Form Closure: First-Order

Thursday, January 31, 2008
9:12 AM

Definition 1: a grasp has <u>form closure</u> iff it cannot be moved when the fingers are locked

$$\boxed{G_n^T \nu \geq 0 \Rightarrow \nu = 0}$$

Definition 2: a grasp has <u>form closure</u> iff it is possible for frictionless fingers to generate any wrench.

$$\boxed{\left.\begin{array}{r} G_n \lambda_n = g \\ \lambda_n \geq 0 \end{array}\right\} \forall g \in \mathbb{R}^{n_\nu}} \equiv \boxed{\exists\, \lambda_n > 0 \ni G\lambda_n = 0}$$

~~Will use~~ this for computation

Necessary condition $\text{rank}(G_n) = n_\nu$!

Necessary condition $n_c \geq n_\nu + 1$

Example: Object in $\mathbb{R}^2$

$$\nu = \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix}$$

$$G_n = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix}$$

<u>Twist Test</u>

$$G_n^T \nu \geq 0 \Rightarrow \left.\begin{array}{r} \dot{x} \geq 0 \\ \dot{y} \geq 0 \\ -\dot{x} \geq 0 \\ -\dot{y} \geq 0 \end{array}\right\} \Rightarrow \begin{array}{l} \dot{x} = 0 \\ \dot{y} = 0 \end{array} \Rightarrow \text{Form Closure exists!}$$

<u>Wrench Test</u>

$$G_n \lambda_n = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} f_{1n} \\ f_{2n} \\ f \end{bmatrix} = \begin{bmatrix} f_{1n} - f_{3n} \\ f - f \end{bmatrix} = \begin{bmatrix} -F_x \\ F_y \end{bmatrix} = -g$$

Wrench Test

$$G_n \lambda_n = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} f_{1n} \\ f_{2n} \\ f_{3n} \\ f_{4n} \end{bmatrix} = \begin{bmatrix} f_{1n} - f_{3n} \\ f_{2n} - f_{4n} \end{bmatrix} = \begin{bmatrix} -F_x \\ -F_y \end{bmatrix} = -g$$

Note that given any g, we can find $\lambda_n \ni G_n \lambda_n = -g$!

$\therefore$ Form closure exists.

Other interpretations:

Velocity constraints

Each contact eliminates a half-space of body twists

Form closure means that no twists are possible

Assume Triangle can only translate in the plane.

Wrench generation

Nonnegative span of columns of G_n is $\mathbb{R}^{n_v}$

Geometric interpretation in Wrench Space

Convex hull of columns of G_n strictly includes origin.

Geometric interpretation

Geometric interpretation
in work space (planar only)

If $\exists$ Cones, C_1 & C_2, formed with pairs of normals, and line segment L connecting the cone apexes $\ni$ L lies entirely in $C_1 \cap C_2$ or $-C_1 \cap -C_2$, then the grasp has form closure.

Computation Tests for Form Closure.

<u>Linear program</u>

$$\min_x \; c^T x$$
$$\text{s.t.} \quad Ax \geq b$$

Matlab has good solver.

Form closure requires $\text{rank}(G) = n_v$ <u>and</u> the existence of $\lambda_n > 0 \ni G_n \lambda_n = 0$

<u>Second part</u>

We must satisfy $G_n \lambda_n = 0$
$\lambda_n > 0$

How do we "encourage" λ_n to be positive?

$$G_n \lambda_n = 0$$
$$\lambda_n - \text{slack} \geq 0 \longleftarrow \lambda_n \geq \text{slack}$$

slack ≥ 0

Maximize the slack variable

L.P. $\max_{\lambda_n, d} \; d$ ← d is a measure of how far the grasp is from being form closure

s.t: $G_n \lambda_n = 0$

$I\lambda_n - \mathbb{1} d \geq 0$

$d \geq 0$

$\mathbb{1}^T \lambda_n \leq n_c$ ← prevent unboundedness

Test for form closure.

1 Rank $G = n_v$

If not, stop. No frictional form closure

2 Compute solution to LP.

If $d > 0$, then frictional form closure exists!

$G_n = \begin{bmatrix} 1 & -1 \end{bmatrix}$

$\lambda_n = \begin{bmatrix} \lambda_{1n} \\ \lambda_{2n} \end{bmatrix} = \begin{bmatrix} f_{1n} \\ f_{2n} \end{bmatrix}$

$$\max_{f_{1n}, f_{2n}, d} \; d$$

s.t.

$$f_{1n} - f_{2n} = 0$$
$$f_{1n} - d \geq 0$$
$$f_{2n} - d \geq 0$$
$$d \geq 0$$
$$f_{1n} + f_{2n} \leq 2$$

Feasible Solutions

$f_{1n} = f_{2n}$	$d \leq f_{1n}$
0	0
0.5	[0, 0.5]
1	[0, 1]
~~1.1~~	~~[0, 1.1]~~

Without last constraint, f_{1n} could increase without bound – infinitely tight squeezing

Planar Example

Figure 28.17

Form closure exists for

$1.052 < \alpha < \pi/2$

No form closure if $\alpha = \pi/2$

See plot in Figure 28.18