & -\frac{\sqrt{2}}{2}(1-q_3) & \sqrt{2}/2 \\ 0 & 1 & 0 \end{bmatrix}$$

$$H_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

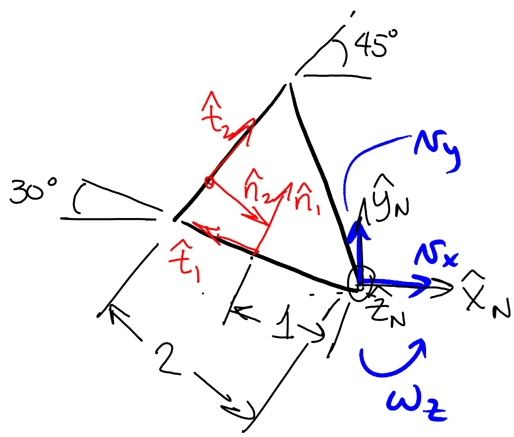
2D selection matrix

$$\Rightarrow \boxed{J_2 = \begin{bmatrix} 0 & -\frac{\sqrt{2}}{2}(1+q_3) & \sqrt{2}/2 \\ 0 & -\frac{\sqrt{2}}{2}(1-q_3) & \sqrt{2}/2 \end{bmatrix}}$$

$$J = \begin{bmatrix} J_1 \\ J_2 \end{bmatrix}_{(4 \times n_q)}$$

Now construct G_1^T & G_2^T

Given $\nu = \begin{bmatrix} \nu_x \\ \nu_y \\ \omega_z \end{bmatrix}$



Consider effects of ν_x, ν_y, ω_z separately on ν_1 & ν_2. Build $L\tilde{G}_1^T$ & $L\tilde{G}_2^T$ analogously to $L\tilde{J}_1$ & $L\tilde{J}_2$

$$L\tilde{G}_1^T = \begin{bmatrix} v_{1n} \\ v_{1t} \\ \omega_{1\theta} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} v_x & \frac{\sqrt{3}}{2} v_y & -\omega_z \\ -\frac{\sqrt{3}}{2} v_x & \frac{1}{2} v_y & 0 \\ 0 & 0 & \omega_z \end{bmatrix}$$

2D Jacobian

$$\Longrightarrow \boxed{L\tilde{G}_1^T = \begin{bmatrix} 1/2 & \sqrt{3}/2 & -1 \\ -\sqrt{3}/2 & 1/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}}$$

use 2D selecction matrix

$H_1 L\tilde{G}_1^T$ = first two rows of $L\tilde{G}_1^T$

$$\text{Now } L\tilde{G}_2^T = \begin{bmatrix} \sqrt{2}/2\, v_x & -\frac{\sqrt{2}}{2} v_y & 0 \\ \sqrt{2}/2\, v_x & \sqrt{2}/2\, v_y & -2\omega_z \\ 0 & 0 & \omega_z \end{bmatrix}$$

$$H_2 = H_1 \Longrightarrow \boxed{H_2 L\tilde{G}_2^T = \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & -2 \end{bmatrix} = G_2^T}$$

$$G^T = \begin{bmatrix} G_1^T \\ G_2^T \end{bmatrix}_{(4\times 3)}$$

Could also construct $\tilde{J}_1$, $\tilde{J}_2$, $\tilde{G}_1^T$, $\tilde{G}_2^T$ in 3D, then apply wrench transmission matrices H_1 & H_2 then drop to 2D by eliminating out-of-plane components.

Here are G_1^T and G_2^T in 3D.

$$G_i^T = \begin{bmatrix} \hat{n}_i^T & (r_i \times \hat{n}_i)^T \\ \hat{t}_i^T & (r_i \times \hat{t}_i)^T \\ \hat{o}_i^T & (r_i \times \hat{o}_i)^T \end{bmatrix}_{(3\times 6)} = H_i \tilde{G}_i^T \qquad i = 1, 2$$

$$r_1 \triangleq \begin{bmatrix} -\sqrt{3}/2 \\ 1/2 \\ 0 \end{bmatrix} \qquad r_2 \triangleq \begin{bmatrix} -\sqrt{2} \\ \sqrt{2} \\ 0 \end{bmatrix}$$

Column Labels

$$L\tilde{G}_1^T = G_1^T = \begin{array}{c} \begin{array}{cccccc} v_x & v_y & v_z & \omega_x & \omega_y & \omega_z \end{array} \\ \begin{bmatrix} 1/2 & \sqrt{3}/2 & 0 & 0 & 0 & -1 \\ -\frac{\sqrt{3}}{2} & 1/2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \end{array} \begin{array}{l} v_{1n} \\ v_{1t} \\ v_{1o} \end{array}$$

Row Labels

$$L\tilde{G}_2^T = G_2^T = \begin{bmatrix} \sqrt{2}/2 & -\sqrt{2}/2 & 0 & 0 & 0 & 0 \\ \sqrt{2}/2 & \sqrt{2}/2 & 0 & 0 & 0 & -2 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

Now drop to 2D be eliminating out-of-plane information

$$(G_2^T)_{2D} = \begin{bmatrix} \sqrt{2}/2 & -\sqrt{2}/2 & 0 \\ \sqrt{2}/2 & \sqrt{2}/2 & -2 \end{bmatrix}$$

What about doing the same thing with J_2 ?

Column Labels

$$
L\tilde{J}_2 = J_2 = \begin{array}{c|l}
\begin{array}{ccc} \dot{q}_1 & \dot{q}_2 & \dot{q}_3 \end{array} & \\
\begin{bmatrix} 0 & -\sqrt{2}/2\,(1+q_3) & \sqrt{2}/2 \\ 0 & -\sqrt{2}/2\,(1-q_3) & \sqrt{2}/2 \\ 0 & 0 & 0 \end{bmatrix} & \begin{array}{l} N_{2n} \\ N_{2t} \\ N_{2o} \end{array}
\end{array}
$$

Row Labels

Eliminate out-of-plane stuff

$$
(J_2)_{2D} = \begin{bmatrix} 0 & -\frac{\sqrt{2}}{2}(1+q_3) & \frac{\sqrt{2}}{2} \\ 0 & -\frac{\sqrt{2}}{2}(1-q_3) & \frac{\sqrt{2}}{2} \end{bmatrix}
$$