background concepts for Robotics II

Saturday, January 12, 2008
8:15 PM

Fundamentals of robot kinematics (Craig Chapters 1-6)

1. Representations of 3D coordinate frames
2. Transformation of vector representation from frame to frame
3. Forward kinematic map construction
4. Inverse kinematics problem solution
5. Manipulator Jacobian construction
6. Identification of singular configurations
7. Manipulator dynamics (and equilibrium)

Fundamentals of linear algebra & linear inequalities

Given n vectors of length m,

1. Determine number of linearly independent vectors
2. Span of a set of vectors (a vector space)
3. Determine a basis for a vector space.

Given $A \in \mathbb{R}^{m \times n}$, $y \in \mathbb{R}^m$

1. Rank A
2. Null space of A
3. Row space of A
4. Transpose of A
5. Inverse of A
6. Pseudoinverse of A

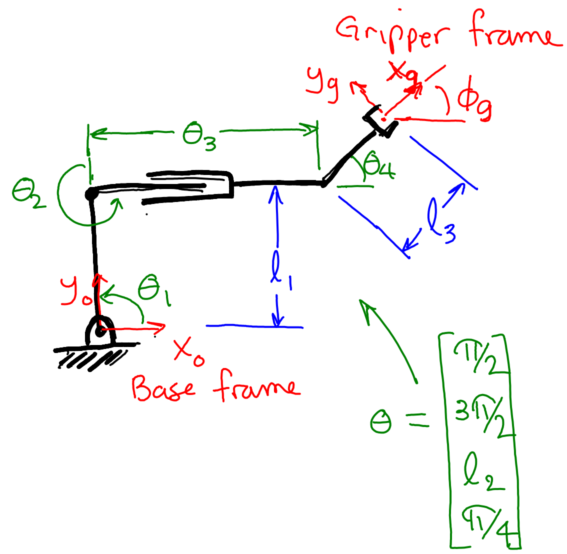
$Ax = y$ represents a system of linear equations in the unknown $x \in \mathbb{R}^n$

1. Solution existence
2. Solution uniqueness
3. Graphical interpretation

$Ax \geq y$ represents a system of linear inequalities

1. Solution existence 2. Solution uniqueness
3. Graphical interpretation

A few refresher questions



1. Construct the Jacobian matrix relating the 4 joint velocities to the velocity of the gripper frame.

Let $\nu = \begin{bmatrix} \dot{x}_g \\ \dot{y}_g \\ \dot{\phi}_g \end{bmatrix}$ $\quad \dot{\theta} = \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \\ \dot{\theta}_4 \end{bmatrix}$

By inspection construct ${}^0J_G$ and ${}^4J_G$

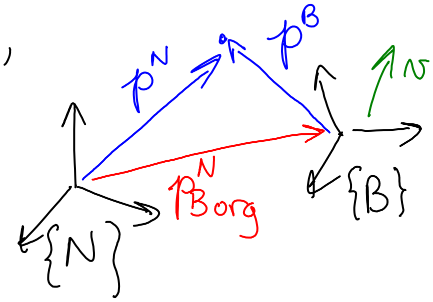
$\left.\begin{aligned} {}^0J_G \dot{\theta} &= {}^0\nu \\ {}^4J_G \dot{\theta} &= {}^4\nu \end{aligned}\right\}$ where ${}^i\nu$ is the velocity of the origin of the gripper frame expressed in frame i.

2. Given $A = \begin{bmatrix} 1 & 0 & 2 \\ 1 & 1 & 1 \end{bmatrix}$, $y = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

a.) sketch the row space of A
b.) what is the null space of A
c.) Find all solutions to $Ax = y$ (you'll need matrices

c.) find all solutions to $Ax = y$ (you'll need matrices representing the row space and null space of A.

3.a.) Given a point expressed in $\{B\}$, determine its representation in $\{N\}$,



i.e. given p^B,

p^N_{Borg}, and R^N_B;

determine p^N

Note: R^N_B is the rotation matrix whose column are the unit direction of $\{B\}$ in frame $\{N\}$, that is:

$$R^N_B = \begin{bmatrix} \hat{x}^N_B & \hat{y}^N_B & \hat{z}^N_B \end{bmatrix}_{(3\times3)}$$

b. Given a free vector v expressed in $\{B\}$, determine its expression in $\{N\}$.