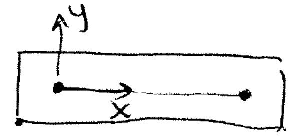
Homework Chap 3, # 14. Implementation 3/24/08

Represent Links in $\mathbb{R}^2$

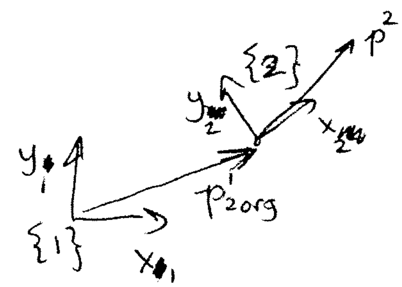


Cond prob 6a

$\{(-1,-1),(L+1,-1),(L+1,1),(-1,1)\}$

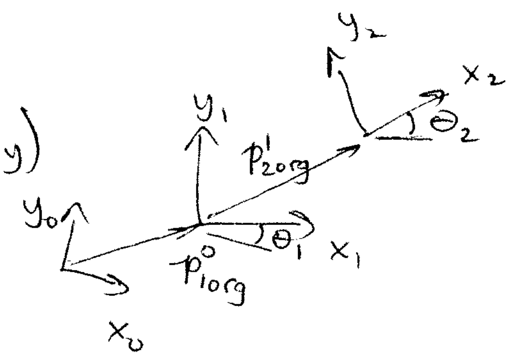
$^{B}p = (L, 0)$

$$T_2^1 = \begin{bmatrix} \hat{x}_2^1 & \hat{y}_2^1 & p_{2org}^1 \\ 0 & 0 & 1 \end{bmatrix}$$



$$p^1 = T_2^1 \, p^2$$

$$T_2^1 = \begin{bmatrix} c_{\theta_1} & -s_{\theta_1} & d_{1x} \\ s_{\theta_1} & c_{\theta_1} & d_{1y} \\ 0 & 0 & 1 \end{bmatrix} = T_2^1(\theta_1, d_{1x}, d_{1y})$$



To implement revolute joint

fix $\underline{d_{1x}, d_{1y}}$ and $\boxed{\text{vary } \theta_1}$

To implement prismatic joint

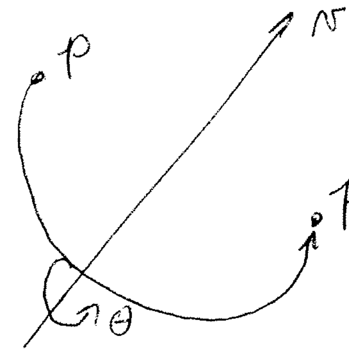
fix $\underline{\theta_1}$ and $\boxed{\text{vary } d_{1x}, d_{1y}}$ to lie along a line

i.e. $d_{1y} = 3d_{1x} + 7$

Don't think of quaternion as operating on a rigid body rather as operating on a point

If there is a point, then you specify (v, θ) $\longrightarrow$

If you specify $(-v, -\theta)$ then p moves to p' also.



Meet w/ Pat to start project. Don't leave off until end.

Forgot to mention:

1. C-space in general has # of components that is exponential in the dimension of $\mathcal{C}$.

2. Constructing all of $\mathcal{C}_{obs}$ is impractical even in SE(3). ~~Com~~

3. Number of features of $\mathcal{C}_{obs}$ is polynomial in # of verts, edge, faces of $\mathcal{A}$ & $\mathcal{O}$.

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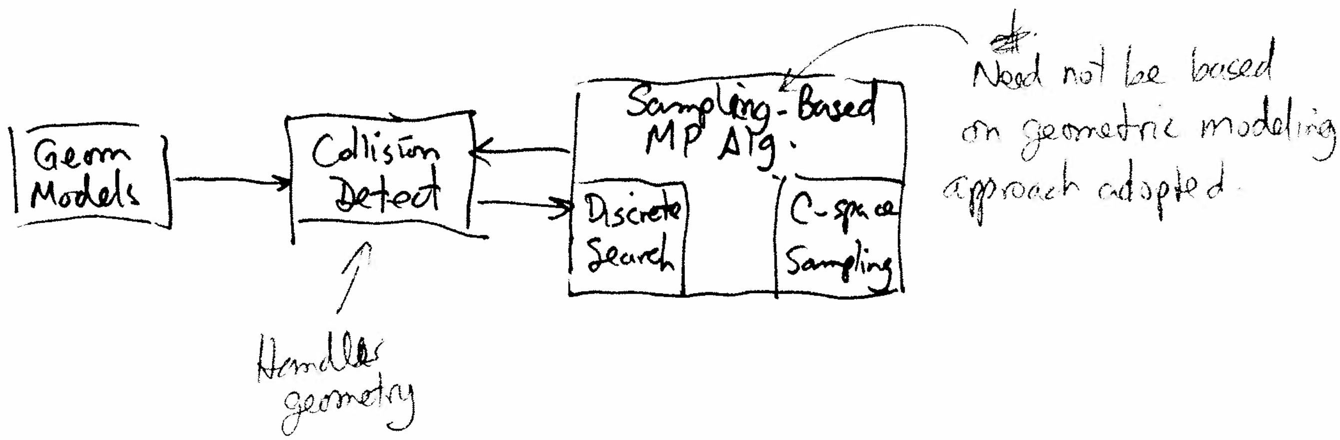
Chapter 5: Sampling-Based MP

This took about 70 min
Need to give general sampling alg. first!

Avoid explicit construction of C_{obs}

Even with polyhedra
\# of components of C_{free} in general is exp the dimension of C.

\# of facets of C_{obs} = At least quadratic where k approx geom. complexity
edges, faces, vertices



This approach assume geometry determines goodness of plan.
Maybe energy use should be included.
Other things.

Completeness — Alg. is complete if it returns soln in finite time if one exists, or it returns non-existance in finite time

Sampling-Based Methods are not complete but are more practical typically

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Resolution Complete

If samples cover C-space densely as time $\rightarrow \infty$, then Sample-Based Methods are resolution complete.

Probabilistically Complete

If sampling is based on a probability dist. and if the sampling is dense over C, then it is prob. complete.

Rate of convergence becomes important, but hard to establish!

5.1 Distance & Volume

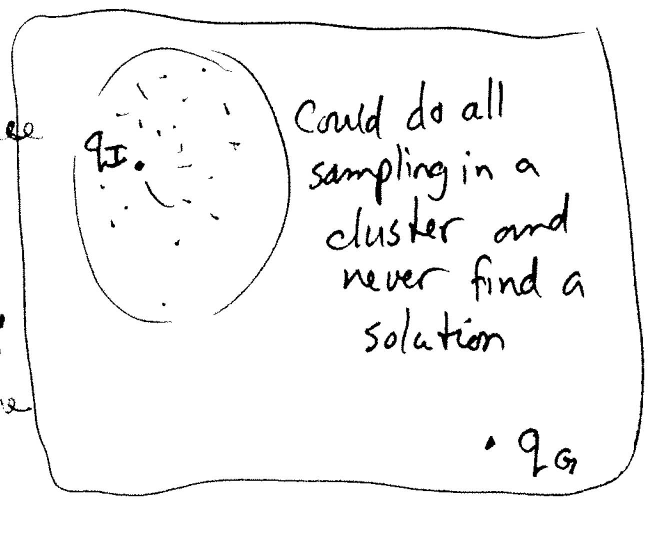
We require a distance function on C for effective sampling

C becomes a <u>metric space</u>

Means you can have a notion of distance

Some measure of volume is also helpful $\Rightarrow$ C is a <u>measure space</u>

Means you can have a notion of volume



Metric Spaces

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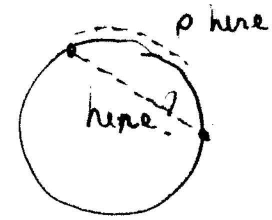
$\mathbb{R}^n$ — the usual Euclidean norm

$\|X_1 - X_2\|$ is a distance metric

$\sqrt{(x_1-x_2)^2 + (y_1-y_2)^2 + \cdots}$



On curved spaces, ρ, we must have

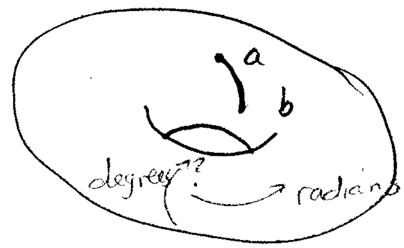
Non neg. $\rho(a,b) \geq 0$ $\quad a, b \in X$

Reflexive $\rho(a,b) = 0$ only if $a = b$

Symmetric $\rho(a,b) = \rho(b,a)$

Triangle Ineq $\rho(a,b) + \rho(b,c) \geq \rho(a,c)$

Should we use geodesic ~~distance~~? Distance metric affects geodesics



Lp Norms

$\boxed{\text{Lp Metrics on } \mathbb{R}^n}$ $\Rightarrow$ $\rho = \left(\sum_{i=1}^{n} |(x_i - x_i')|^p \right)^{1/p}$, $p \geq 1$

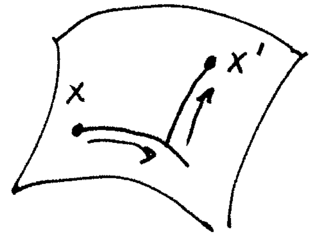
where $x, x' \in \mathbb{R}^n$

L_2 = usual Euclidean Metric

L_1 = Manhattan

$L_\infty = \max_{i=1}^{n} \{ |x_i - x_i'| \}$

Limit as $p \to \infty$ is



Note: Metric Space is (X, ρ) = a topological space w/ a metric.

If you have a metric on SO(3) & $\mathbb{R}^3$, then you have a metric on SE(3)

A product of metric spaces is a metric space.

$X = X_1 \times X_2$ let $c_1, c_2 > 0 \Rightarrow \rho = c_1\rho_1 + c_2\rho_2$

Parametrization & Metric are Important

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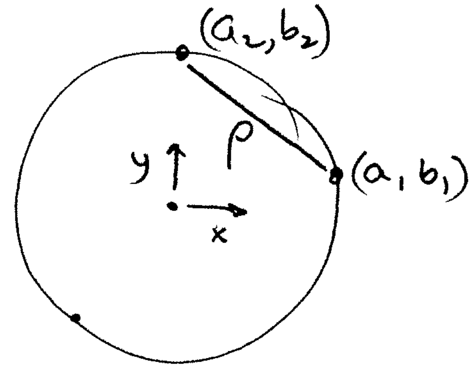
Metric on $SO(2) = \{(a,b) \in \mathbb{R}^2 \mid a^2+b^2=1\}$

L_2 metric: $\rho(a_1,b_1;a_2,b_2) = \sqrt{(a_1-a_2)^2+(b_1-b_2)^2}$

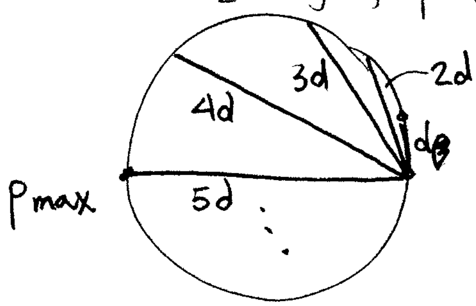


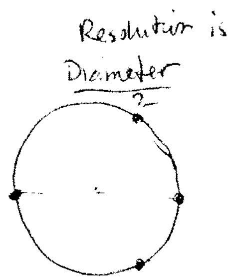
Suppose we sample uniformly?

What does that mean?

It means samples are equally distant, i.e., depends on metric chosen

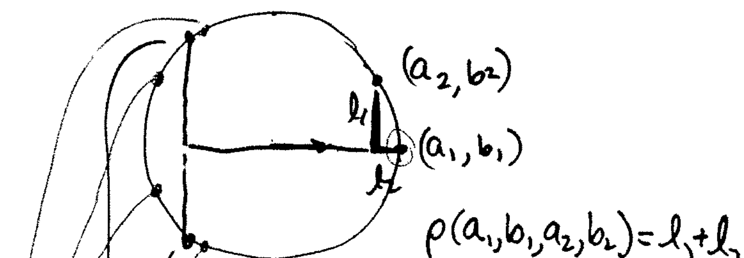
L_2 using a,b parametrization





opposite points are maximally distant

2 points equally distant



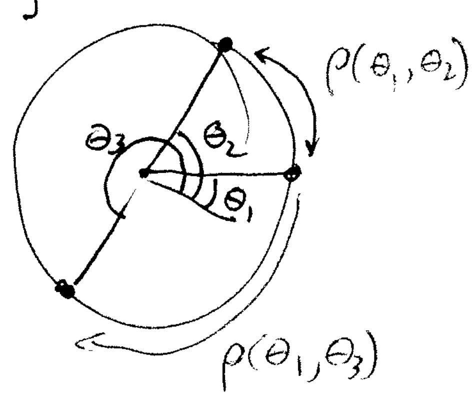
$\rho(a_1,b_1,a_2,b_2) = l_1+l_2$

Two maximally distant points

4 points equally distant from (a_1,b_1).

$$\rho(\theta_1,\theta_2) = \min\{|\theta_1-\theta_2|,\ 2\pi-|\theta_1-\theta_2|\}$$

This assume $\theta_1,\theta_2 \in [0, 2\pi]$



Really you need

$$\boxed{\rho(\theta_1,\theta_2) = \min\{\mathrm{mod}(|\theta_1-\theta_2|, 2\pi), \ldots \ldots 2\pi - \mathrm{mod}(|\theta_1-\theta_2|)\}}$$

Equiv: $\rho(a_1,b_1,a_2,b_2) = \cos^{-1}(a_1a_2+b_1b_2)$

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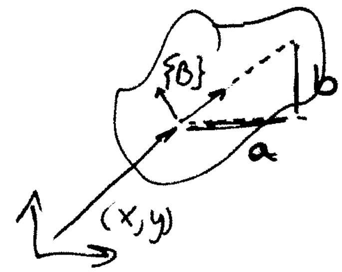
Metric on SE(2)



Let $q \in \mathbb{R}^4$, $q = (x, y, a, b)$

L_2 metric on $\mathbb{R}^4$

$$\rho = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (a_1 - a_2)^2 + (b_1 - b_2)^2}$$

alternate metric $\rho = c_1\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} + c_2\sqrt{(a_1 - a_2)^2 + (b_1 - b_2)^2}$

Could use L_2 in $\mathbb{R}^2$ and L_1 in other $\mathbb{R}^2$

L_p in $\mathbb{R}^2$ and L_k in S^1

⋮

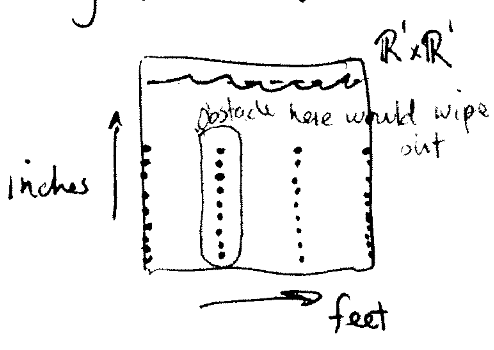
Note: mismatch L in $\mathbb{R}^2$ is in feet, meters, ...

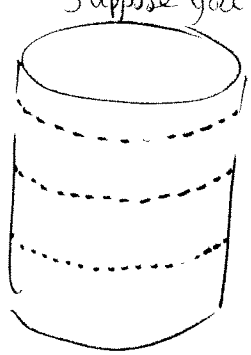
L in S^1 is in radians, degrees, ...

Can be helpful to multiply L in S^1 by a characteristic distance or divide L in $\mathbb{R}^2$ by a characteristic distance

This can be tricky to chose "right" characteristic length.

Imagine $S^1 \times \mathbb{R}^1$





Suppose you grid C for motion planning using constant distances between columns & rows.

Is this uniform?

Depends on metric.

Metric on SO(3)

Let h_i be a unit quaternion

$$\rho(h_1, h_2) = \min\{\|h_1 - h_2\|, \|h_1 + h_2\|\}$$

takes care of wrap-around identification issue

This metric has analogous problem as the metric on SO(2) that cuts straight across the circle.

Fix problem with spherical linear interpolation (analogous to measuring distance along the great circle between two points on S^3 (embedded in $\mathbb{R}^4$).

Let $\rho_s(h_1, h_2) = \cos^{-1}(a_1 a_2 + b_1 b_2 + c_1 c_2 + d_1 d_2)$

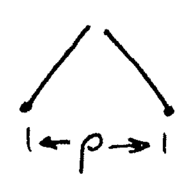
Then a good metric is:

$$\rho(h_1, h_2) = \min\{\rho_s(h_1, h_2), \rho_s(h_1, -h_2)\}$$

The Metric you choose should be an important quantity for the system in question.

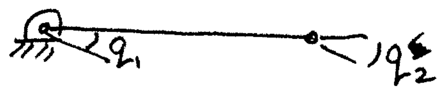
Ex. 5.6 Robot Displacement (max displacement of point)

$$\rho(q_1, q_2) = \max_{a \in A} \{a(q_1) - a(q_2)\}$$



Ex 5.7 Metric on $\mathbb{T}^n$

There are n arbitrary coefficients

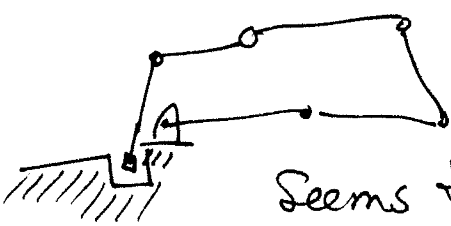


$$\rho = c_1\sqrt{(q_1-q_1')^2} + c_2\sqrt{(q_2-q_2')^2}$$

Maybe c_1 should be very small since $\Delta q_1 \Rightarrow$ big motion at tip.

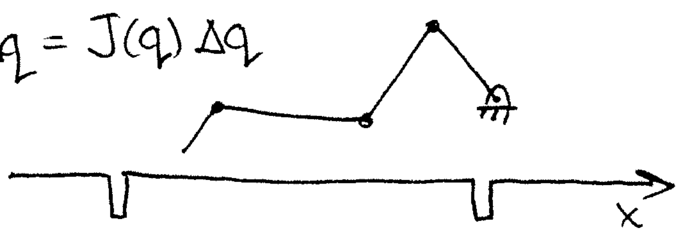
Should c_1 be ~~big~~ small here? peg

What if robot is working close to base?



Seems that metrics may need to be configuration dependent if goal is to sample the workspace uniformly. But would we retain the metric property?

$$\Delta x = J\Delta q = J(q)\,\Delta q$$

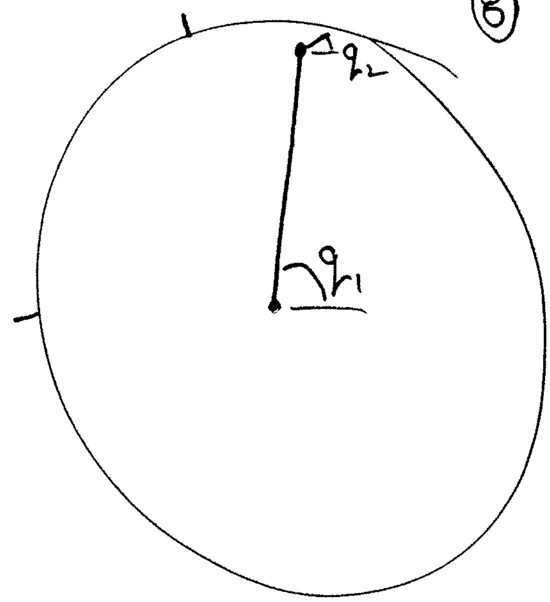


Suppose we want to sample X uniformly to quickly find holes?

Suppose we sample q_1

every $\pi/2$, $\{0, \pi/2, \pi, 3\pi/2\}$

Even if we sample q_2 finely we will never find holes.



Metrics for $SE(3)$.

Same issue with units of $\mathbb{R}^3$ & $SO(3)$

Can add any $SO(3)$ metric to an $\mathbb{R}^3$ metric.

Sampling Theory

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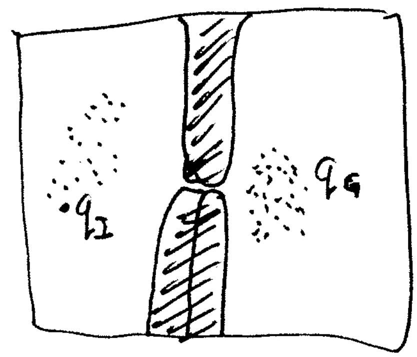
C-space is a set with an uncountably infinite # of elements

But sampling-based planning must terminate after a finite # of samples have been considered.

∴ Sampling techniques should be carefully designed.

Since samples are discrete, the planning/search algs of Ch 2 can be used.

Clearly performance is a function of sampling alg.

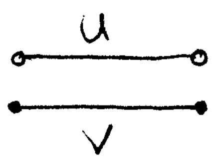


Denseness

A set U is dense in V if

$cl(U) = V$

e.g.



$(0,1)$
$[0,1]$

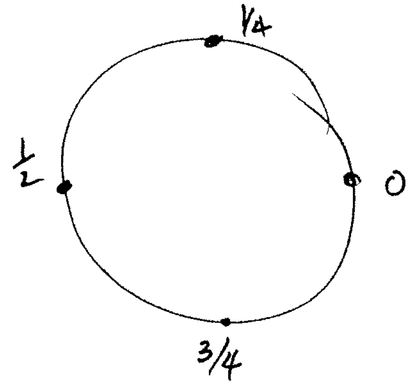
~~If~~ Suppose V is C-space and U is set of samples. To guarantee ~~solution~~ completeness, samples must get arbitrarily close to every point in V.

* $\boxed{\text{Sampling methods must produce a dense set of samples}}$

The van der Corput Sequence

Let $C = [0,1]/\sim \xrightarrow{\text{homeomorphic}}$ to $SO(2) = S^1$

Suppose you want 4 evenly-spaced samples $\{0, 1/4, 1/2, 3/4\}$

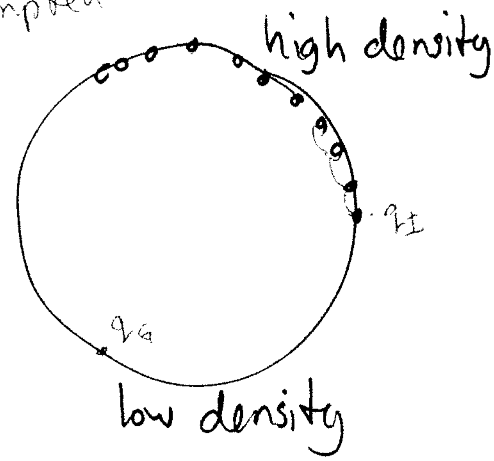


Order of samples	Binary Rep	
	0.00	0
	0.01	1/4
	0.10	1/2
	0.11	3/4

What really matters is order of ~~If we tok all samples first~~ attempted connections'

Suppose you want 16 samples

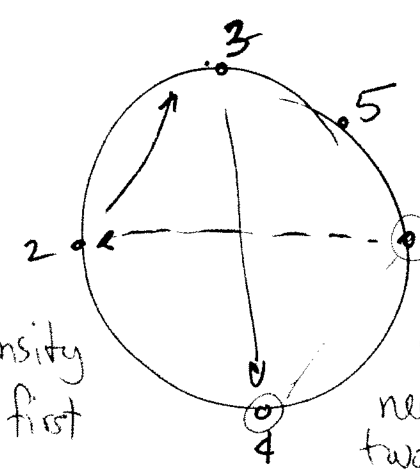
0.0000	0	march around the circle
0.0001	1/16	
0.0010	2/16	
0.0011	3/16	
⋮	⋮	
0.1111	15/16	



van der Corput — flipped the bits

0.0000	0
0.1000	1/2
0.0100	1/4
0.1100	3/4
0.0010	1/8
⋮	

Maintains sampling density in breadth-first manner!



Suppose you only needed these two pts for a solution.

van der Corput sequence benefits

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~~Can~~ Need not determine # of samples in advance.

Sequence is dense in $[0,1]/\sim$

Random Sampling.

Suppose we pick k points at random $U[0,1]$.

The probability that no points fall in ~~the~~ any interval of length e is $(1-e)^k$. This probability $\to 0$ as $k \to \infty$.

$\therefore$ random sampling is dense (w/ probability 1).

Random Sampling of $\mathcal{C}$

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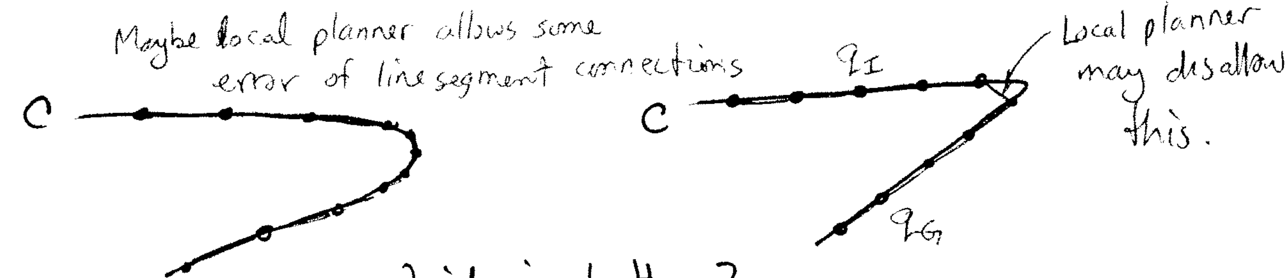
(2)

<u>Goal</u>: Prob(q) is constant over $\mathcal{C}$.

Is that even good?

· It may be the best that can be done w/o knowing more about the problem or the structure of $\mathcal{C}$ (which we don't know in advance in detail).



Which is better?

Uniform sampling of SO(3):

Uniform sampling of Euler angles does not give uniform sampling of orientations in SO(3).

Uniform sampling of unit quaternions does!

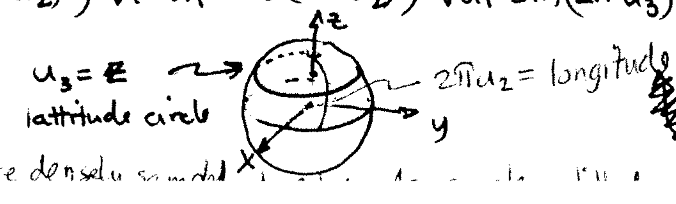
Here's how. Choose $u_1, u_2, u_3 \in [0,1]$ uniformly at random.

A uniform random quaternion is given by:

$$h = \left(\sqrt{1-u_1}\sin(2\pi u_2),\ \sqrt{1-u_1}\cos(2\pi u_2),\ \sqrt{u_1}\sin(2\pi u_3),\ \sqrt{u_1}\cos(2\pi u_3)\right)$$

eq.(5.15)

Imagine sampling uniformly of lattitude & longitude. Sphere will be more densely sampled



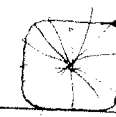
If $\sin(2\pi u_2) < 0$, replace $\sin(2\pi u_2)$ with $-\sin(2\pi u_2)$.

Now, we can sample uniformly (deterministically or at random) over the interval, I, S^1, $SO(3)$.

Since uniform random sample of $X \times Y$ is just (x,y) where x is a uniform rand. sample of X and
y " " " " " " Y

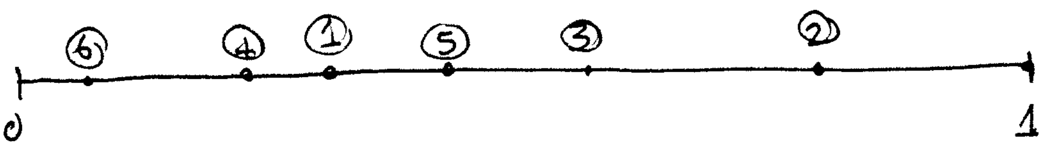
$\therefore$ We can sample uniformly of any $\mathcal{C}$ composed as a product of intervals, circles, & $SO(3)$s.

This is pretty much everything, but not all circles are equal.
If circle is a square, samples will be denser at the points closer to origin.



Question?

Is it better to choose k sample at random on I or is it better to choose them on the largest current subinterval?



.... What's the probability of the greatest distance being more than a given fraction of the interval?