Force Closure & Frictional Form Closure

Thursday, January 31, 2008
3:04 PM

Differences between form closure & force closure.

Form closure is completely geometric - requires n_v+1 contacts.

Force closure adds:

1) Use of friction to reduce # of contacts
2) Use of finger structure/capabilities.

Planar Case: $\lambda_i = [f_{in}, f_{it}]^T$

Dry (Coulomb) friction

$-\mu_i f_{in} \leq f_{it} \leq \mu_i f_{in}$

μ = coeff of friction ≥ 0

$$\mathcal{F} = \{(f_{in}, f_{it}) \mid -\mu_i f_{in} \leq f_{it} \leq \mu_i f_{in},\ \mu_i \geq 0\}$$

- Model implies that $f_{in} \geq 0$
- Contact force must lie in the cone, $\mathcal{F}$
- if contact force is in the interior of $\mathcal{F}$, $\mathrm{Int}(\mathcal{F})$, the contact is sticking/rolling
- if contact force is on boundary of $\mathcal{F}$, $\partial\mathcal{F}$, then sliding can occur
- if sliding is occurring, then $\lambda_i \in \partial\mathcal{F}$

Spatial Case: $\lambda_i = [f_{in}\ f_{it}\ f_{io}]^T$

$$\mathcal{F} = \{(f_{in}, f_{it}, f_{io}) \mid \mu_i^2 f_{in}^2 - f_{it}^2 - f_{io}^2 \geq 0,\ \mu_i \geq 0\}$$

Coulomb friction cone constraint on λ_i in 3D is quadratic cone.

First we study frictional form closure.

Definition: A grasp has frictional form closure iff:

1.) $\text{rank}(G) = n_v$

2.) $\exists\, \lambda > 0 \ni G\lambda = 0,\ \lambda \in \text{Int}\,\mathcal{F}$

Planar Case:

4 HF $\Longleftrightarrow$ 8 PwoF

Equivalent

Cone of possible contact forces $\left\{ \begin{array}{c} s_1\sigma_1 + s_2\sigma_2 \\ \sigma_1, \sigma_2 \geq 0 \end{array} \right\} \equiv \left\{ \begin{array}{c} \hat{n}_1\lambda_{1n} + \hat{n}_2\lambda_{2n} \\ \lambda_{1n}, \lambda_{2n} \geq 0 \end{array} \right\}$

cone of possible contact $\left\{ \begin{bmatrix} \overbrace{\begin{matrix} s_1 & s_2 \\ r_1 \times s_1 & r_2 \times s_2 \end{matrix}}^{S_1} \end{bmatrix} \overbrace{\begin{bmatrix} \sigma_1 \\ \sigma_2 \end{bmatrix}}^{\sigma_1} \right. \equiv \begin{bmatrix} \hat{n}_1 & \hat{n}_2 \\ r_1 \times \hat{n}_1 & r_2 \times \hat{n}_2 \end{bmatrix} \begin{bmatrix} \lambda_{1n} \\ \lambda_{2n} \end{bmatrix}$

wrenches $\sigma_1, \sigma_2 \geq 0$ $\lambda_{1n}, \lambda_{2n} \geq 0$

S_i contains two wrenches:
one for each edge of friction cone at contact i.

However,
All we really care about is a constraint that allows the contact wreach to be anything in the friction cone.

A more convenient way is:

$$\left.\begin{aligned} w_{i1} \cdot \lambda_i &\geq 0 \\ w_{i2} \cdot \lambda_i &\geq 0 \end{aligned}\right\} \text{These guarantee contact force is in friction cone.}$$

$$\begin{bmatrix} \cdot_1^T \\ \cdot_2^T \end{bmatrix} \lambda_i = F_i \lambda_i \geq 0$$

$$\max_{\lambda, d} \; d$$

$$\text{s.t.:} \quad G\lambda = 0$$

$$F\lambda - \mathbb{1} d \geq 0$$

$$d \geq 0$$

In chapter, e is defined such that $e\lambda \leq n_c \iff \sum_{i=1}^{n_c} \lambda_{in} \leq n_c$

If $d^* > 0$, then frictional form closure exists.

$$\begin{bmatrix} & \vdots & \\ & \vdots & \end{bmatrix}$$

2.) If frictional form closure exists, then

$\mathcal{N}(G) \cap \mathcal{N}(J^T) \overset{\text{must}}{=} 0$, ie. all internal forces must be controllable.

Check this by solving

Check this by solving

LP3:

$$\max_{\lambda, d} \; d$$

$$\text{s.t: } G\lambda = 0$$

$$J^T\lambda = 0$$

$$E\lambda - \mathbb{1} d \geq 0$$

$$d \geq 0$$

$$e\lambda \leq n_c \iff \sum_{i=1}^{n_c} \lambda_{in} \leq n_c$$

where $E \triangleq \text{block diag}(e_1, \cdots, e_{n_c})$