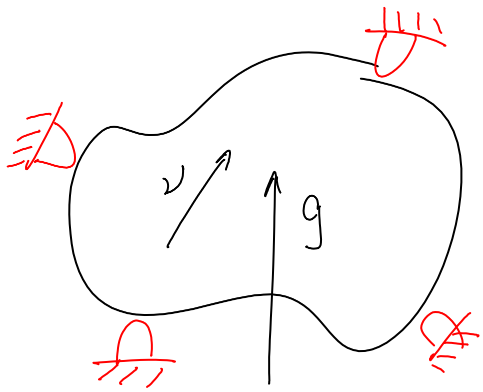
Form Closure: First-Order

Thursday, January 31, 2008
9:12 AM

Definition 1: a grasp has <u>form closure</u> iff it cannot be moved when the fingers are locked



$$\boxed{G_n^T \nu \geq 0 \Rightarrow \nu = 0}$$

Definition 2: a grasp has <u>form closure</u> iff it is possible for frictionless fingers to generate any wrench.

~~Will use~~ this for computation

$$\boxed{\left.\begin{aligned} G_n \lambda_n &= g \\ \lambda_n &\geq 0 \end{aligned}\right\} \forall g \in \mathbb{R}^{n_\nu}} \equiv \boxed{\exists\, \lambda_n > 0 \ni G\lambda_n = 0}$$

Necessary condition $\operatorname{rank}(G_n) = n_\nu$!

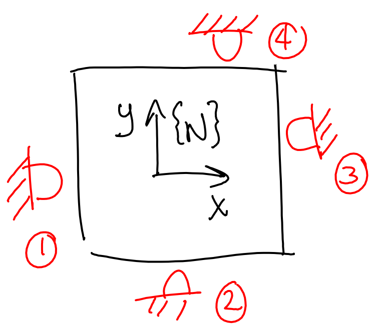
Necessary condition $n_c \geq n_\nu + 1$

$\therefore$ Need at least $\boxed{7}$ contacts to form close a $\boxed{3D}$ object.

Need at least $\boxed{4}$ contacts to form close a $\boxed{2D}$ object.

Example: Object in $\mathbb{R}^2$

$$\nu = \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} \Leftarrow \left.\text{assume translation only}\right\} \nu \in \mathbb{R}^2$$



$$G_n = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix}$$

<u>Twist Test</u>

$$G_n^T \nu \geq 0 \Rightarrow \left.\begin{aligned} \dot{x} &\geq 0 \\ \dot{y} &\geq 0 \\ -\dot{x} &\geq 0 \\ -\dot{y} &\geq 0 \end{aligned}\right\} \Rightarrow \begin{aligned} \dot{x} &= 0 \\ \dot{y} &= 0 \end{aligned} \Rightarrow \boxed{\text{Form Closure exists!}}$$

$$G_n^T \nu \geq 0 \Rightarrow \left.\begin{array}{l} \dot{x} \geq 0 \\ \dot{y} \geq 0 \\ -\dot{x} \geq 0 \\ -\dot{y} \geq 0 \end{array}\right\} \Rightarrow \begin{array}{l} \dot{x} = 0 \\ \dot{y} = 0 \end{array} \Rightarrow \boxed{\text{Form Closure exists!}}$$

Wrench Test

$$G_n \lambda_n = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} f_{1n} \\ f_{2n} \\ f_{3n} \\ f_{4n} \end{bmatrix} = \begin{bmatrix} f_{1n} - f_{3n} \\ f_{2n} - f_{4n} \end{bmatrix} = \begin{bmatrix} -F_x \\ -F_y \end{bmatrix} = -g$$

$$f_{1n}, f_{2n}, f_{3n}, f_{4n} \geq 0$$

Note that given any g, we can find $\lambda_n \ni G_n \lambda_n = -g$!

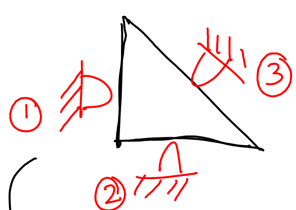
$\boxed{\therefore \text{ Form closure exists.}}$

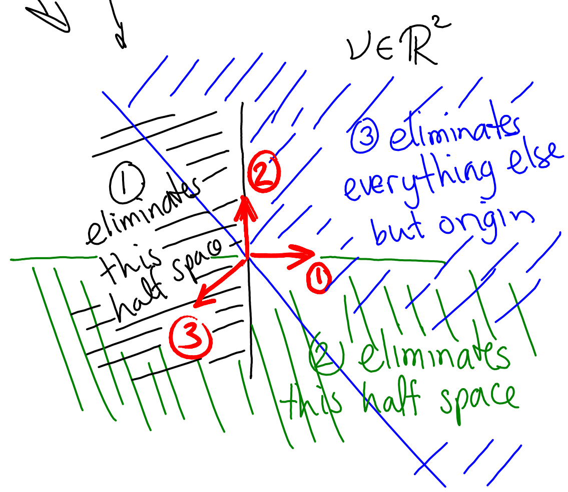
Other interpretations:

Velocity constraints

Each row of $\tilde{G}\nu \geq 0$ eliminates a half-space in the twist space

Assume Triangle can only translate in the plane.



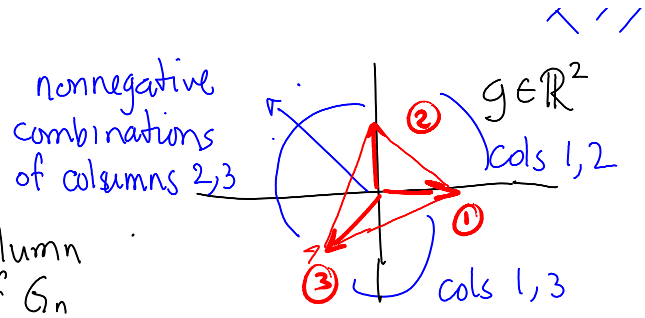


Form closure means that no non-trivial twists are possible!

Wrench generation

Non negative span of

Columns of G_n must equal $\mathbb{R}^{n_w}$.



Example:

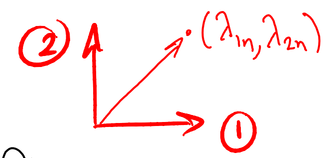
positive span of 1 column of G_n

$G_{1n}\lambda_{1n},\ \lambda_{1n} \geq 0$

this is a ray labeled 1

$\begin{bmatrix}1\\0\end{bmatrix}\lambda_{1n},\ \lambda_{1n} \geq 0.$

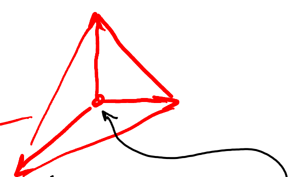
Positive span of 2 columns



$$\left.\begin{matrix}\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}\begin{bmatrix}\lambda_{1n}\\ \lambda_{2n}\end{bmatrix}\\ \lambda_{1n}, \lambda_{2n} \geq 0\end{matrix}\right\} = \text{any point in first quadrant of wrench space}$$

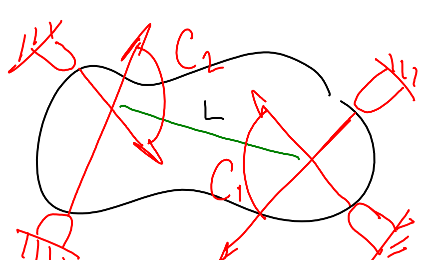
So the positive span includes the two rays and everything between them!

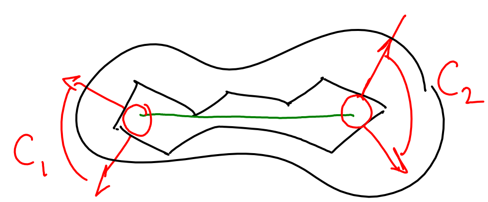
Geometric interpretation in Wrench Space



Convex hull of columns of G_n strictly includes origin.

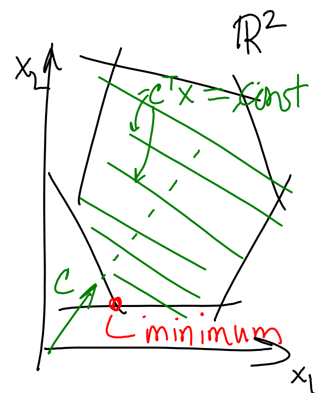
Geometric interpretation in work space (planar only)





If $\exists$ cones, C_1 & C_2, formed with pairs of normals, and line segment L connecting the cone apexes $\ni L$ lies entirely in $C_1 \cap C_2$ or $-C_1 \cap -C_2$, then the grasp has form closure.

Computation Tests for Form Closure.



<u>Linear program</u>

Matlab has good solver.

$$\min_{x} \; c^T x$$

$$\text{s.t.} \quad A x \geq b$$

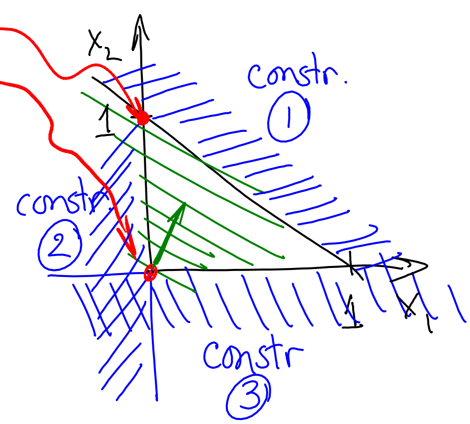
Example:

$$\text{Min} \begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

max

min

$$\text{s.t.} \quad \begin{bmatrix} -1 & -1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \geq \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}$$



Form closure requires $\text{rank}(G) = n_\nu$ <u>and</u> the existence of $\lambda_n > 0 \ni G_n \lambda_n = 0$

<u>Second part</u>

We must satisfy $G_n \lambda_n = 0$

$$\lambda_n > 0$$

How do we "encourage" λ_n to be positive?

$$G_n \lambda_n = 0$$

$$\lambda_n - d \geq 0 \quad \leftarrow \quad \lambda_n \geq 0$$

"slack variable"

$$d \geq 0$$

Maximize the slack variable, d.

L.P.

$$\max_{\lambda_n, d} \; d$$

d is a measure of how far the grasp is from losing form closure

$$\text{s.t.:} \quad G_n \lambda_n = 0$$

$$I \lambda_n - \mathbb{1} d \geq 0$$

$$d \geq 0$$

$$\mathbb{1}^T \lambda_n \leq n_c$$

$\leftarrow$ prevent unboundedness

Test for form closure.

1 $\quad \text{Rank}(G_n) = n_\nu$

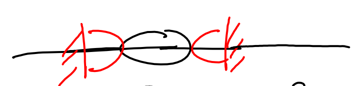
If not, stop. Form closure does not exist.

If yes, continue

2 $\quad$ Compute solution to LP1

If $d^* > 0$, then form closure exists,

Simple example – bead on wire



$$G_n = \begin{bmatrix} 1 & -1 \end{bmatrix}$$

$$\lambda_n = \begin{bmatrix} \lambda_{1n} \\ \lambda_{2n} \end{bmatrix} = \begin{bmatrix} f_{1n} \\ f_{2n} \end{bmatrix}$$

$$\max_{f_{1n}, f_{2n}, d} \; d$$

$$\text{s.t.} \quad f_{1n} - f_{2n} = 0$$

$$f_{1n} - d \geq 0$$

Feasible Solutions

$$f_{2n} - d \geq 0$$
$$d \geq 0$$
$$f_{1n} + f_{2n} \leq 2$$

$f_{1n} = f_{2n}$	domain of d
0	0
0.5	[0, 0.5]
1	[0, 1.0]
~~1.1~~	~~[0, 1.1]~~

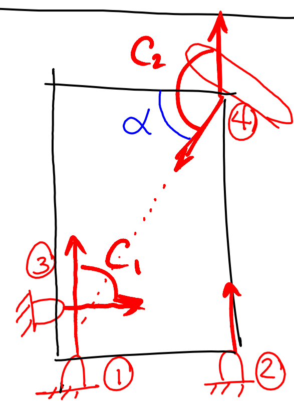
d^*

Without last constraint, f_{in} could increase without bound – infinitely tight squeezing. Form closure requires ability to squeeze, which is evident without letting $d \to \infty$.

Without the bound $(\mathbf{1}^T \lambda_n \leq n_c)$, d^* for all form closure grasps would be ∞ and then d^* could not be used to compare different form closure grasps.

Planar Example

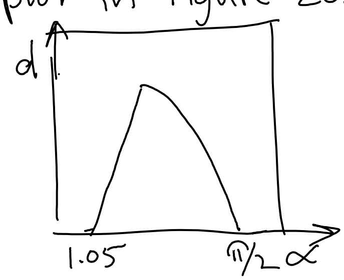
Figure 28.17



Form closure exists for $1.052 < \alpha < \pi/2$

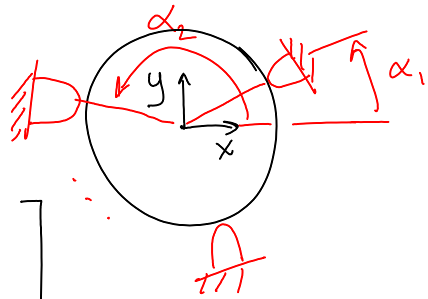
No form closure if $\alpha = \pi/2$

See plot in Figure 28.18



Partial Form Closure – can't prevent all twists, but can prevent a subspace of all twists.

Example: Disk $\longrightarrow$



$$G_n = \begin{bmatrix} -\cos(\alpha_1) & -\cos(\alpha_2) & \cdots \\ -\sin(\alpha_1) & -\sin(\alpha_2) & \cdots \\ 0 & 0 & \cdots \end{bmatrix}$$

$\max(\text{Rank}(G_n)) = 2 \implies$ greatest number of d.o.f. constrained is 2.

For this problem, as long as the angle between each pair of contacts is less than π, the disk is form closed in the (v_x, v_y) subspace of ν.

Can't form close ω_z dimension
with any # of contacts.