1.) A planar object is grasped with two hard fingers. The Coefficient of friction at both contact points is 1.0.

(2 pts)

a.) Find a location for contact ② such that a 2-fingered grasp has frictional form closure.

(2 pts)

b.) You know contact ① is somewhere on edge ④, but its precise location is not known. Find a finite region on the polygon such that placing contact ② anywhere in that region, will form a grasp with frictional form closure.
2. Show analytically that the grasp shown on the right does not have form closure.

3. **3D Problem!**

Contact 1 (on left) is a soft finger contact. Contact 2 (on right) is a hard finger contact.

α) Construct G & J using the (x-y-z) reference frame shown.
b.) For the correct $G$ and $J$, bases of the four null spaces are:

\[
N(G) = \begin{bmatrix}
-1 \\
0 \\
-1 \\
0
\end{bmatrix}, \quad N(G^T) = \begin{bmatrix}
0 \\
0 \\
0 \\
0
\end{bmatrix}, \quad N(J) = \begin{bmatrix}
0 \\
0 \\
0 \\
0
\end{bmatrix}, \quad N(J^T) = \begin{bmatrix}
0 \\
0 \\
0 \\
0
\end{bmatrix}
\]

Complete the picture below, i.e. identify the dimensions of the various subspaces.
For the next two problems you might find the following quantities helpful.

\[
J^T G^+ = \begin{bmatrix}
0 & 0 & 2.1 & 0 & -2.1 & 0 \\
-0.5 & 0 & 0 & 0 & 0 & 0 \\
-0.8 & -0.6 & 0 & 0 & 0 & -0.6 \\
-0.8 & 0.6 & 0 & 0 & 0 & -0.6 \\
0.5 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -2.1 & 0 & -2.1 & 0
\end{bmatrix}
\]

\[
G N(J^T) = \begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix}
\]

\[
G(J^T)^+ = \begin{bmatrix}
0 & -1 & 0 & 0 & 1 & 0 \\
0 & 4/3 & -5/6 & 5/6 & 4/3 & 0 \\
5/2 & 0 & 0 & 0 & 0 & -5/2 \\
0 & 0 & 0 & 0 & 0 & 0 \\
-5/2 & 0 & 0 & 0 & 0 & -5/2 \\
0 & 4/3 & -5/6 & -5/6 & 4/3 & 0
\end{bmatrix}
\]

\[
J^T N(G) = \begin{bmatrix}
0 \\
-\sqrt{2}/2 \\
-1.13 \\
1.13 \\
-\sqrt{2}/2 \\
0
\end{bmatrix}
\]

c.) Use the relationships \( \tau = J^T \lambda \) and \( g_{app} = G \lambda \) to determine which joint torques do not change in response to changes in the internal wrench.
d.) Use the relationships \( \tau = J^T \lambda \) and \( g_{app} = G \lambda \) to determine which component of the external wrench cannot be controlled by adjusting joint torques.

4.) A particle is close to circular and linear obstacles \( x^2 + y^2 \geq R^2 \) and \( x \geq -1 \).

a.) Assume \( u = 0, \ M = h = 1 \).

Determining \( u, v, \) and \( p_n \) at \( t = 1 \) and \( t = 2 \).

b.) Assuming \( \mu_1, \mu_2 \neq 0 \), give the definitions of \( G_n, G_e, E, U, M, \) and \( \frac{\partial \psi_n}{\partial t} \) for the first time step.

c.) What is the size of the LCP if both obstacles are incorporated and friction is not zero?