Lecture 3 Basic Grasp

Review ~~Force/Moment~~ Equations

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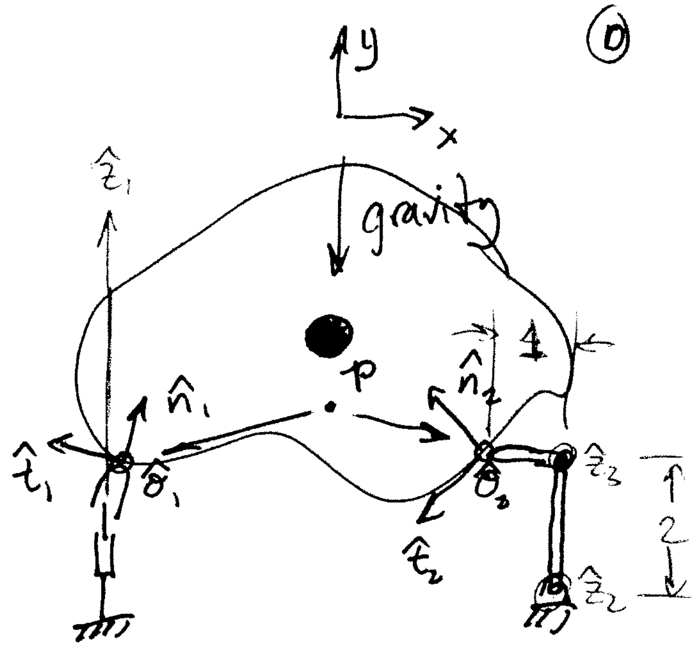
1

$$J\dot{q} - G^T \nu = \nu_{cc}$$

$$\begin{bmatrix} J^T \\ -G \end{bmatrix} \lambda = \begin{bmatrix} \tau \\ g \end{bmatrix}$$



$\boxed{\text{Assume Planar Case}}$

Let $c_1 - p = \begin{bmatrix} -3.5 \\ -0.5 \end{bmatrix} = r_1$

$c_2 - p = \begin{bmatrix} 2.0 \\ -0.3 \end{bmatrix} = r_2$

$$G = \begin{bmatrix} \hat{n}_1 & \hat{t}_1 & \hat{n}_2 & \hat{t}_2 \\ r_1 \otimes \hat{n}_1 & r_1 \otimes \hat{t}_1 & r_2 \otimes \hat{n}_2 & r_2 \otimes t_2 \end{bmatrix}$$

remove columns if contacts are frictionless

$$G \simeq \begin{bmatrix} .4 & -.9 & -\sqrt{2}/2 & -\sqrt{2}/2 \\ .9 & .4 & \sqrt{2}/2 & -\sqrt{2}/2 \\ -3.2 & -1.0 & .8 & -2 \end{bmatrix}_{(3\times 4)} \qquad \lambda = \begin{bmatrix} f_{1n} \\ f_{1t} \\ f_{2n} \\ f_{2t} \end{bmatrix}$$

$$J = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ \cancel{0} & \cancel{0} & \cancel{0} \\ \hline 0 & -2 & 0 \\ 0 & -1 & -1 \\ \cancel{0} & & \end{bmatrix} \begin{matrix} \left.\begin{matrix} \\ \\ \end{matrix}\right\} \text{HF cont.} \\ \\ \left.\begin{matrix} \\ \\ \end{matrix}\right\} \text{HF cont} \\ \\ \end{matrix} \qquad g = \begin{bmatrix} 0 \\ -mg \\ 0 \end{bmatrix} \qquad \tau = \begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \end{bmatrix} =$$

Planar Case

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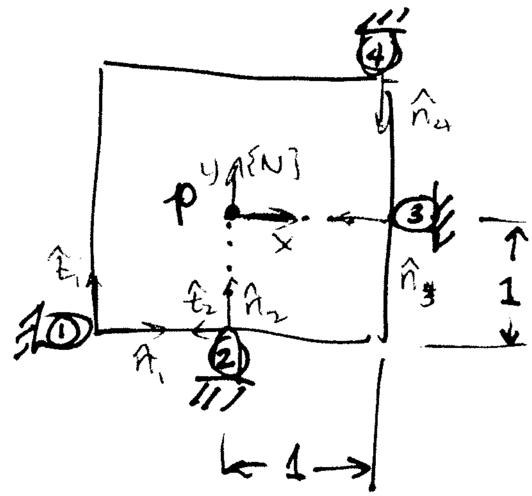
2

Approximate G & J by inspection



Assume contact models

1 — HF

2 — SF

3 — PwoF

4 — PwoF

$$G = \left[\begin{array}{cc|cc|c|c} 1 & 0 & 0 & -1 & -1 & 0 \\ 0 & 1 & 1 & 0 & 0 & -1 \\ 1 & -1 & 0 & 1 & 0 & 1 \end{array}\right]$$

J = empty

Planar Case

Approximate G & J by inspection

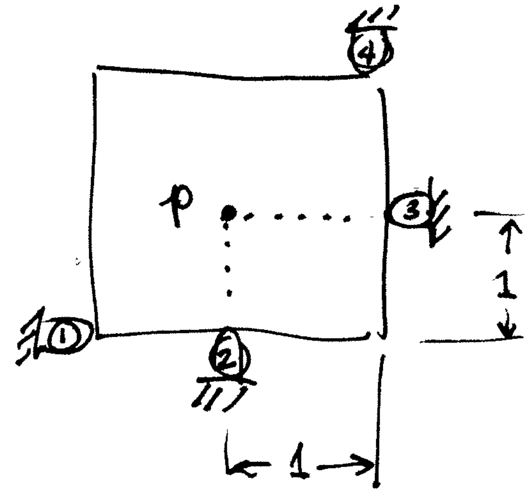
Assume contact models

1 — HF

2 — SF

3 — PwoF

4 — PwoF



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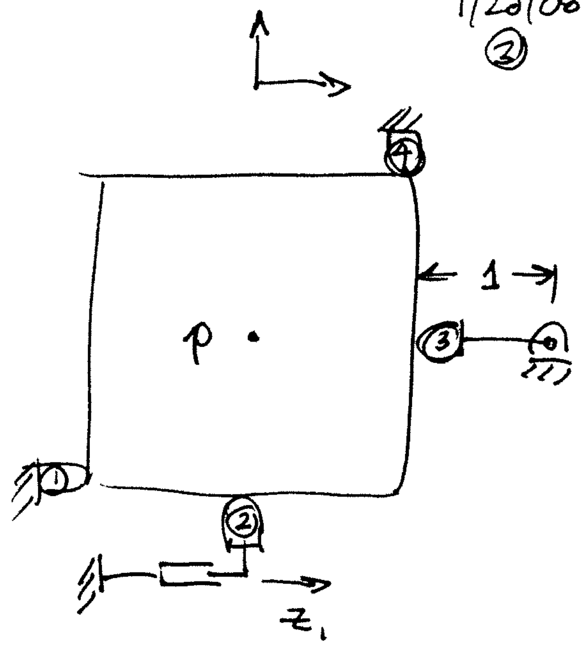
②

Add a 1-joint finger for contacts ② & ③.

Approximate J & G by inspection



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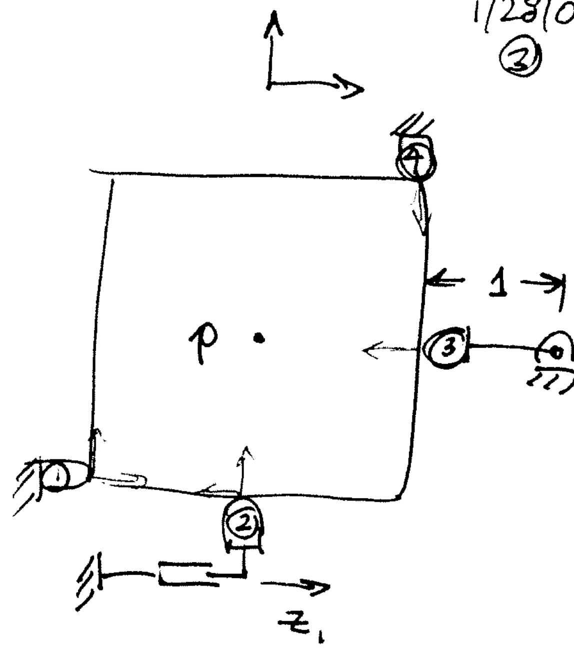
Add a 1-joint finger
for contacts 2 & 3.

Approximate J & G
by inspection



G is same as previous page

$$J \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} = \begin{bmatrix} \nu_{1n} \\ \nu_{1t} \\ \nu_{2n} \\ \nu_{2t} \\ \nu_{3n} \\ \nu_{4n} \end{bmatrix}$$

$$J = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ -1 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Controllable Twists & Wrenches

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④

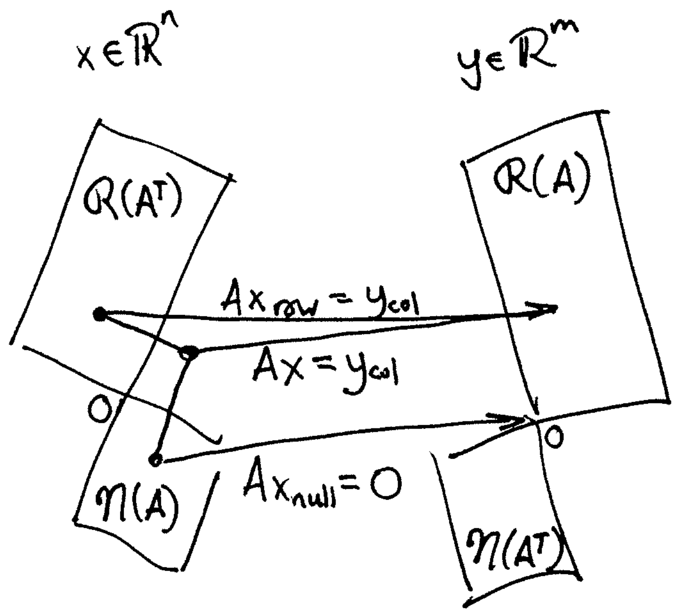
$A \in \mathbb{R}^{m \times n}$

$\mathcal{R}(A)$ = all possible linear combinations of columns of A

$\mathcal{R}(A^T)$ = " " " " " rows of A

$N(A)$ = all vectors $\ni$ $Ax = 0$, $x \in \mathbb{R}^n$

$N(A^T)$ = all vectors $\ni$ $A^T y = 0$, $y \in \mathbb{R}^m$



$\text{Rank}(A) = r$

$\text{Dim}(\mathcal{R}(A)) = r$

$\text{Dim}(\mathcal{R}(A^T)) = r$

$\text{Dim}(\mathcal{N}(A)) = n - r$

$\text{Dim}(\mathcal{N}(A^T)) = m - r$

$$Ax = A(x_{row} + x_{null}) = A x_{row} = y$$

If $y \in \mathcal{R}(A)$, then many x's satisfy $Ax = y$

If $y \in \mathcal{N}(A^T)$, then no solution exists

$\mathcal{N}(A) \cap \mathcal{R}(A^T) = 0$, $\mathcal{N}(A) \cup \mathcal{R}(A^T) = \mathbb{R}^n$

i.e. $\mathcal{N}(A)$ & $\mathcal{R}(A^T)$ are orthogonal complements!

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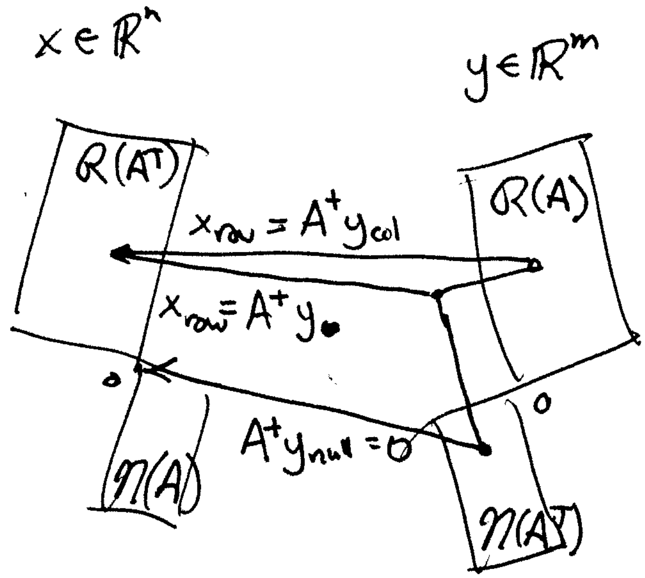
Reverse Mapping



Mappings A, A^+ between $\mathcal{R}(A)$ & $\mathcal{R}(A^T)$ are one-to-one & onto.

$$Ax = y$$

If soln exists, then:

$$x = A^+ y + N(A)\alpha$$

Matlab easily computes:

determinant	det(A)
null space	N(A)
pseudo inverse	pinv(A)

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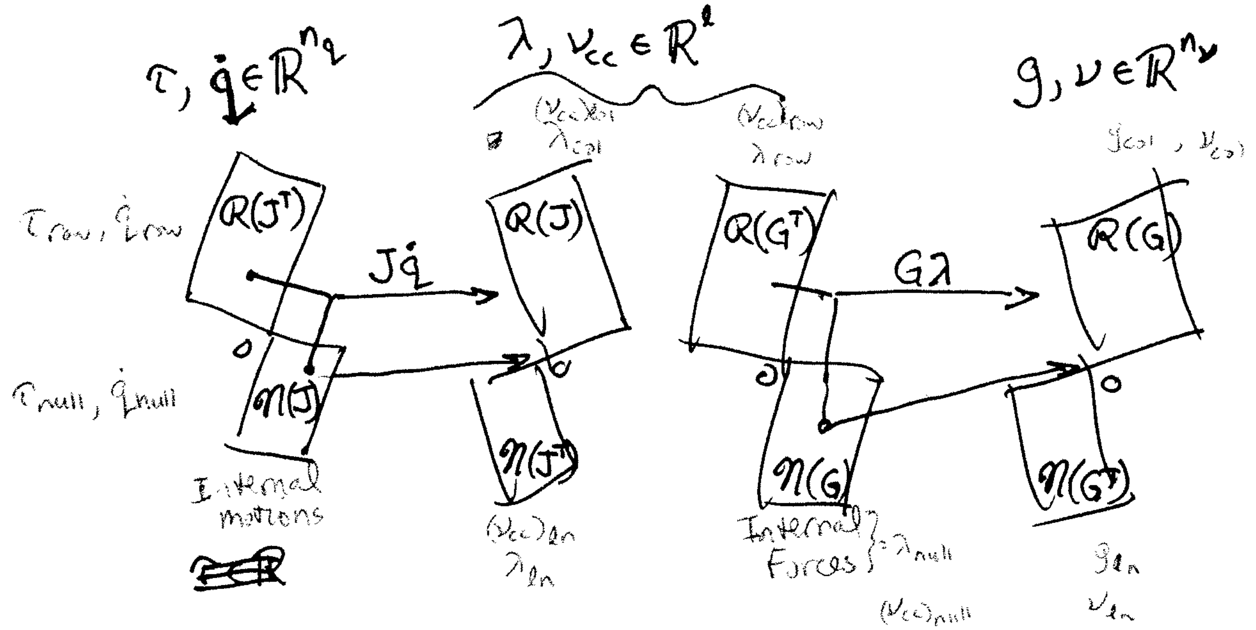
(6)

Linear subspaces in Grasping



Cannot cause $\nu_{cc} \in \mathcal{N}(J^T)$

$\tau \in \mathcal{N}(J)$ is joint torques that don't effect contact wrench

$\dot{q} \in \mathcal{N}(J)$ is ~~redundant~~ joint motion that does not impact finger contact twist.

$\dot{q}_{col} \in \mathcal{R}(J^T)$ is the part that causes the motion

$\tau \in \mathcal{R}(J^T)$ directly impact contact wrenches.

Cannot cause $\nu \in \mathcal{N}(G^T)$

$\lambda \in \mathcal{R}(G^T)$ generates wrench, g

$\nu_{cc} \in \mathcal{R}(G^T)$ imparts the object motion, ν

$\nu_{cc} \in \mathcal{N}(G)$ does not effect object motion, ν

~~$\lambda \in \mathcal{R}(G^T)$~~

$\lambda \in \mathcal{N}(G)$ generates internal forces

Dexterous Manipulation 1/28/08

Task Requirements

All object twists & wrenches possible $\leftarrow$ $\operatorname{rank}(G) = n_v$

All object twists & " controllable $\leftarrow$ $\begin{cases} \operatorname{rank}(G) = n_v \\ \operatorname{rank}(GJ) = n_v \end{cases}$

All internal forces controllable $\leftarrow$ $\mathcal{N}(G) \cap \mathcal{N}(J^T) = 0$

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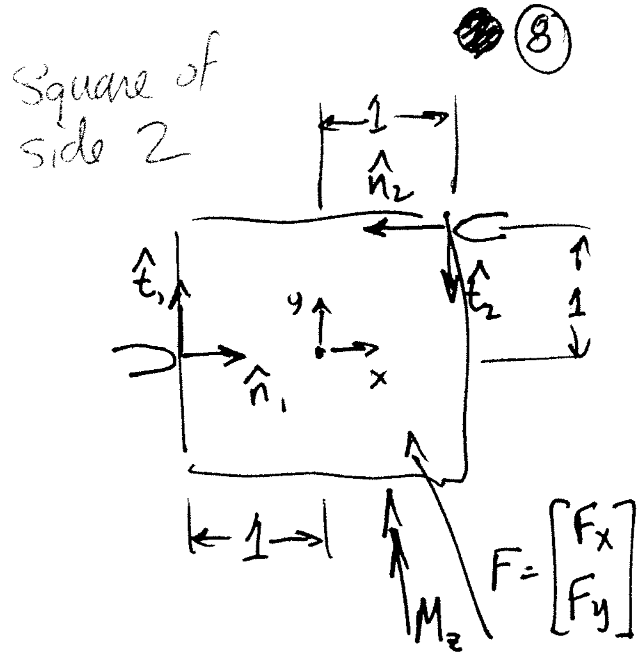
Planar example



Assume 2 hard fingers

$$G = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & -1 & 1 & -1 \end{bmatrix}$$

Columns 1, 3 are li

Column 2 is li from 1 & 3

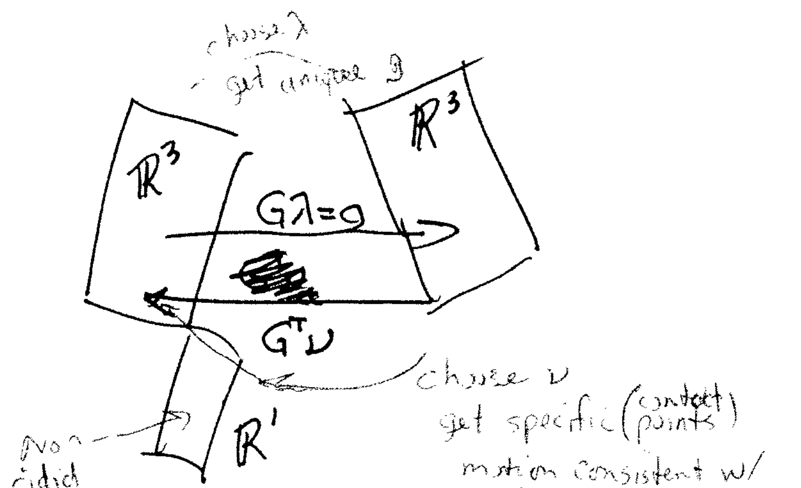
Determinant of cols 1,2,3 is $1 \neq 0$

$\therefore\ \mathcal{N}(G^T) = 0 \in \mathbb{R}$ $\Longrightarrow$ All object twists & wrenches can be caused by fingers that are sufficiently dexterous

$\mathcal{R}(G) = \mathbb{R}^3$

$\text{rank}(G) = 3 = n_\nu$

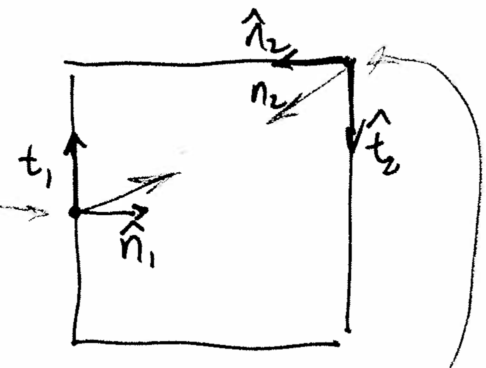
$\mathcal{N}(G) = \mathbb{R}^1$ $\Longrightarrow$ Internal forces may be present.



Internal force space exists

What are the internal forces?

$$\text{null}(G) = \begin{bmatrix} 0.6 \\ 0.3 \\ 0.6 \\ 0.3 \end{bmatrix}$$



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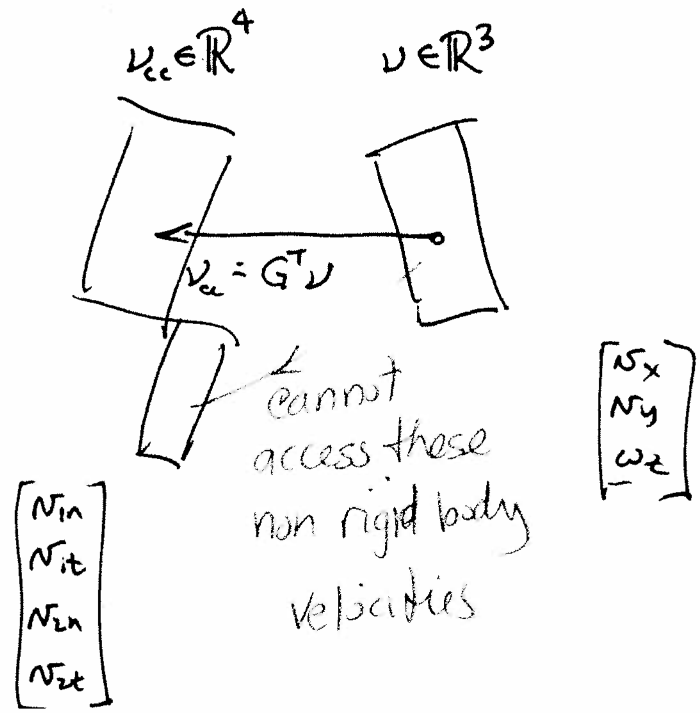
Two interpretations:

1. can squeeze along internal force vectors w/o causing acceleration of ~~force~~ object
2. cannot move contacts toward each other (i.e. the body is rigid).



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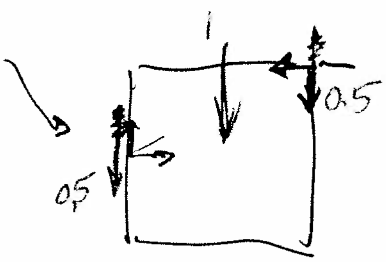
Many grasp forces to satisfy/produce one ext. wrench

$$G\lambda = -g$$

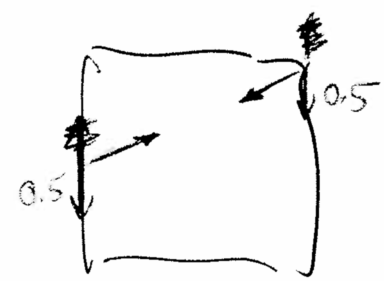
$$\lambda = -G^{+}g + N(G)\gamma$$

$$\lambda = -\begin{bmatrix} 0.6 & 0 & 0.2 \\ -0.2 & 0.5 & -0.4 \\ -0.4 & 0 & 0.2 \\ -0.2 & -0.5 & -0.4 \end{bmatrix} \begin{bmatrix} F_x \\ F_y \\ M_z \end{bmatrix} + \begin{bmatrix} 0.6 \\ 0.3 \\ 0.6 \\ 0.3 \end{bmatrix} \gamma = \begin{bmatrix} f_{1n} \\ f_{1t} \\ f_{2n} \\ f_{2t} \end{bmatrix}$$

Let $g = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ $\quad \gamma = 0 \Rightarrow \quad \lambda = \begin{bmatrix} 0 \\ -0.5 \\ 0 \\ 0.5 \end{bmatrix}$



$\gamma = 1 \Rightarrow \quad \lambda = \begin{bmatrix} 0.6 \\ -0.2 \\ 0.6 \\ 0.8 \end{bmatrix}$



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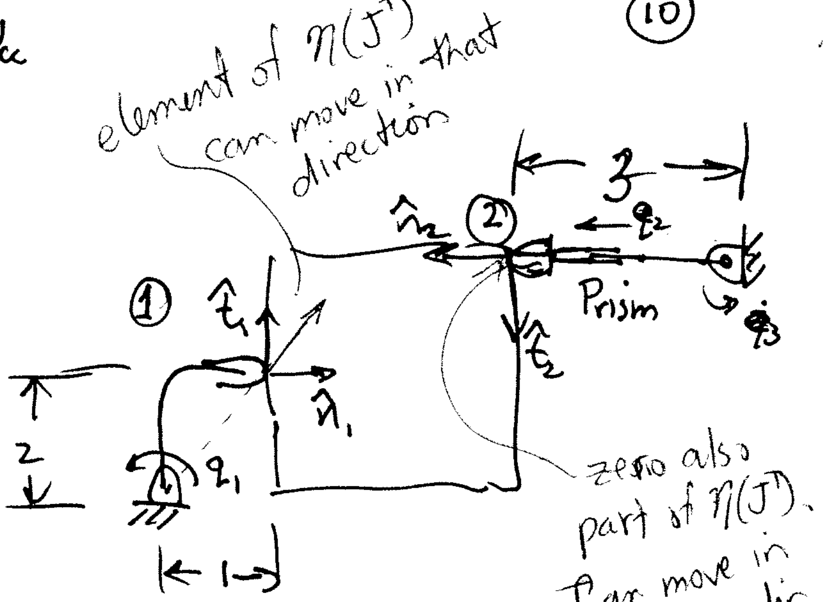
Add some fingers $\quad J\dot{q} = \nu_{cc}$

$$J\dot{q} = \begin{bmatrix} -2 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix} = \begin{bmatrix} \nu_{1n} \\ \nu_{1t} \\ \nu_{2n} \\ \nu_{2t} \end{bmatrix}$$



$\text{Rank}(J) = 3$

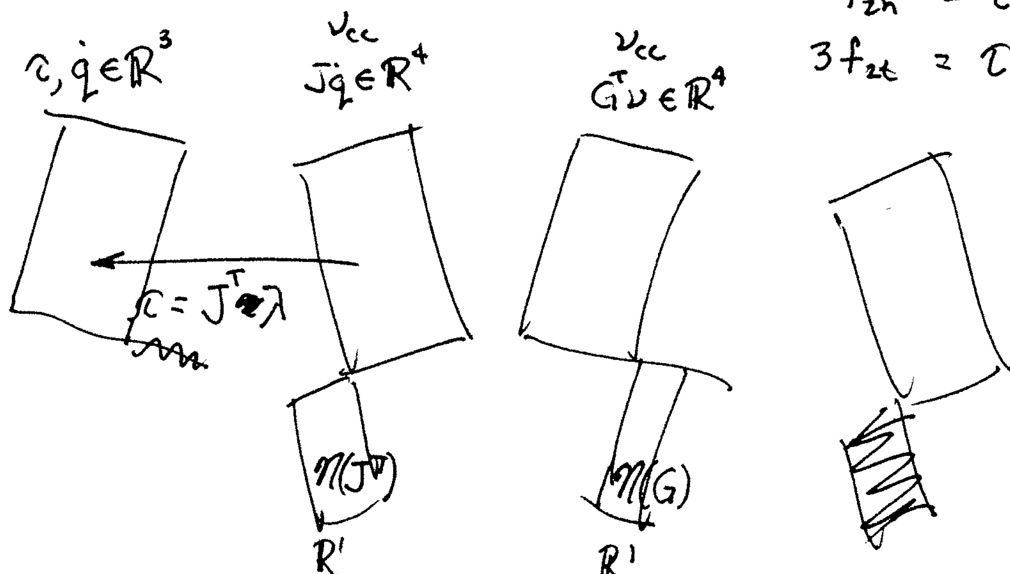
$\therefore \ \mathcal{N}(J) = 0 \quad \Rightarrow$ any nonzero finger motion will cause nonzero ν

$\mathcal{R}(J) = \mathbb{R}^3 \quad$ contact wrenches uniquely determine τ

$\mathcal{R}(J^T) = \mathbb{R}^3$

$\mathcal{N}(J^T) = \mathbb{R}^1$

$$J^T\lambda = \tau \qquad \begin{aligned} -2f_{1n} + f_{1t} &= \tau_1 \\ f_{2n} &= \tau_2 \\ 3f_{2t} &= \tau_3 \end{aligned}$$



$$\mathcal{N}(J^T) = \begin{bmatrix} 1 \\ 2 \\ 0 \\ 0 \end{bmatrix} \qquad \mathcal{N}(G) = \begin{bmatrix} 2 \\ 1 \\ 2 \\ 1 \end{bmatrix}$$

Can we control all ~~[illegible]~~ object wrenches & twists?

$$\text{rank}(G) = n_v = 3$$

$$\text{rank}(GJ) \overset{?}{=} 3$$

$$GJ = \begin{bmatrix} -2 & -1 & 0 \\ 1 & 0 & -3 \\ -1 & 1 & -3 \end{bmatrix}$$

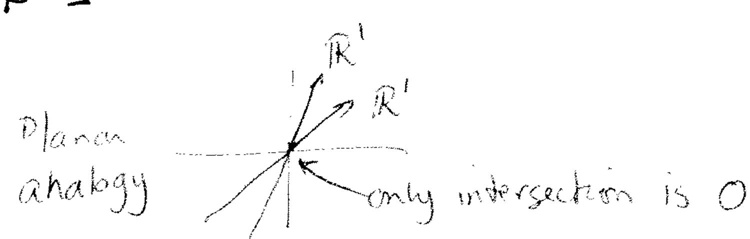
~~[illegible]~~ $\text{Det}(GJ) = -12$

$\therefore\ \text{rank}(GJ) = n_v = 3$

Can we control all internal forces?

$$\mathcal{N}(G) \cap \mathcal{N}(J^T) \overset{?}{=} 0$$

$$\begin{bmatrix} 2 \\ 1 \\ 2 \\ 1 \end{bmatrix} \cap \begin{bmatrix} 1 \\ 2 \\ 0 \\ 0 \end{bmatrix} = 0$$ Yes. Internal force is controllable!



How could we change the fingers so that the internal force would not be controllable?

violate $\mathcal{N}(G) \cap \mathcal{N}(J^T) \neq 0$

What about making it impossible to control ν & g?

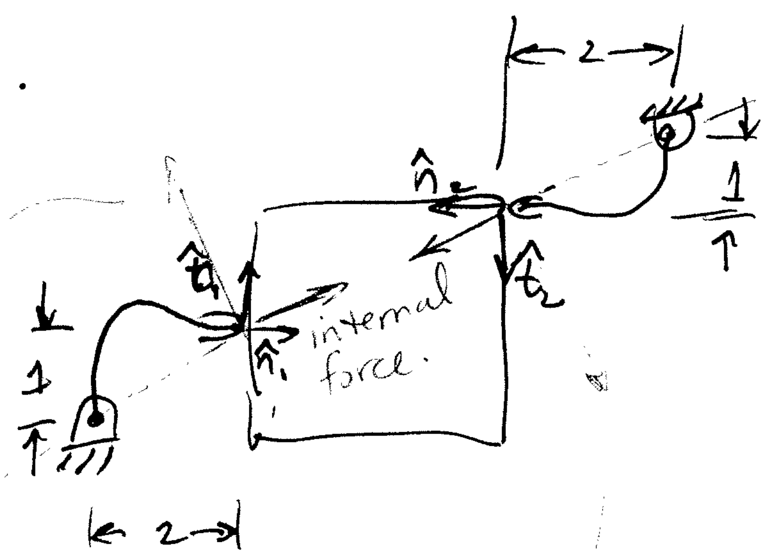
make $J \ni \text{rank}(GJ) < 3$

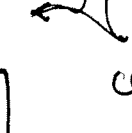
$G_{(3\times 2)} J_{(2\times n_q)}$

How? make $n_q < 3$, i.e., get rid of joints.

Consider changing to 2 joints.

Now make motion in direction of internal force impossible





New $J = \begin{bmatrix} -1 & 0 \\ 2 & 0 \\ 0 & -1 \\ 0 & 2 \end{bmatrix}$

could add more joints and still have same lack of control.

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What is $\mathrm{rank}(GJ) \stackrel{?}{=} \underline{\underline{2}} < 3$ $\therefore$ ν, g not controllable

$$GJ = \begin{bmatrix} -1 & 1 \\ 2 & -2 \\ 2 & -3 \end{bmatrix}$$

What is $\mathcal{N}(G) \cap \mathcal{N}(J^T)$?

$$N(G) = \begin{bmatrix} 2 \\ 1 \\ 2 \\ 1 \end{bmatrix} \qquad N(J^T) = \begin{bmatrix} 2 & -4 \\ 1 & -2 \\ 4 & 2 \\ 2 & 1 \end{bmatrix}$$

What is $\mathcal{N}(G) \cap \mathcal{N}(J^T)$? i.e. is there $\alpha, \beta \in \mathbb{R}$ $\ni$ $N(J^T)\begin{bmatrix} \alpha \\ \beta \end{bmatrix} = N(G)$

$$\begin{bmatrix} 2 & -4 \\ 1 & -2 \\ 4 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 2 \\ 1 \end{bmatrix} \Rightarrow \begin{matrix} \left.\begin{matrix} 2\alpha - 4\beta = 2 \\ \alpha - 2\beta = 1 \end{matrix}\right\} \text{same} \\ \left.\begin{matrix} 4\alpha + 2\beta = 2 \\ 2\alpha + \beta = 1 \end{matrix}\right\} \text{same} \end{matrix}$$

solve these two

$$\begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} 3/5 \\ -1/5 \end{bmatrix}$$

$\therefore$ $\mathcal{N}(G) \cap \mathcal{N}(J^T) \neq 0$ can't control internal forces!