Lecture 1 Velocity Kinematics 1/9/08

Grasping: Definitions & ~~Models~~

Goal: Build model of grasp system sufficient to successfully design and execute grasps.

Refer to Figure 28.2
and Table 28.1
Introduce terminology/notation.

Need to understand how contact between finger & object constrain the object.

① How does moving a joint move the object? - Jacobian - J_{ij}

② How does locking a joint constrain the object?

③ How does joint torque apply force? - Jacobian

④ What about slipping? - Friction models

Notation

n_c = # of contacts – assume points

n_q = # of joint dof –

n_v = # of object dof = $6 \cdot n_b$

$q \in \mathbb{R}^{n_q}$ = joint displacements – really $\underbrace{S^1 \times S^1 \times \cdots S^1}_{\substack{\#\text{ of} \\ \text{revolute} \\ \text{joints}}} \times \underbrace{\mathbb{R}^1 \times \cdots \mathbb{R}^1}_{\substack{\#\text{ of} \\ \text{prismatic} \\ \text{joints}}}$

$\dot{q} \in \mathbb{R}^{n_q}$ = ~~Ang~~ joint velocities

$\tau \in \mathbb{R}^{n_q}$ = noncontact joint loads = fully actuated

$u \in \mathbb{R}^{n_u}$ = config of ~~body~~ object (or bodies) – really $\mathbb{R}^3 \times SO(3)$

$\nu \in \mathbb{R}^{n_v}$ = twist of ~~body~~ object = $\begin{bmatrix} v \\ \omega \end{bmatrix}$

$g \in \mathbb{R}^{n_v}$ = non-contact load on object

$\{B\}$ = frame fixed in body

$\{C\}_i$ = frame fixed at contact point i

$\{N\}$ = inertial frame – nonaccelerating

Screws,
Twists, Wrenches

1/9/08

②

Chasle's Thm:

Every spatial displacement is the composition of translation along an axis and rotation about that axis.

Twist $\in \mathbb{R}^6$, ν.

Instantaneous screw.

$$\nu = \begin{bmatrix} v \\ \omega \end{bmatrix}^N$$

v — velocity of point p exp. in $\{N\}$ by def.

ω — angular velocity of body exp. in $\{N\}$

Poinsot's Thm:

Every system of forces is equivalent to a single force plus a couple w/ moment parallel to the force

Wrench $\in \mathbb{R}^6$, $g = \begin{bmatrix} f \\ m \end{bmatrix} = \begin{bmatrix} f_1 + f_2 + f_3 \\ m_1 + m_2 + r_1 \times f_1 + r_2 \times f_2 + r_3 \times f_3 \end{bmatrix}$

Contact Twist on Object – vel of object at contact pt.

$$\begin{bmatrix} v^c_{i,obj} \\ \omega^c_{obj} \end{bmatrix} = \nu_{i,obj} = \underset{(6\times 6)}{\tilde{G}_i^T} \nu \qquad (28.4)$$

by definition. Express in $\{C_i\}$

c_i = ith contact point

Given:

$v^N = \dot{p}$

ω^N = ang. vel of body

Determine:

v_i^c & ω_i^c

Angular velocity is same at all points

$\therefore \ \omega_i^{c_i} = R_N^{c_i} \omega_i^N$

Translation velocity of contact point in $\{N\}$

$$v_i^N = v^N + \omega_i^N \times (c_i^N - p^N)$$

$$= v^N - \underbrace{(c_i^N - p^N)}_{v} \times \omega_i$$

DEFINE CROSS PROD MATRIX

$$\begin{bmatrix} 0 & -z & y \\ z & 0 & -x \\ -y & x & 0 \end{bmatrix}$$

$$\begin{bmatrix} v_i^N \\ \omega_i^N \end{bmatrix} = \underbrace{\begin{bmatrix} I & \tilde{S}^T(c_i - p) \\ 0 & I \end{bmatrix}}_{P^T \ (6\times 6)} \begin{bmatrix} v^N \\ \omega^N \end{bmatrix}$$

Final step just a frame change.

$$\nu_{i,obj} = \begin{bmatrix} v_i^c \\ \omega_i^c \end{bmatrix} = \begin{bmatrix} R_i^T & \\ & R_i^T \end{bmatrix} \begin{bmatrix} I & \tilde{S}^T(c_i - p) \\ 0 & I \end{bmatrix} \begin{bmatrix} v^N \\ \omega^N \end{bmatrix} = \tilde{R}_i^T P_i^T \nu = \tilde{G}_i^T \nu$$

$${}^N_{c_i}T = \left[\begin{array}{c|c} R^N_{c_i} & c_i^N \\ \hline 0\ 0\ 0 & 1 \end{array}\right]$$

Next

$$v_i^N = R^N_{c_i} v_i^{c_i}$$

$$\therefore \ v_i^{c_i} = R^{c_i}_N v_i$$

The contact fra

$$R^N_{c_i} = \left[\hat{n}_i^N \ \hat{t}_i^N \ \hat{o}_i^N\right]$$

Skip to next page

Review (B) Change of coordinates via Rb

$$v_i^c = \begin{bmatrix} v_{in}^c \\ v_{it}^c \\ v_{io}^c \end{bmatrix} \qquad \omega_i^c = \begin{bmatrix} \omega_{it}^c \\ \omega_{io}^c \end{bmatrix}$$

$$\tilde{G}_i^T = \begin{bmatrix} R_i^T & R_i^T S^T(c_i - p) \\ 0 & R_i^T \end{bmatrix} = \begin{bmatrix} \hat{n}_i^T & ((c_i-p)\times \hat{n}_i)^T \\ \hat{t}_i^T & ((c_i-p)\times \hat{t}_i)^T \\ \hat{o}_i^T & ((c_i-p)\times \hat{o}_i)^T \\ 0_{(3\times1)}^T & \hat{n}_i^T \\ 0_{(3\times1)}^T & \hat{t}_i^T \\ 0_{(3\times1)}^T & \hat{o}_i^T \end{bmatrix}_{(6\times6)}$$

(B) How does one change frame of expression of a vector?

given a vector a^N

convert coordinates to a^{c_i}

$$\begin{bmatrix} (a^{c_i})_{n_i} = a\cdot\hat{n}_i = (\hat{n}_i^T a) \\ (a^{c_i})_{t_i} = a\cdot\hat{t}_i = \hat{t}_i^T a \\ (a^{c_i})_{o_i} = a\cdot\hat{o}_i = o_i^T a \end{bmatrix}$$

any frame of expression is fine

$$\Rightarrow \underbrace{\begin{bmatrix} (\hat{n}_i^N)^T \\ (\hat{t}_i^N)^T \\ (\hat{o}_i^N)^T \end{bmatrix}}_{(R_{c_i}^N)^T = R_N^{c_i}} a^N = a^{c_i} \Bigg\} = \Rightarrow \underline{a^{c_i} = R_N^{c_i} a^N}$$

Contact Twist on Hand

$$\boxed{\underset{(6\times 1)}{\nu_i^{hnd}} = \underset{(6\times n_q)}{\tilde{J}_i}\,\dot{q} \qquad (28.10)}$$

by definition, express in frame i

same Jacobian derived in Robotics I all the way out to the tool tip!

$$\nu_{i,hnd}^N = \begin{bmatrix} v_{i,hnd}^N \\ \omega_{i,hnd}^N \end{bmatrix} = Z_i\, q_i \qquad (28.7)$$

$$Z_i = \begin{bmatrix} \cdots & d_{ij} & \cdots \\ \cdots & l_{ij} & \cdots \end{bmatrix}$$

This is just a Jacobian as learned in Robotics I.

i is index of contact

j is index of joint

Physical Interp.

d_{ij} velocity induced at contact point i due to joint velocity at joint j. IN FRAME {N}

l_{ij} angular velocity induced at contact i by joint j. IN FRAME {N}.

Put all contacts together

$$\boxed{\begin{aligned} \underset{(6n_c\times 1)}{\nu_{c,obj}} &= \underset{(6n_c\times n_\nu)}{\tilde{G}^T}\,\underset{(n_\nu\times 1)}{\nu} \qquad (28.12) \\ \underset{(6n_c\times 1)}{\nu_{c,hnd}} &= \underset{(6n_c\times n_q)}{\tilde{J}}\,\underset{(n_q\times 1)}{\dot{q}} \qquad (28.13) \end{aligned}}$$

Let

$$\nu_{c,obj} = \underset{(6n_c\times 1)}{\begin{bmatrix} \nu_{1,obj} \\ \nu_{2,obj} \\ \vdots \\ \nu_{n_c,obj} \end{bmatrix}} = \underset{(6n_c\times n_\nu)}{\begin{bmatrix} \tilde{G}_1^T \\ \tilde{G}_2^T \\ \vdots \\ \tilde{G}_{n_c}^T \end{bmatrix}} \underset{(n_\nu\times 1)}{\nu}$$

$$\therefore \quad \tilde{G}^T = \begin{bmatrix} \tilde{G}_1^T \\ \tilde{G}_2^T \\ \vdots \\ \tilde{G}_{n_c}^T \end{bmatrix}_{(6n_c\times 6)}$$

Similarly $\quad \tilde{J} = \begin{bmatrix} \tilde{J}_1 \\ \vdots \\ \tilde{J}_{n_c} \end{bmatrix}_{(6n_c\times q)}$

Ex1, P1

Saturday, January 12, 2008
8:18 PM

Two contacts

$i = \{1, 2\}$

First Determine R_i, $i = 1, 2$

$$R_1 \triangleq \begin{bmatrix} \hat{n}_1^N & \hat{t}_1^N & \hat{o}_1^N \end{bmatrix}$$

By inspection:

$$R_1 \cong \begin{bmatrix} 0.9 & -0.4 & 0 \\ 0.4 & 0.9 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad R_2 \cong \begin{bmatrix} -0.93 & -0.3 & 0 \\ 0.3 & -0.93 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Recall $\bar{R}_i = \mathrm{diag}(R_i, R_i)$

Now compute $\tilde{G} = \begin{bmatrix} \tilde{G}_1 & \tilde{G}_2 \end{bmatrix} \equiv \tilde{G}^T = \begin{bmatrix} \tilde{G}_1^T \\ \tilde{G}_2^T \end{bmatrix}$

$$\tilde{G}_1^T = \bar{R}_1^T P_1^T$$

$$P_1 = \begin{bmatrix} I & 0 \\ S(c_1 - p) & I \end{bmatrix}_{6\times 6}$$

By inspection

$$c_1 - p \simeq \begin{bmatrix} -1.8 \\ -0.8 \\ 0 \end{bmatrix} \qquad S(c_1 - p) = \begin{bmatrix} 0 & 0 & -0.8 \\ 0 & 0 & 1.8 \\ 0.8 & -1.8 & 0 \end{bmatrix}$$

$$c_2 - p \approx \begin{bmatrix} 1.86 \\ -0.6 \\ 0 \end{bmatrix} \qquad S(c_2 - p) = \begin{bmatrix} 0 & 0 & -0.6 \\ 0 & 0 & -1.86 \\ 0.6 & 1.86 & 0 \end{bmatrix}$$

$$\tilde{G}_1^T = R_1^T P_1^T = \begin{bmatrix} 0.9 & 0.4 & 0 & & & \\ -0.4 & 0.9 & 0 & & 0 & \\ 0 & 0 & 1 & & & \\ & & & 0.9 & 0.4 & 0 \\ & 0 & & -0.4 & 0.9 & 0 \\ & & & 0 & 0 & 1 \end{bmatrix} \left[\begin{array}{c|ccc} I & 0 & 0 & 0.8 \\ & 0 & 0 & -1.8 \\ & -0.8 & 1.8 & 0 \\ \hline 0 & & I & \end{array}\right]$$

$$\tilde{G}_1^T \simeq \begin{bmatrix} 0.9 & 0.4 & 0 & 0 & 0 & 0 \\ -0.4 & 0.9 & 0 & 0 & 0 & -2 \\ 0 & 0 & 1 & -0.8 & 1.8 & 0 \\ & & & 0.9 & 0.4 & 0 \\ & 0 & & -0.4 & 0.9 & 0 \\ & & & 0 & 0 & 1 \end{bmatrix}$$

$$\tilde{G}_1^T = \begin{bmatrix} \hat{n}_1^T & (c_1 - p) \times \hat{n}_1 \\ \hat{t}_1^T & (c_1 - p) \times \hat{t}_1 \\ \hat{o}_1^T & (c_1 - p) \times \hat{o}_1 \\ 0 & \hat{n}_1^T \\ 0 & \hat{t}_1^T \\ 0 & \hat{o}_1^T \end{bmatrix}$$

Estimate $\tilde{G}_2^T$ by inspection

Recall physical interpretation

$$\tilde{G}_i^T \nu^N = \nu^C_{i,obj}$$

$$\begin{bmatrix} \| & \| & \| & \| & \| & \| \end{bmatrix} \begin{bmatrix} N_x \\ N_y \\ N_z \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \begin{bmatrix} N_{in} \\ N_{it} \\ N_{io} \\ \omega_{in} \\ \omega_{it} \\ \omega_{io} \end{bmatrix}$$

$$\tilde{G}_2^T \simeq \left[\begin{array}{ccc|ccc} -0.93 & 0.3 & 0 & 0 & 0 & 0 \\ -0.3 & -0.93 & 0 & 0 & 0 & -2 \\ 0 & 0 & 1 & -0.6 & -1.86 & 0 \\ \hline & & & -0.93 & 0.3 & 0 \\ & 0 & & -0.3 & -0.93 & 0 \\ & & & 0 & 0 & 1 \end{array}\right]$$

$$\tilde{G}^T = \begin{bmatrix} \tilde{G}_1^T \\ \tilde{G}_2^T \end{bmatrix}_{(12 \times 6)}$$

Now estimate $\tilde{J} = \begin{bmatrix} \tilde{J}_1 \\ \tilde{J}_2 \end{bmatrix}$

$$\tilde{J}_1 = \bar{R}_1^T Z_1 \qquad Z_1 = \begin{bmatrix} d_{11} & d_{12} & \cdots & d_{15} \\ l_{11} & l_{12} & \cdots & l_{15} \end{bmatrix}$$

There are 5 jnts

Contact 1 is on finger 1 which has joints 1 & 2.

$$\therefore \quad d_{13} = d_{14} = d_{15} = l_{13} = l_{14} = l_{15} = d_{21} = d_{22} = l_{21} = l_{22} = 0_{(3\times1)}$$

What about others?

$$d_{11} \simeq \begin{bmatrix} -4.5 \\ -1.0 \\ 0 \end{bmatrix} \quad d_{12} \simeq \begin{bmatrix} -1.3 \\ -0.6 \\ 0 \end{bmatrix} \quad d_{23} = \begin{bmatrix} 0 \\ 0 \\ 2.5 \end{bmatrix} \quad d_{24} \simeq \begin{bmatrix} 0 \\ 0 \\ 3.5 \end{bmatrix} \quad d_{25} \simeq \begin{bmatrix} -0.4 \\ -0.9 \\ 0 \end{bmatrix}$$

$$l_{11} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad l_{12} \simeq \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad l_{23} \simeq \begin{bmatrix} 0.6 \\ 0.8 \\ 0 \end{bmatrix} \quad l_{24} \simeq \begin{bmatrix} 0.9 \\ 0.4 \\ 0 \end{bmatrix} \quad l_{25} \simeq \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$Z_1 = \begin{bmatrix} -4.5 & -1.3 & \text{jnts 3,4,5, do not} \\ -1.0 & -0.6 & \text{affect vel. of} \\ 0 & 0 & \text{contact point 1} \\ 0 & 0 & \\ 0 & 0 & \\ 1 & 1 & \end{bmatrix}_{(6\times5)} \qquad (6\times3)$$

$$Z_2 = \begin{bmatrix} \text{jnts 1,2} & & & & \\ \text{don't impact} & 0 & 0 & -0.4 \\ \text{vel at contact} & 0 & 0 & -0.9 \\ \text{point 2} & 2.5 & 3.5 & 0 \\ & 0.6 & 0.9 & 0 \\ & 0.8 & 0.4 & 0 \\ & 0 & 0 & 1 \end{bmatrix}_{(6\times5)} \qquad (6\times2)$$

$$\tilde{J}_1 = \underset{(6\times6)}{\bar{R}_1^T} \underset{(6\times5)}{Z_1} \qquad \tilde{J}_2 = \underset{(6\times6)}{\bar{R}_2^T} \underset{(6\times5)}{Z_2} \implies \tilde{J}_i \text{ is } (6\times5)$$

$$\tilde{J} = \begin{bmatrix} \tilde{J}_1 \\ \tilde{J}_2 \end{bmatrix}_{(12\times5)}$$

Now estimate $\tilde{J}$ by inspection

Recall $\tilde{J}\dot{q} = \nu_{c,hnd}$

$$\begin{bmatrix} -4.9 & -1.5 & & & & & & \\ 0.5 & 0 & & & & & & \\ 0 & 0 & & & & & & \\ 0 & 0 & & & & & & \\ 0 & 0 & & & & & & \\ 1 & 1 & & & & & & \\ & & 0 & 0 & 0 \\ & & 0 & 0 & -1.4 \\ & & 12.5 & 3.5 & 0 \\ & & 0 & -.9 & 0 \\ & & 1 & -.4 & 0 \\ & & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \\ \dot{q}_5 \end{bmatrix} = \begin{bmatrix} v_{1n} \\ v_{1t} \\ v_{1o} \\ \omega_{1n} \\ \omega_{1t} \\ \omega_{1o} \\ v_{2n} \\ v_{2t} \\ v_{2o} \\ \omega_{2n} \\ \omega_{2t} \\ \omega_{2o} \end{bmatrix}$$