Robotics II Lecture 2

1/25/08

Go over changes in grading

assignments -- project demos on
~~lecture~~ last day of class ?

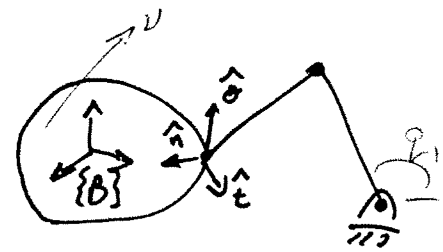
Today cover: contact models for grasp analysis

dynamics

Contact Models

1/9/08

Simplest models base on relative contact twists



$$\underbrace{\tilde{J}_i \dot{q}}_{\nu_{cc_i,hnd}} - \underbrace{\tilde{G}_i^T \nu}_{\nu_{cc_i,obj}} = \underset{(6\times 1)}{\nu_{cc_i}} = \begin{bmatrix} v_{in} \\ v_{it} \\ v_{io} \\ \omega_{in} \\ \omega_{it} \\ \omega_{io} \end{bmatrix}_{\text{relative}}$$

PwoF - only prevent penetration

Require normal component of relative velocity to be zero

All other components are free

Select normal component

Saves row 1 of vector to right

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 & \vdots & 0 & 0 & 0 \end{bmatrix}}_{H_i} \left(\underset{(6\times n_q)}{\tilde{J}_i \dot{q}} - \underset{(6\times 6)}{\tilde{G}_i^T} \nu \right)$$

Note that H_i is simple because we wrote contact twists in the contact frames.

Give example of physical implications of 1, 2, ... etc frictionless contacts

Let $\quad H_i \tilde{J}_i = J_i \quad , \quad H_i \tilde{G}_i^T = G_i^T$

Then the non-penetration constraint is

$$\boxed{J_i \dot{q} - G_i^T \nu = 0} \quad \forall\, i \ni \text{contact } i \text{ is PwoF}$$

Now incorporates only the constrained components.

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HF contact

Same as PwoF except H_i

$$H_i = \begin{bmatrix} 1 & 0 & 0 & \vdots & \\ 0 & 1 & 0 & \vdots & 0 \\ 0 & 0 & 1 & \vdots & \end{bmatrix}_{(3\times 6)}$$

$\forall i \ni$ contact is HF

Give 3D Example with 1, 2, 3, etc contacts

SF contact

$$H_i = \begin{bmatrix} 1 & 0 & 0 & \vdots & 0 & 0 & 0 \\ 0 & 1 & 0 & \vdots & 0 & 0 & 0 \\ 0 & 0 & 1 & \vdots & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & \vdots & 1 & 0 & 0 \end{bmatrix}_{(4\times 6)}$$

$\forall i \ni$ contact is SF

Put all contacts into one constraint eq.

$$\begin{bmatrix} 0 \\ C \\ 0 \end{bmatrix} \quad \begin{bmatrix} \dot{q} \\ \nu \end{bmatrix}$$

$$= \boxed{J\dot{q} - G^T \nu = 0}$$

where $H = \mathrm{diag}(H_1, H_2, H_3)$,

$J = H\tilde{J}$, $G^T = H\tilde{G}^T$

Example #1, Part 2 1/12/08

Jacobian derived before was of size (12 x 5).

Assume two soft contacts New one will be smaller.

$$H_1 = H_2 = \left[\begin{array}{cccc|cc} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{array}\right]$$

$$J = \begin{bmatrix} H_1 & 0 \\ 0 & H_2 \end{bmatrix} \begin{bmatrix} \tilde{J}_1 \\ \tilde{J}_2 \end{bmatrix} \Rightarrow$$

remove rows 5 & 6 from $\tilde{J}_1$

remove " 5 & 6 from $\tilde{J}_2$

$$G^T = \begin{bmatrix} & \text{same} & \end{bmatrix} \begin{bmatrix} \tilde{G}_1^T \\ \tilde{G}_2^T \end{bmatrix} \Rightarrow$$

remove rows 5 & 6 from $\tilde{G}_1^T$

" " " " $\tilde{G}_2^T$

$$J = \begin{array}{c} \\ \\ \end{array} \begin{array}{cc} \overbrace{\begin{array}{cc} \dot{q}_1 & \dot{q}_2 \end{array}} & \overbrace{\begin{array}{ccc} \dot{q}_3 & \dot{q}_4 & \dot{q}_5 \end{array}} \end{array}$$

$$J = \left[\begin{array}{cc|ccc} -4.9 & -1.5 & 0 & 0 & 0 \\ 0.5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1.4 \\ 0 & 0 & 2.5 & 3.5 & 0 \\ 0 & 0 & 0 & -.9 & 0 \end{array}\right] \begin{array}{l} \nu_{1n} \\ \nu_{1t} \\ \nu_{1o} \\ \omega_{1n} \\ \nu_{2n} \\ \nu_{2t} \\ \nu_{2o} \\ \omega_{2n} \end{array}$$

shows that when q_4 moves the finger twists against the surface of the object.

$$\cancel{G^T = \begin{bmatrix} 0.9 & 0.4 & 0 & 0 \\ -0.4 & 0.9 & 0 & 0 \\ 0 & 0 & 1 & -0.8 \\ & & & \end{bmatrix}}$$

$$G^T = \begin{bmatrix} H_1 \tilde{G}_1^T \\ H_2 \tilde{G}_2^T \end{bmatrix}$$

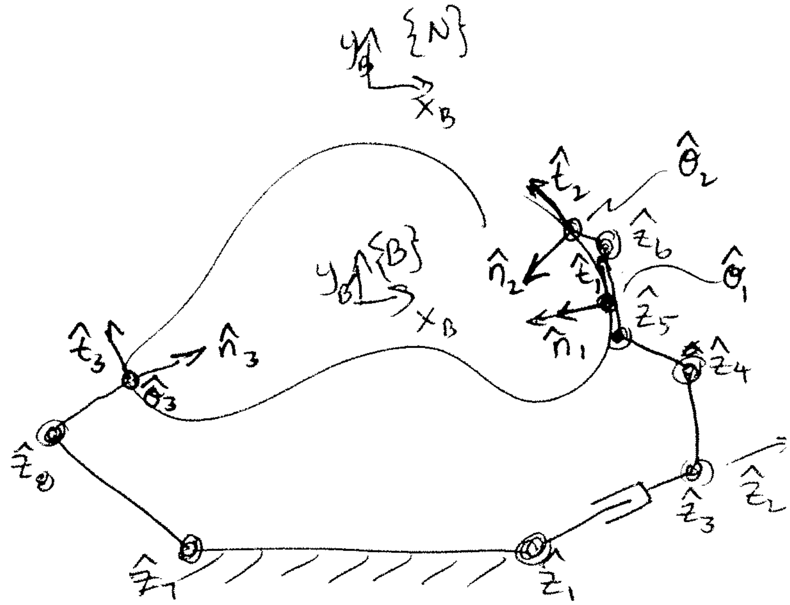
$$G^T = \left[\begin{array}{ccc|ccc} 0.9 & 0.4 & 0 & 0 & 0 & 0 \\ -0.4 & 0.9 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.9 & 0.4 & 0 \\ \hline -0.93 & 0.3 & 0 & 0 & 0 & 0 \\ -0.3 & -0.93 & 0 & 0 & 0 & -2 \\ 0 & 0 & 1 & -0.6 & -1.86 & 0 \\ 0 & 0 & 0 & -0.93 & 0.3 & 0 \end{array}\right]$$

(8×6)

Planar Simplifications

1/24/08

1



① Assign frames with 2 axes in the plane

② Revolute joint axes out of plane

③ Prismatic axes in plane.

$$\tilde{J}_i \dot{q} = \begin{bmatrix} v_{in} \\ v_{it} \\ v_{io} \\ \omega_{in} \\ \omega_{it} \\ \omega_{io} \end{bmatrix}$$

Eliminate out of plane motions.

Selection matrix

$$L = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}_{(3\times 6)}$$

$$L\tilde{J}_i \dot{q} = \underbrace{\begin{bmatrix} \text{rows} \\ 1,2,6 \\ \text{of } \tilde{J}_i \end{bmatrix}}_{(3\times n_q)} \dot{q} = \begin{bmatrix} v_{in} \\ v_{it} \\ \omega_{io} \end{bmatrix}$$

What happens to the contact models

PwoF : spatial case

$$H = \begin{array}{cccccc} v_{in} & v_{it} & v_{io} & \omega_{in} & \omega_{it} & \omega_{io} \\ \end{array}$$
$$H = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

keep row 1

keep v_{in} component

planar case

$$\begin{array}{ccc} v_{in} & v_{it} & \omega_{io} \end{array}$$
$$H = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$

$$\underbrace{H L \tilde{J}_i}\dot{q} = v_{in}$$

$$J_i \dot{q} = v_{in}$$

HF : spatial case, keep v_{in}, v_{it}, v_{io}

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & \\ 0 & 1 & 0 & 0_{3\times3} \\ 0 & 0 & 1 & \end{array}\right]$$

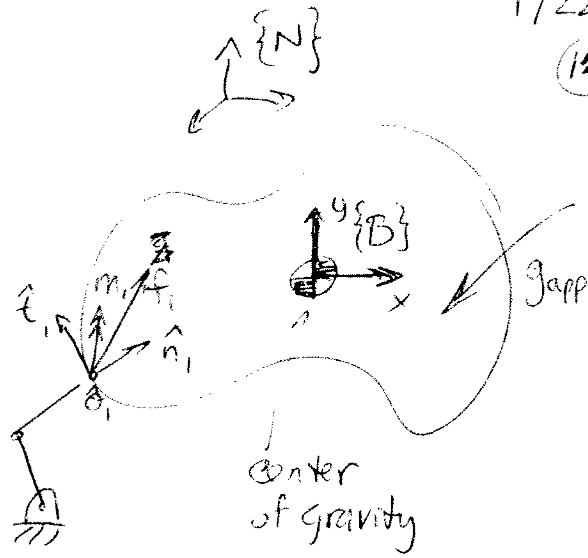
planar case; $H = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$ there is no v_{io} to keep!

same

SF : spatial case, keep $v_{in}, v_{it}, v_{io}, \omega_{in}$

planar case: $H = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$ there is no ω_{in} to keep!

Dynamics



Notation

$\tilde{\lambda}_i$ is contact force

$$\tilde{\lambda}_i = \begin{bmatrix} f_{in} \\ f_{it} \\ f_{io} \\ m_{in} \\ m_{it} \\ m_{io} \end{bmatrix}^{c_i}.$$

$\tilde{\lambda}_i$ is expressed in $\{C_i\}$ for simplicity

Newton Euler Equations

$$M_{hand}(q)\,\ddot{q} + b_{hnd}(q,\dot{q}) + J^T\lambda = \tau_{app}$$

$$M_{obj}(u)\,\dot{\nu} + b_{obj}(u,\nu) + \underset{(6n_b \times l)}{G}\,\lambda = \underset{(6n_b \times 1)}{g_{app}}$$

Inertia Terms

$$\lambda_i = H_i \tilde{\lambda}_i$$

$$\lambda = \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \vdots \\ \lambda_{n_c} \end{bmatrix}$$

Note: $\tilde{G}^T$ maps ν to contact frames, $\tilde{G}$ maps $\tilde{\lambda}$ to body frame.

$\tilde{J}$ maps $\dot{q}$ to contact frames, $\tilde{J}^T$ maps $\tilde{\lambda}$ to joint frames.

G & J do same, but restricted to the active components of the contact models.