In-class_LCP_soln

Monday, February 21, 2011
2:34 PM

1. Find all $z \ni z$ satisfies the LCP(A, b), where

$$A = \begin{bmatrix} 1 & 3 \\ -2 & 0 \end{bmatrix} \quad \text{and} \quad b = \begin{bmatrix} 6 \\ 4 \end{bmatrix}$$

$$0 \leq z \perp Az + b \geq 0$$

Expand ....

$$0 \leq z_1 \perp z_1 + 3z_2 + 6 \geq 0$$
$$0 \leq z_2 \perp -2z_1 + 4 \geq 0$$

Notice that the r.h.s. inequalities are always strictly positive. $\therefore$ $z_1 = z_2 = 0$

Notice also that the solution is unique even though A is not P.D, so there's no guarantee of solution if we change b.

2. Repeat the problem with $b = \begin{bmatrix} -6 \\ -4 \end{bmatrix}$

Check $z_1 = z_2 = 0$.

$$\Longrightarrow \quad 0 \leq \begin{bmatrix} 0 \\ 0 \end{bmatrix} \perp \begin{bmatrix} -6 \\ -4 \end{bmatrix} \geq 0 \quad \text{FAIL!}$$

$$\Longrightarrow \quad 0 \le \begin{bmatrix} 0 \\ 0 \end{bmatrix} \perp \begin{bmatrix} 0 \\ -4 \end{bmatrix} \ge 0 \qquad \text{FAIL!}$$

Try setting the r.h.s. to zero

$$Az + b = 0 \implies z = -A^{-1}b = \begin{bmatrix} 2 \\ -8/3 \end{bmatrix} \quad \text{FAIL!}$$

Try setting one r.h.s. ineq. to zero, i.e. $0 \le \begin{bmatrix} + \\ 0 \end{bmatrix} \perp \begin{bmatrix} 0 \\ + \end{bmatrix} \ge 0$

Enforce the equations:

$$z_2 = 0 \;, \quad z_1 + 3z_2 - 6 \ge 0 \implies z_1 \ge 6$$

Test the inequalities

$$z_1 \ge 0 \quad \text{Yes.} \qquad -2z_1 - 4 \ge 0 \qquad \text{No!}$$

Last chance for solution $\quad 0 \le \begin{bmatrix} 0 \\ + \end{bmatrix} \perp \begin{bmatrix} + \\ 0 \end{bmatrix} \ge 0$

Eqs:

$$z_1 = 0 \;, \quad -2z_1 - 4 = 0 \quad \Leftarrow$$

Notice 2 equations gave us only one piece of info!

Ineqs:

$$-4 \ge 0 \qquad \text{Fail!}$$

We could have seen this at the start.

$$0 \le z_1 \perp \sim\!\sim\!\sim \ge 0$$

$$0 \le \sim \perp -2z_1 - 4 \ge 0$$

— will always violate

$0 \leq w \perp \underbrace{-2z_1 - 4}_{\text{Will always violate nonnegativity}} \geq 0$

Will always violate nonnegativity

3. Repeat the problem with $b = \begin{bmatrix} -6 \\ 4 \end{bmatrix}$.

$$0 \leq z_1 \perp z_1 + 3z_2 - 6 \geq 0$$
$$0 \leq z_2 \perp -2z_1 + 4 \geq 0$$

Notice: $z_1 = z_2 = 0$ can't work because of the -6.

If we let z_1 & z_2 grow enough, the -6 will be balanced.

If we let z_1 grow too much we'll cause infeasibility of the second r.h. inequality.

Case 1: $\begin{bmatrix} 0 \\ 0 \end{bmatrix} \perp \begin{bmatrix} + \\ + \end{bmatrix}$ FAIL!

Case 2: $\begin{bmatrix} + \\ + \end{bmatrix} \perp \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ $z = -A^{-1}\begin{bmatrix} -6 \\ 4 \end{bmatrix} = \begin{bmatrix} 2 \\ 4/3 \end{bmatrix}$. Success!

Case 3: $\begin{bmatrix} + \\ 0 \end{bmatrix} \perp \begin{bmatrix} 0 \\ + \end{bmatrix}$

Eqs.: $z_2 = 0$, $z_1 + 3\cancelto{0}{z_2} - 6 = 0 \Rightarrow z_1 = 6$

Ineqs: $z_1 \geq 0$, $-2z_1 + 4 \geq 0$ FAIL!

Case 4: $\begin{bmatrix} 0 \\ + \end{bmatrix} \perp \begin{bmatrix} + \\ 0 \end{bmatrix}$

Eqs. $z_1 = 0$, $-2z_1 + 4 = 0 \Rightarrow z_1 = 2$ Contradiction!

Graphical interpretation of solution

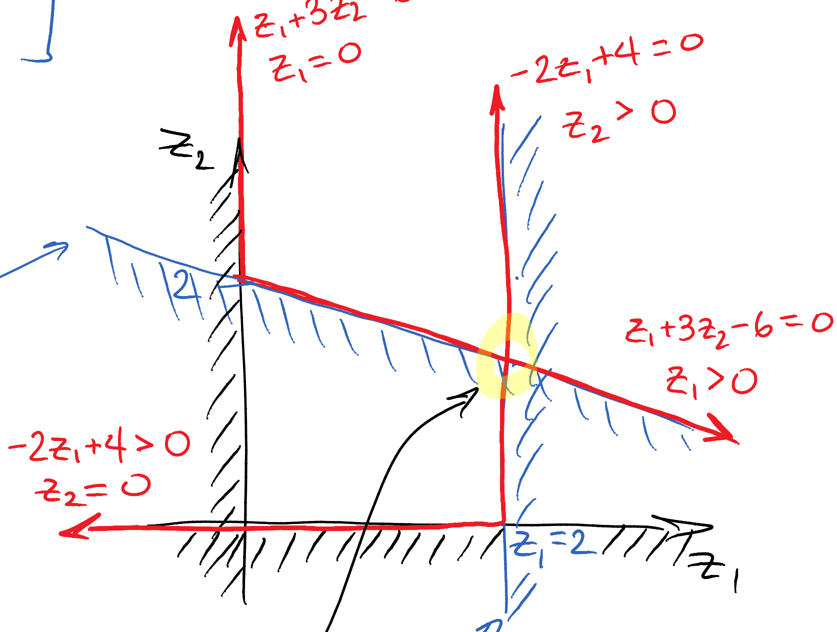
$$0 \le \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} \perp \begin{bmatrix} z_1 + 3z_2 - 6 \\ -2z_1 + 4 \end{bmatrix} \ge 0$$

$z_1 + 3z_2 - 6 \geq 0$

$z_2 \geq -\frac{1}{3} z_1 + 2$

$-2z_1 + 4 \geq 0$

$z_1 \leq 2$



SOL(LCP(A,b)) is unique