Dynamics of Grasps

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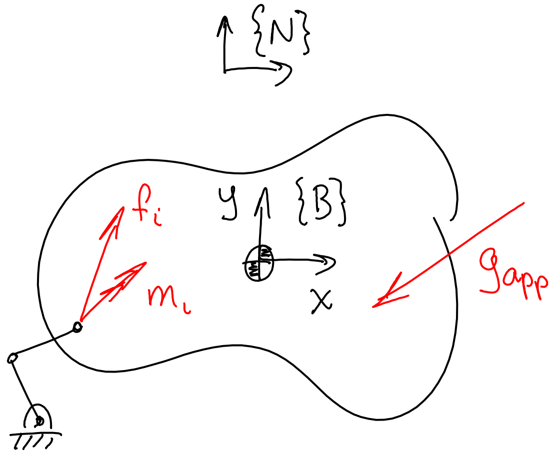

Some definitions:

Let $\tilde{\lambda}_i = g_i^{c_i}$

$$\tilde{\lambda}_i = \begin{bmatrix} f_{in} \\ f_{it} \\ f_{io} \\ m_{in} \\ m_{it} \\ m_{io} \end{bmatrix}^{c_i} \qquad \tilde{\lambda} = \begin{bmatrix} \tilde{\lambda}_1 \\ \vdots \\ \tilde{\lambda}_{n_c} \end{bmatrix} \qquad \boxed{\lambda = H\tilde{\lambda}}$$

Important! $\tilde{\lambda}_i$ is expressed in $\{C_i\}$ by definition.

Also $\lambda_i = H_i \tilde{\lambda}_i$

$$\lambda = \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \vdots \\ \lambda_{n_c} \end{bmatrix}$$

Now we can write the Newton-Euler equations in matrix form

$$\begin{aligned} J^T \lambda &= \tau \\ -G\lambda &= g \end{aligned} \quad \equiv \quad \begin{bmatrix} J^T \\ -G \end{bmatrix} \lambda = \begin{bmatrix} \tau \\ g \end{bmatrix} \qquad \text{(eq. 28.21)}$$

Expand:

(eq. 28.22)

$$M_{hnd}(q)\,\ddot{q} + b_{hnd}(q,\dot{q}) + J^T\lambda = \tau_{app}$$

applied actuator efforts

$$M_{obj}(u)\,\dot{\nu} + b_{obj}(u,\nu) - G\lambda = g_{app}$$

inertia terms

applied wrench

M_{obj}, M_{hnd} are symmetric and positive definite.

$\therefore$ M_{obj}^{-1} and M_{hnd}^{-1} exist.

Matrix form:

$$\begin{array}{c c}
n_q \{ & \\
n_\nu \{ & \\
l \{ &
\end{array}
\underbrace{\begin{bmatrix} M_{hnd} & 0 & J^T \\ 0 & M_{obj} & -G \\ J & -G^T & 0 \end{bmatrix}}_{\substack{n_q \quad n_\nu \quad l \\ \text{Call this matrix } A}}
\begin{bmatrix} \ddot{q} \\ \dot{\nu} \\ \lambda \end{bmatrix}
=
\begin{bmatrix} \tau_{app} - b_{hnd} \\ g_{app} - b_{obj} \\ \dot{J}\dot{q} - \dot{G}^T\nu \end{bmatrix}$$

For purposes of simulation, we need to produce $q(t)$ and $u(t)$. To get $u(t)$, we need one more eq:

$$\dot{u} = B(u)\,\nu$$

where B is a matrix relating the object twist to the time derivatives of the position and orientation parameter of the object.

When $u = (x, y, z, \epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4)$ ... B is (7×6)

When $u = (x, y, z, \underbrace{\epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4}_{\text{Euler params}})$, then B is (7×6).

Assume $\dot{q}$ and ν are available from sensors.

Assume τ_{app} and g_{app} are known.

b_{obj}, b_{hnd}, $\dot{J}$, $\dot{G}$ are known functions of $u, q, \nu, \dot{q}$

Then, if A^{-1} exists, then one can solve for the accelerations and contact forces.

Sometimes you can solve for $\ddot{q}, \dot{\nu}$ uniquely, but not λ. In this case, $N(A)$ has special structure

$$\begin{bmatrix} \ddot{q} \\ \dot{\nu} \\ \hdashline \lambda \end{bmatrix} = A^{+} \begin{bmatrix} \tau_{app} - b_{hnd} \\ g_{app} - b_{obj} \\ \dot{J}\dot{q} - \dot{G}\dot{\nu} \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ \hdashline \underline{\text{nonzero}} \end{bmatrix}}_{N(A)} \delta$$

where δ is an arbitrary vector and "nonzero" is the block of $N(A)$ corresponding to λ.

Suppose tactile sensors can measure λ. Then we can solve for the accelerations.

$$\ddot{q} = M_{hnd}^{-1} \left(\tau_{app} - b_{hnd} - J^{T} \lambda \right)$$

$$\dot{\nu} = M_{obj}^{-1}\left(g_{app} - b_{obj} + G\lambda\right)$$