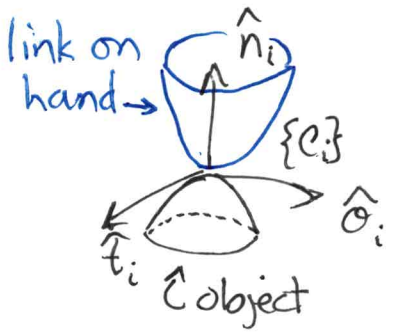
Contact Modeling for Grasping

Most easily thought about & expressed in contact frames.

Relative velocity at contact i.



$$\nu_{cc,i} = \nu_{i,hand} - \nu_{i,obj}$$

$$\nu_{cc,i} = \tilde{J}_i \dot{q} - \tilde{G}_i^T \nu$$

$$\nu_{cc,i} = \begin{bmatrix} \nu_{in} \\ \nu_{it} \\ \nu_{io} \\ \omega_{in} \\ \omega_{it} \\ \omega_{io} \\ \end{bmatrix}$$

ν_{in} ← setting = to zero means contact cannot separate

ν_{it}, ν_{io} ← setting = to zero means contact cannot slide

ω_{in} ← setting = to zero means bodies can't twist relative to each other about the contact normal.

3 Standard models: PwoF, HF, SF

PwoF – Point without Friction

- set $v_{in} = 0$
- let other rel. vel. components be free
- remove 1 d.o.f. from system
- enforce 1 equation

$$H_i \, v_{cc,i} = v_{in} = 0$$

where $H_i = \underset{(1\times 6)}{\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}}$

HF – Hard Finger (Point with Friction):

- set $v_{in} = v_{it} = v_{io} = 0$
- let other rel. vel. components be free
- remove 3 d.o.f. from system
- enforce 3 equations

$$H_i \, v_{cc,i} = \begin{bmatrix} v_{in} \\ v_{it} \\ v_{io} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

where $H_i = \underset{(3\times 6)}{\begin{bmatrix} I_{(3\times 3)} & O_{(3\times 3)} \end{bmatrix}}$

SF – Soft Finger:

- set $v_{in} = v_{it} = v_{io} = 0 = \omega_{in}$
- let other rel. vel. components be free
- remove 4 d.o.f. from system
- enforce 4 equations

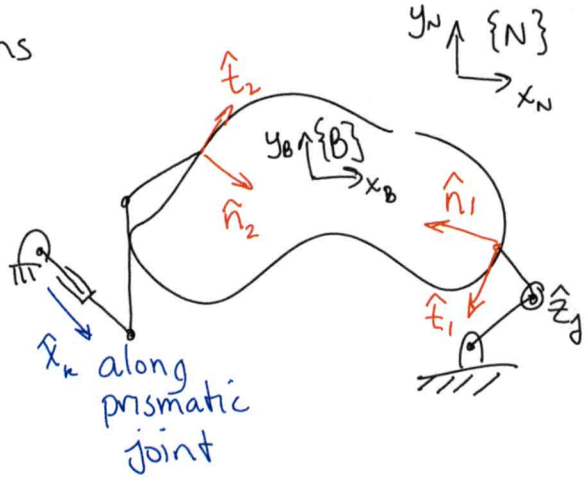
$$\begin{bmatrix} v_n \\ \end{bmatrix} \quad \begin{bmatrix} \\ \end{bmatrix}$$

$$H_i \nu_{cc,i} = \begin{bmatrix} \nu_{in} \\ \nu_{it} \\ \nu_{io} \\ \omega_{in} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

where $H_i = \underset{(4\times6)}{\left[I_{(4\times4)} \quad 0_{(4\times2)} \right]}$

Planar Simplifications

Put all $\hat{x},\hat{y}$ axes & $\hat{n},\hat{t}$ axes in the plane



Now use a selection matrix L to eliminate unwanted components

$$\nu_i = \begin{bmatrix} \nu_{in} \\ \nu_{it} \\ \nu_{io} \\ \omega_{in} \\ \omega_{it} \\ \omega_{io} \end{bmatrix} \text{ eliminate } \Rightarrow \quad L\nu_i = \begin{bmatrix} \nu_{in} \\ \nu_{it} \\ \omega_{io} \end{bmatrix}$$

where $L = \underset{(6\times3)}{\left[\begin{array}{cc:c:c} 1 & 0 & & 0 \\ 0 & 1 & 0_{3\times3} & 0 \\ 0 & 0 & & 1 \end{array}\right]}$

$$L\tilde{J}_i \dot{q} = \begin{bmatrix} v_{in} \\ v_{it} \\ \omega_{io} \end{bmatrix}_{hnd} = v_{i,hnd} \qquad v_{i,obj} = \begin{bmatrix} v_{in} \\ v_{it} \\ \omega_{io} \end{bmatrix}_{obj} = L\tilde{G}_i^T \nu$$

Now with the smaller Jacobian $(L\tilde{J}_i)_{(3 \times n_q)}$ & $(L\tilde{G}_i^T)_{(3\times 3)}$ we must still select transmitted components of the contact twists.

$$H_i \begin{bmatrix} v_{in} \\ v_{it} \\ \omega_{io} \end{bmatrix} \Rightarrow \quad J_i = H_i L\tilde{J}_i \qquad G_i^T = H_i L\tilde{G}_i^T$$

PwoF: $v_{in} = 0 \implies H_i = [1\ 0\ 0]$

HF: $v_{in} = v_{it} = 0 \implies H_i = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

SF: soft finger makes no sense, since can't rotate about $\hat{n}_i$

Kinematic Grasp Modeling Process

1. Choose contact types that are reasonable for your situation
2. Enforce contact types by

$$H_i \nu_{cc,i} = 0 \quad \forall\ i$$

$(l_i \times 6)$ where l_i = # of d.o.f. removed = # of constraint eqs.

Expand

$$H_i\left(v_{i,hnd} - v_{i,obj}\right) = 0$$

$$H_i\begin{bmatrix}\tilde{J}_i & -\tilde{G}_i^T\end{bmatrix}\begin{bmatrix}\dot{q}\\ \nu\end{bmatrix} = 0$$

$$\begin{bmatrix}H_i\tilde{J}_i & -H_i\tilde{G}_i^T\end{bmatrix}\begin{bmatrix}\dot{q}\\ \nu\end{bmatrix} = 0$$

$$\begin{bmatrix}J_i & -G_i^T\end{bmatrix}\begin{bmatrix}\dot{q}\\ \nu\end{bmatrix} = 0$$

typo

where $J_i = H_i\tilde{J}_i$

$G_i^T = H_i\tilde{G}_i^T$

Put all contacts together. Select constrained d.o.f.s from all contacts in one big eq.

$$H\left(v_{c,hnd} - v_{c,obj}\right) = 0$$

where $H = \text{diag}(H_1, H_2, \cdots, H_{n_c})$

$$H\begin{bmatrix}\tilde{J} & -\tilde{G}^T\end{bmatrix}\begin{bmatrix}\dot{q}\\ \nu\end{bmatrix} = 0$$

$$\begin{bmatrix}J & -G^T\end{bmatrix}\begin{bmatrix}\dot{q}\\ \nu\end{bmatrix} = 0$$

← velocity-level constraint

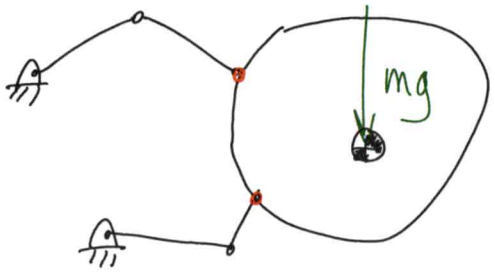
where $J = H\tilde{J}$ & $G^T = H\tilde{G}^T$

Physical Interpretation

$J_i \dot{q} = v_{i,hnd}$ = twist components transmitted to object by hand at contact i under contact model choice.

$G_i^T v = v_{i,obj}$ = twist components transmitted to hand by object at contact i under contact model choice.

Some grasping examples



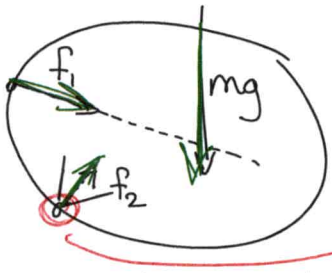
HF, HF – palming a Bball

Our kinematic model will enforce contact maintenance.

In a real system, contact constraints can be maintained by controller if friction high enough.

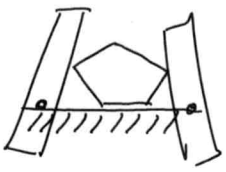
What if we change one contact model to PwoF ?

Then equilibrium will not be satisfied, but kinematic model would enforce contact constraints anyway



sum moments here

No way to satisfy equilib. Look at horizontal comp. of contact forces.



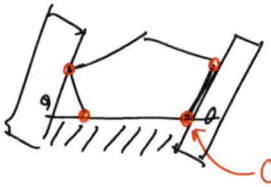
$n_b = 4$

$n_{joint} = 6 =$ 2 fngr & 4 contacts

Assume PwoF $\Rightarrow$ $F \geq 3(4-6-1) + 2(1) + 4(2)$

$F \geq -9 + 10 = 1$

What if system slides until there is a 5th cnt?



$F \geq 3(4-7-1) + 2(1) + 5(2)$

$F \geq -12 + 12 = 0$

count twice (on finger & palm)

Planning & control are easier with few d.o.f., that's why TAMU dexterous hand/object interface was teflon.

What if we assume HF contacts?

$F \geq 3(4-6-1) + 2(1) + 4(1) = -1$

Then $F = 0$