Robotics II Mid-Term Exam Spring 2008 - Solution

Wednesday, March 05, 2008
10:20 AM

1.a.) 10 pts

$$G = \begin{bmatrix} \hat{n}_1 & \hat{t}_1 & \hat{n}_2 & \hat{t}_2 \\ r_1 \otimes \hat{n}_1 & r_1 \otimes \hat{t}_1 & r_2 \times \hat{n}_2 & r_2 \times \hat{t}_2 \end{bmatrix}$$

$$G = \begin{bmatrix} 0.8 & -0.6 & -1 & 0 \\ 0.6 & 0.8 & 0 & 1 \\ 0 & -d_1 & 0 & d_2 \end{bmatrix}$$

$\text{rank}(G) = 3$ (since $\text{Det}(\text{cols } 2,3,4) = d_1 + 0.8 d_2$ and $d_1, d_2 > 0$)

$\boxed{\text{Yes}}$

1.b.) 10 pts

$$J = \begin{bmatrix} 0.6 & 0 & 0 \\ 0.8 & l_1 & 0 \\ 0 & 0 & l_2\sqrt{2}/2 \\ 0 & 0 & -l_2\sqrt{2}/2 \end{bmatrix}$$

$$GJ = \begin{bmatrix} 0 & -0.6 l_1 & -\frac{\sqrt{2}}{2} l_2 \\ 1 & 0.8 l_1 & -\frac{\sqrt{2}}{2} l_2 \\ -0.8 d_1 & -l_1 d_1 & -\frac{\sqrt{2}}{2} l_2 d_2 \end{bmatrix}$$

$\text{rank}(GJ) = 3 \quad \therefore \quad \boxed{\text{Yes}}$

$$\det(GJ) = -(0.6)(0.8)\left(\tfrac{\sqrt{2}}{2}\right) d_1 l_1 l_2 + \left(\tfrac{\sqrt{2}}{2}\right) l_2 d_1 - (0.8)^2 \left(\tfrac{\sqrt{2}}{2}\right) l_1 l_2 d_1 - (0.6)\left(\tfrac{\sqrt{2}}{2}\right) d_2 l_1 l_2 \overset{?}{=} 0$$

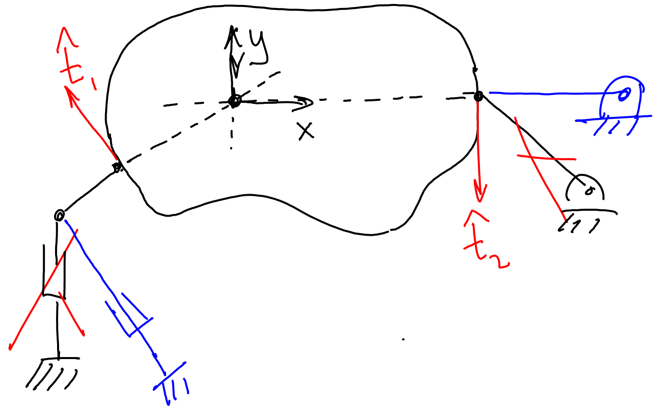
Not zero except for special values of l_1, l_2, d_1, d_2.

1.c.) Look for J such that $\text{rank}(GJ) < 3$.

5 pts This will be true if $\operatorname{rank}(J) < 3$.

One approach, make 2 rows of J equal to zero.

Move fingers to config shown, then the first and third rows of J will become zero.



2.) 15 pts

$\lambda_{1n} + a\lambda_{3n} = g_x$ for any g_x in $\mathbb{R}$ including $g_x < 0$

$\lambda_{2n} + b\lambda_{3n} = g_y$ for any g_y in $\mathbb{R}$ including $g_y < 0$

$\lambda_n \geq 0$

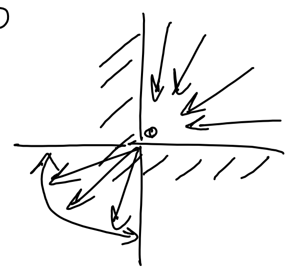
$\lambda_{1n} = g_x - a\lambda_{3n} \Rightarrow$ if $g_x < 0$, then $a < 0$

if $g_x \geq 0$, a is free

same logic for other equation

$\therefore$ we must have $a < 0$, $b < 0$

contact normals must be in the interior of the cone shown



2.b.) 10 pts

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} \geq 0 \overset{?}{\Rightarrow} \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} a & b \end{bmatrix} \begin{bmatrix} y \end{bmatrix} \qquad \begin{bmatrix} y \end{bmatrix} \quad \begin{bmatrix} 0 \end{bmatrix}$$

$$\left.\begin{aligned} \dot{x} &\geq 0 \\ \dot{y} &\geq 0 \\ a\dot{x} + b\dot{y} &\geq 0 \end{aligned}\right\}$$ if $a \geq 0$, then $\dot{x} > 0$ is possible regardless of b & $\dot{y}$

$\therefore \ a < 0$

same logic on $\dot{y} \Rightarrow b < 0$

q.e.d.

3.a.) Case 1

5 pts

$$\begin{bmatrix} p_1 \\ p_2 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} \geq \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$\Rightarrow \ b_2 \geq b_1$ AND $2b_2 \geq b_1$

5 pts

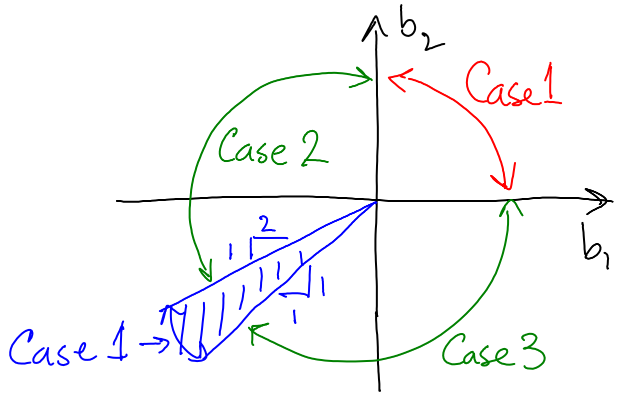
Case 2: $p_2 = 0$

$\Rightarrow p_1 = -b_1/2 \geq 0$

and $p_1 + b_2 \geq 0$

$\Rightarrow \ 2b_2 \leq b_1$

$b_1 \leq 0$



Cases include the boundries of their cones.

Case 3: $p_1 = 0$

4 pts

$\Rightarrow \ b_1 \geq b_2$ AND $b_2 \leq 0$

Case 4: $p_1 = p_2 = 0$

4 pts

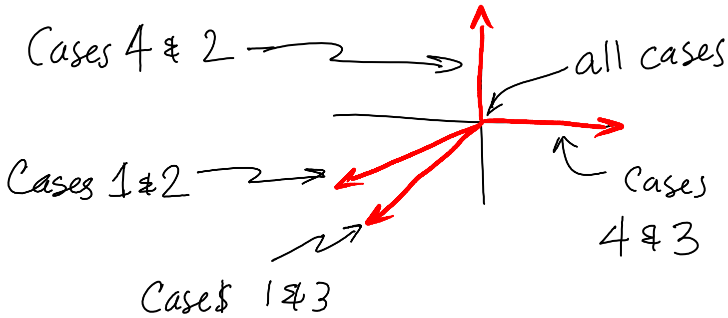
$\Rightarrow b_1 \geq 0$ AND $b_2 \geq 0$

3.b.) Yes. See plot above.

3 pts

3.c.) Yes. The boundaries of the 4 case cones.

4 pts



4. 25 pts

$$\begin{bmatrix} \psi_{1n} \\ \psi_{2n} \end{bmatrix} = \begin{bmatrix} y \\ x - at \end{bmatrix} \qquad \frac{\partial \psi_n}{\partial t} = \begin{bmatrix} 0 \\ -a \end{bmatrix} \qquad E = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{bmatrix}$$

$$U = \begin{bmatrix} \mu_1 & 0 \\ 0 & \mu_2 \end{bmatrix} \qquad G_n = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$G_f = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$