Complementarity Problems

Saturday, February 09, 2008
10:45 AM

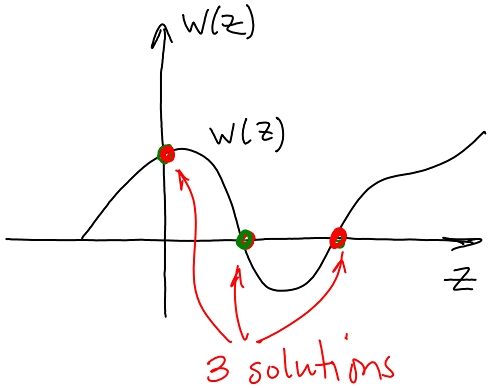
Let w and z be vectors of length n. Further let w(z) be a given function of length n. The complementarity problem is, find z satisfying:

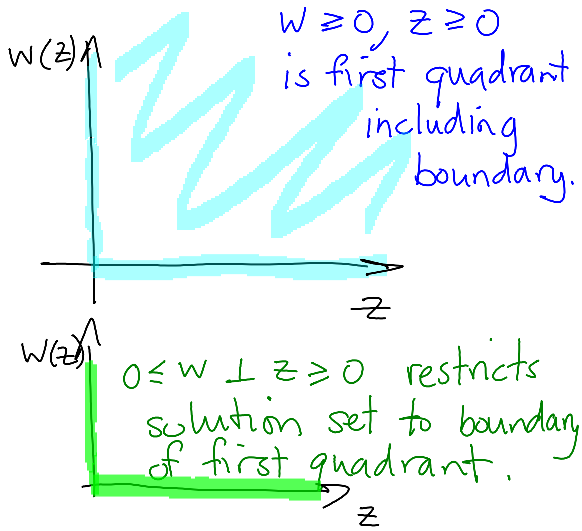
$$z \geq 0, \quad w(z) \geq 0, \quad w(z)^T z = 0$$

We will use the following symmetric notation:

$$\boxed{0 \leq z \perp w(z) \geq 0}$$

C.P. of size 1 (i.e., $w, z \in \mathbb{R}^1$)





Note solution set is not convex. This makes it difficult to characterize all solutions

Solution Methods:

The most robust free solution algorithm is "path" (available at cpnet.org)

dVC uses path, which contains a proprietary component owned by Stanford or a slower non-proprietary solver obtained from the Siconos group in France.

group in France.

Linear Complementarity Problem (LCP)

(Notes taken from "The Linear Complementarity Problem," by Cottle, Pang, Stone, 1992.)

An LCP is just a complementarity problem with
$w(z) = Bz + b$
where $B \in \mathbb{R}^{n \times n}$, $b \in \mathbb{R}^n$

Denote by $LCP(B,b)$, the LCP with $w(z) \triangleq Bz+b$

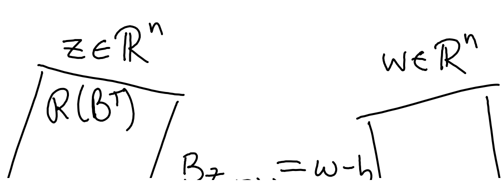
Denote by $SOL(LCP(B,b))$, the solution set of the $LCP(B,b)$.

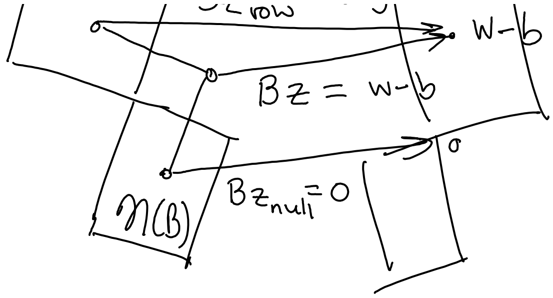
$SOL(LCP(B,b))$ is unique iff B is a matrix of class Q_0 and Lemke's algorithm is guaranteed to find the solution in finite time

Lemke's algorithm is a pivoting method analogous to the simplex method for solving linear programs.

If B is positive definite, then $B \in Q_0$.

Frictionless rigid body systems lead to LCPs whose solutions are "w-unique." Meaning there is a unique solution for $w = Bz+b$, but z is <u>not</u> unique.





Notice that if $\mathcal{N}(B) \neq 0$, then if $w = Bz + b$ has one solution then it has a space of solutions of dimension $n-r$, where $r = \text{rank}(B)$. However to have multiple LCP solutions, we still need $0 \leq z \perp w \geq 0$.