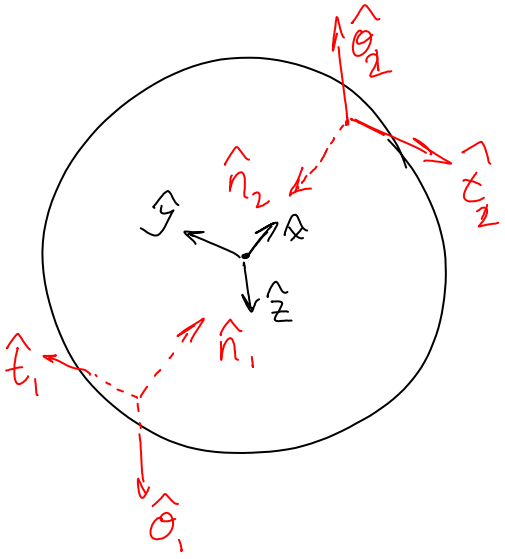
Soln in-class 2: friction cone representation

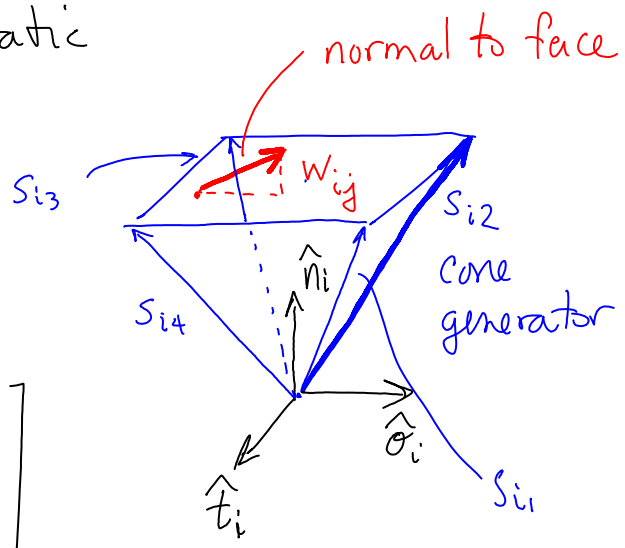
Thursday, February 05, 2009
10:37 AM

Formulate F for a sphere grasped at two diametrically opposite points.

Let the origin of {N} be at the center of the sphere and let $(\hat{x},\hat{y},\hat{z})$ align with $(\hat{n}_1, \hat{t}_1, \hat{o}_1)$



Choose 4-sided pyramids to approximate the quadratic friction cone.



By inspection:

$$s_{ij}'s = \underbrace{\begin{bmatrix} \underline{s_{i1}} & \underline{s_{i2}} & \underline{s_{i3}} & \underline{s_{i4}} \\ | & | & | & | \\ \mu_i & -\mu_i & -\mu_i & \mu_i \\ \mu_i & \mu_i & -\mu_i & -\mu_i \end{bmatrix}}$$

These generators form a pyramid that circumscribes

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the quadratic cone.

We can get w_{ij} by $w_{ij} = s_{ij} \times s_{i(j+1)}$

$$\begin{vmatrix} \hat{n} & \hat{t} & \hat{o} \\ 1 & \mu_i & \mu_i \\ 1 & -\mu_i & \mu_i \end{vmatrix} = \begin{bmatrix} 2\mu_i^2 \\ 0 \\ 2\mu_i \end{bmatrix} \propto \begin{bmatrix} \mu \\ 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} w_{i1}^T \\ w_{i2}^T \\ w_{i3}^T \\ w_{i4}^T \end{bmatrix} = \begin{bmatrix} \mu & 0 & 1 \\ \mu & 1 & 0 \\ \mu & 0 & -1 \\ \mu & -1 & 0 \end{bmatrix} = F_i$$

Note that $F_1 = F_2$

$$\therefore \quad F = \begin{bmatrix} F_1 & 0 \\ 0 & F_2 \end{bmatrix}_{(8\times 6)}$$