Kin. Models of Grasping

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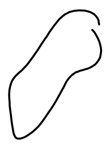
Kinematic contact models of grasps are equivalent to adding constraints to reduce # of d.o.f.

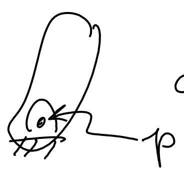
Consider d.o.f. in planar systems first

How many d.o.f. for one free body?



d.o.f. = 3 (in plane)

Now add a revolute joint at point p and fix to ground at origin of $\{N\}$

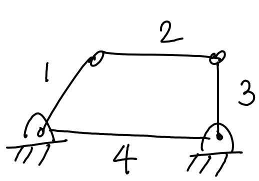


d.o.f. = 1

2 constraint equations $p_x = 0$
$p_y = 0$

3 d.o.f. + 2 constrs = 1 d.o.f.

Consider 4-bar linkage



d.o.f.

(3 dof) × (4 bodies) = 12 dof

Constraints:

Connection between links 1 & 2 $\left.\begin{aligned}(J_{11})_x &= (J_{12})_x \\ (J_{11})_y &= (J_{12})_y\end{aligned}\right\}$ 2 constraints

There are 4 joint connections $\Rightarrow$ 8 constraints

One link is fixed to ground $\Rightarrow$ 3 constraints

$$12 \text{ d.o.f.} + 11 \text{ constr} = 1 \text{ d.o.f.}$$

What happens in 3D systems?

Each body starts with 6 d.o.f.

Each joint allowing 1 relative d.o.f. imposes 5 constr.

Grüblers Formula

$$\boxed{\underline{\text{spatial}} \qquad F \geq 6(n_b - n_{jnt} - 1) + \sum_{j=1}^{n_{jnt}} n_{f_j}}$$

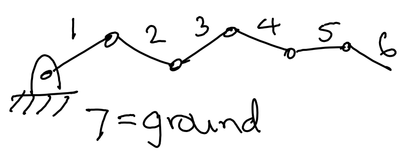
$$\boxed{\underline{\text{planar}} \qquad F \geq 3(n_b - n_{jnt} - 1) + \sum_{j=1}^{n_{jnt}} n_{f_j}}$$

n_b = # of bodies counting ground

n_{jnt} = # of joints

n_{f_j} = # of freedoms afforded to each joint

Consider a Puma robot



$n_{jnt} = 6$

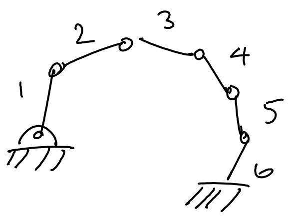
$n_b = 7$

$n_{f_j} = 1 \quad \forall j$

$$F \geq 6(7-6-1) + 6 = 6$$

$$\boxed{F = 6}$$

Suppose we fix the end effector to ground?



Now link 6 is part of the ground link $\Rightarrow n_b = 6$

$$\therefore\ F = 6(6-6-1)+6 = 0$$

$$\boxed{F=0}$$

Does this make sense?

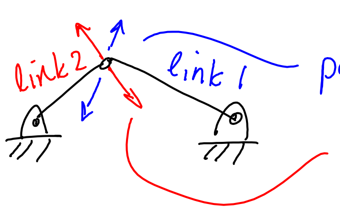
Yes, because inverse kinematic solutions of the Puma are isolated!

Why is Grübler's formula stated as an inequality?

(1.) It can yield negative numbers for F.

$$F<0 \equiv F=0$$

(2.) In some singular configurations, extra d.o.f.s are possible.



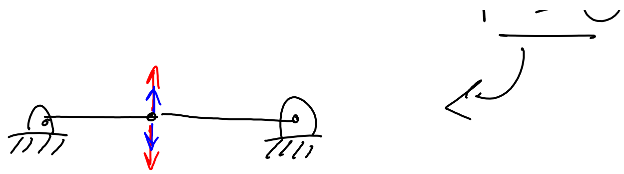
possible velocities of tip of link 1

possible vel. of tip of link 2

The instantaneous velocities only match when they are zero. $\therefore$ d.o.f. $= 0$

$$F \geq 3(3-3-1)+3 = 0$$

Same inequality applies if we spread the base of the 3-bar, i.e. $\underline{F \geq 0}$



Now instantaneous velocities of joint centers on links line up. $\therefore$ d.o.f $= 1$

Easiest to think about contact modeling with the relative velocity at the contact in $\{C_i\}$.

Relative velocity $\triangleq$ $\quad \nu_{cc} = \nu_{c,hnd} - \nu_{c,obj}$

At contact i $\quad \nu_{cc,i} = \nu_{i,hnd} - \nu_{i,obj}$

Kinematic contact modeling adds constraints

$$\nu_{cc,i} = \begin{bmatrix} v_{in} \\ v_{it} \\ v_{io} \\ \omega_{in} \\ \omega_{it} \\ \omega_{io} \end{bmatrix}$$

relative velocities

setting $v_{in} = 0 \implies$
can't lose contact
can't penetrate
i.e. contact is maintained!

Also setting $v_{it} = v_{io} = 0 \implies$
contact points stick together, i.e. contact acts like spherical joint!

Also setting $\omega_{in} = 0 \implies$
contact points can't twist about contact normal.

3 Standard models: PwoF, HF, SF

PwoF – Point without Friction

- set $v_{in} = 0$
- let other rel. vel. components be free
- remove 1 d.o.f. from system
- enforce 1 equation

$$H_i v_{cc,i} = v_{in} = 0$$

where $H_i = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

HF – Hard Finger (Point with Friction):

- set $v_{in} = v_{it} = v_{io} = 0$
- let other rel. vel. components be free
- remove 3 d.o.f. from system
- enforce 3 equations

$$H_i v_{cc,i} = \begin{bmatrix} v_{in} \\ v_{it} \\ v_{io} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

where $H_i = \begin{bmatrix} I_{(3\times3)} & O_{(3\times3)} \end{bmatrix}_{(3\times6)}$

SF – Soft Finger:

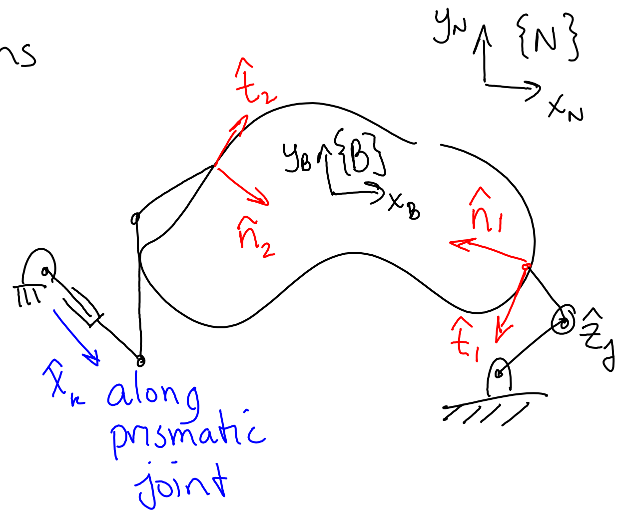
- set $v_{in} = v_{it} = v_{io} = 0 = \omega_{in}$
- let other rel. vel. components be free
- remove 4 d.o.f. from system
- enforce 4 equations

$$H_i \, \nu_{ce,i} = \begin{bmatrix} \nu_{in} \\ \nu_{it} \\ \nu_{io} \\ \omega_{in} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

where $H_i = \begin{bmatrix} I_{(4\times4)} & 0_{(4\times2)} \end{bmatrix}_{(4\times6)}$

Planar Simplifications

Put all
$\hat{x}, \hat{y}$ axes &
$\hat{n}, \hat{t}$ axes
in the plane



Now use a selection matrix L to eliminate unwanted components

$$\nu_i = \begin{bmatrix} \nu_{in} \\ \nu_{it} \\ \cancel{\nu_{io}} \\ \cancel{\omega_{in}} \\ \cancel{\omega_{it}} \\ \omega_{io} \end{bmatrix} \text{eliminate} \implies L\nu_i = \begin{bmatrix} \nu_{in} \\ \nu_{it} \\ \omega_{io} \end{bmatrix}$$

$$\text{where } L = \left[\begin{array}{cc:c:c} 1 & 0 & & 0 \\ 0 & 1 & 0_{3\times3} & 0 \\ 0 & 0 & & 1 \end{array}\right]_{(6\times3)}$$

$$L\tilde{J}_i = \begin{bmatrix} v_{in} \\ v_{it} \\ \omega_{io} \end{bmatrix}_{hnd} = \nu_{i,hnd} \qquad \nu_{i,obj} = \begin{bmatrix} v_{in} \\ v_{it} \\ \omega_{io} \end{bmatrix}_{obj} = L\tilde{G}_i^T$$

Now with the smaller Jacobian $(L\tilde{J}_i)_{(3 \times n_q)}$ & $(L\tilde{G}_i^T)_{(3\times 3)}$ we must still select transmitted components of the contact twists.

$$H_i \begin{bmatrix} v_{in} \\ v_{it} \\ \omega_{io} \end{bmatrix} \Longrightarrow J_i = H_i L\tilde{J}_i \qquad G_i^T = H_i L\tilde{G}_i^T$$

<u>PwoF</u>: $v_{in} = 0 \Longrightarrow H_i = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$

<u>HF</u>: $v_{in} = v_{it} = 0 \Longrightarrow H_i = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

<u>SF</u>: soft finger makes no sense, since can't rotate about $\hat{n}_i$

Kinematic Grasp Modeling Process

1. Choose contact types that are reasonable for your situation
2. Enforce contact types by

$$H_i \nu_{cc,i} = 0 \quad \forall\, i$$

$(l_i \times 6)$ where l_i = # of d.o.f. removed = # of constraint eqs.

Expand

$$H_i(\nu_{i,hnd} - \nu_{i,obj}) = 0$$

$$H_i \begin{bmatrix} \tilde{J}_i & -\tilde{G}_i^T \end{bmatrix} \begin{bmatrix} \dot{q} \\ \nu \end{bmatrix} = 0$$

$$\begin{bmatrix} H_i\tilde{J}_i & -H_i\tilde{G}_i^T \end{bmatrix} \begin{bmatrix} \dot{q} \\ \nu \end{bmatrix} = 0$$

$$\begin{bmatrix} J_i & -G_i^T \end{bmatrix} \begin{bmatrix} \dot{q} \\ \nu \end{bmatrix} = 0$$

where $J_i = H_i \tilde{J}_i$

$\tilde{G}_i^T = H_i \tilde{G}_i^T$

Put all contacts together. Select constrained d.o.f.s from all contacts in one big eq.

$$H(\nu_{c,hnd} - \nu_{c,obj}) = 0$$

where $H = \text{diag}(H_1, H_2, \cdots, H_{n_c})$

$$H \begin{bmatrix} \tilde{J} & -\tilde{G}^T \end{bmatrix} \begin{bmatrix} \dot{q} \\ \nu \end{bmatrix} = 0$$

$$\begin{bmatrix} J & -G^T \end{bmatrix} \begin{bmatrix} \dot{q} \\ \nu \end{bmatrix} = 0$$

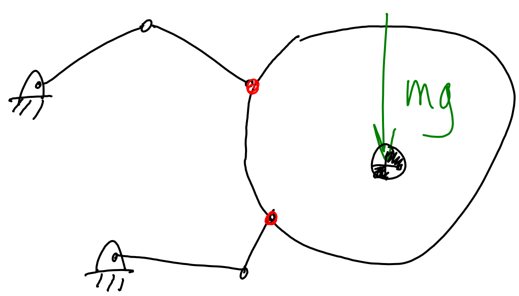
where $J = H\tilde{J}$ & $G^T = H\tilde{G}^T$

Physical Interpretation

$J_i \dot{q} = \nu_{i,hnd}$ = twist components transmitted to object by hand at contact i under contact model choice.

$G_i^T \nu = \nu_{i,obj}$ = twist components transmitted to hand by object at contact i under contact model choice.

Some grasping examples



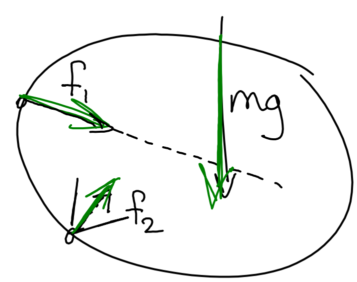
HF, HF – palming a Bball

Our kinematic model will enforce contact maintenance.

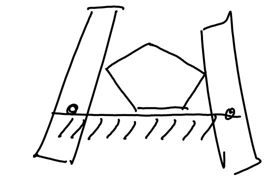
In a real system, contact constraints can be maintained by controller if friction high enough.

What if we change one contact model to PwoF ?

Then equilibrium will not be satisfied, but kinematic model would enforce contact constraints anyway



No way to satisfy equilib. Look at horizontal comp. of contact forces.



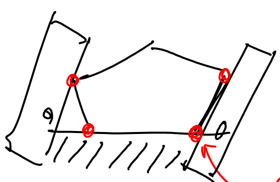
$n_b = 4$

$n_{jnt} = 6 = 2$ fngr & 4 contacts

Assume PwoF $\Rightarrow$ $F \geq 3(4-6-1) + 2(1) + 4(2)$

$F \geq -9 + 10 = 1$

What if system slides until there is a 5th cnt?



$F \geq 3(4-7-1) + 2(1) + 5(2)$

count twice
(on finger & palm)

$F \geq -12 + 12 = 0$

Planning & control are easier with few d.o.f., that's why TAMU dexterous hand/object interface was teflon.

What if we assume HF contacts?

$F \geq 3(4-6-1) + 2(1) + 4(1) = -1$

Then $F = 0$