TetheredBodies

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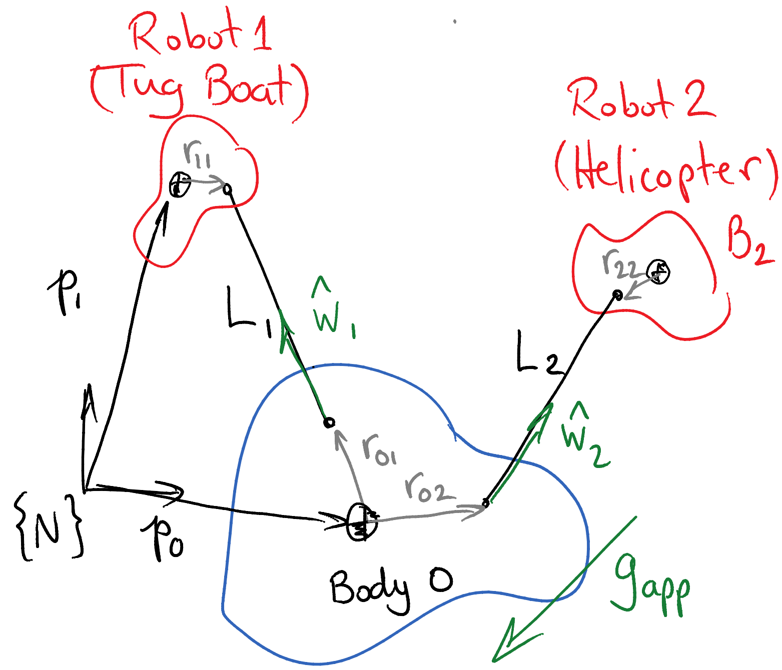
7:27 AM

Body 0 is being towed by cables.



p_i = position of c.g. of body i

$\hat{w}_i$ = unit vector in direction of wire i

L_i = wire length

r_{ij} = vector from c.g. of body j to attachment point of wire i.

Assume wire is massless and inextensible:

$$\|(p_i + r_{ij}) - (p_k + r_{kj})\| \leq L_j \quad \forall j$$

$$\psi_{jw} = L_j - \|(p_i + r_{ij}) - (p_k + r_{kj})\| \geq 0$$

Dynamics of B_0

$$\begin{bmatrix} \hat{w}_1 & \cdots \end{bmatrix}$$

Dynamics of B_0

$$M\dot{\nu} - G_w \lambda_w - g_{app} = 0$$

$$\dot{u} = V\nu$$

$$0 \le \lambda_w \perp \psi_w \ge 0$$

$$G_w = \begin{bmatrix} \hat{w}_1 & \cdots \\ r_{01} \times \hat{w}_1 & \cdots \end{bmatrix}$$

$$\lambda_w = \begin{bmatrix} \lambda_{1w} & \lambda_{2w} & \cdots \end{bmatrix}^T$$

Discretize in time

$$M^\ell \nu^{\ell+1} - G_w^\ell \lambda_w^{\ell+1} - M^\ell \nu^\ell - p_{app} = 0$$

$$0 \le p_w^{\ell+1} \perp \frac{\psi_w^\ell}{h} + (G_w^\ell)^T \nu^{\ell+1} + \frac{\partial \psi_w^\ell}{\partial t} \ge 0$$

$$u^{\ell+1} = u^\ell + V\nu^{\ell+1} h$$

where $p_w = h\lambda_w$

Matrix form:

$$\boxed{\begin{aligned} \begin{bmatrix} 0 \\ \rho_w^{\ell+1} \end{bmatrix} &= \begin{bmatrix} M^\ell & -G_w^\ell \\ (G_w^\ell)^T & 0 \end{bmatrix} \begin{bmatrix} \nu^{\ell+1} \\ p_w^{\ell+1} \end{bmatrix} + \begin{bmatrix} -M^\ell \nu^\ell - p_{app} \\ \frac{\psi_w^\ell}{h} + \frac{\partial \psi_w^\ell}{\partial t} \end{bmatrix} \\ 0 \le p_w^{\ell+1} &\perp \rho_w^{\ell+1} \ge 0 \end{aligned}}$$

MLCP

$$u^{\ell+1} = u^\ell + V\nu^{\ell+1} h$$

Just like dynamics with frictionless contacts.

The matrix has nice properties, so the solution is w-unique $\Rightarrow$ $\nu^{\ell+1}$ is unique, but λ_w might not be unique.

The problem has internal <u>stretching</u> force (opposite of internal squeezing force in grasping).

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w-uniqueness can be seen more easily if the model is written as a std. LCP.

Solve for $\nu^{\ell+1}$ & substitute into the complementarity condition:

$$\boxed{0 \le p_w^{\ell+1} \perp G_w^T M^{-1} G_w \, p_w^{\ell+1} + G_w^T (\nu^\ell + M^{-1} p_{app}) \ge 0}$$

Standard LCP

$G_w^T M^{-1} G_w$ is positive semi-definite

$\therefore$ Solution is w-unique $\forall$ $G_w^T(\nu^l + M^{-1} p_{app})$

Faster solvers will work well, e.g.

projected Gauss Seidel.

If robots are dynamic they can be added in exactly as multiple contacting bodies were.

If robot i is a position controlled source (i.e., is kinematically controlled) then Ψ_{iw} is an explicit fcn of time, $\therefore \frac{\partial \Psi_{iw}}{\partial t} \neq 0$.