Simulation and Experiments in Vibratory Manipulation: Rigid Bodies on a Vibrating Surface

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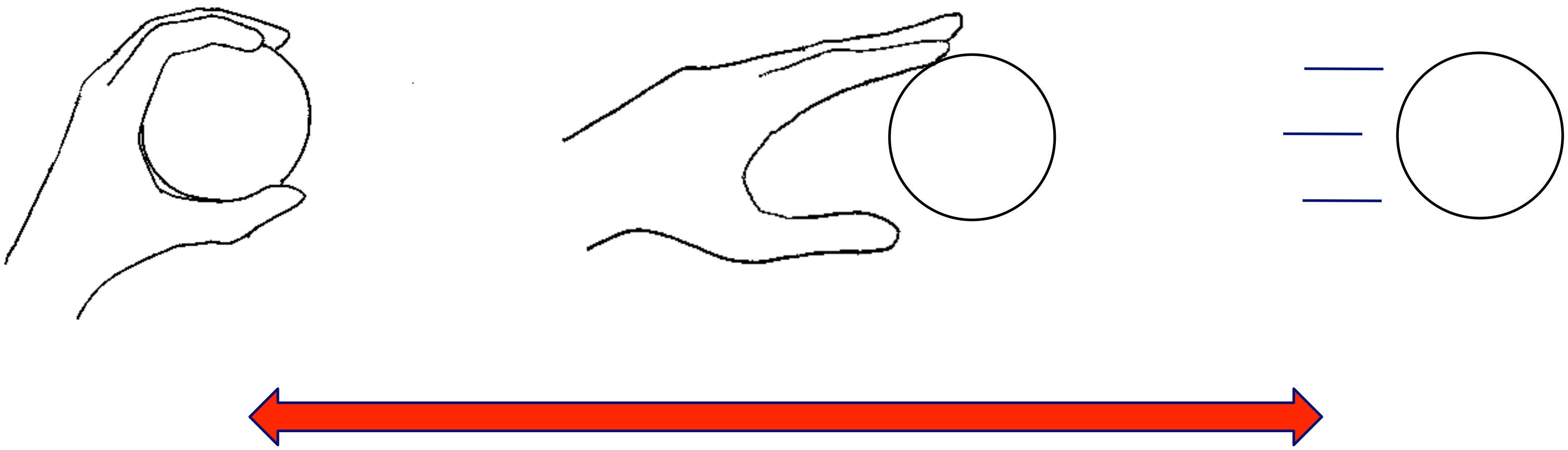


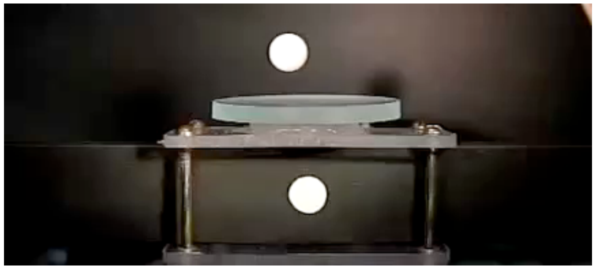
hand controls ball:
grasping

shared control:
nonprehensile manipulation

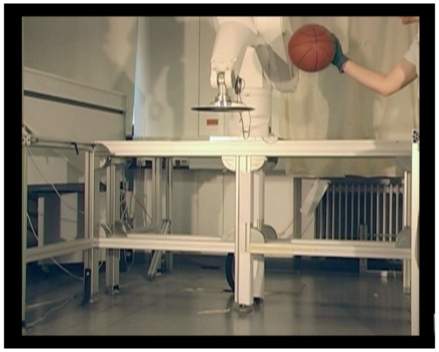
environment
controls ball

Examples

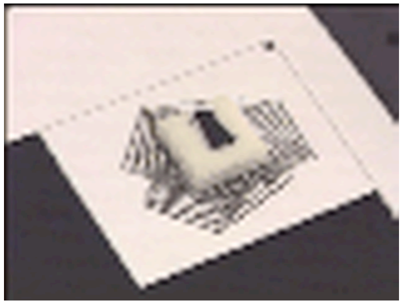
throwing and batting
(U Tokyo)



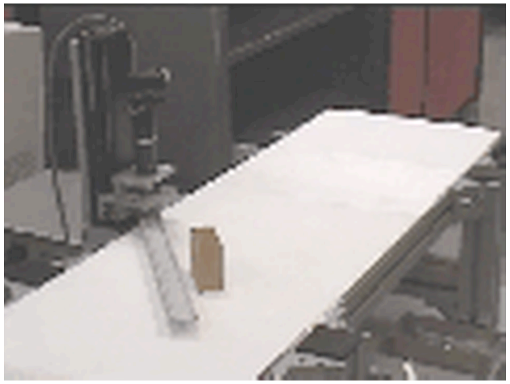
bat juggling



dribbling
(TU Munich)



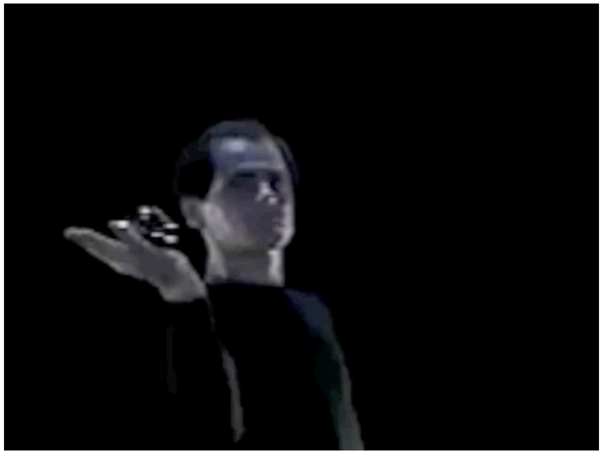
pushing



pushing and toppling

vibratory feeding

rolling
(Michael Moschen)



rolling on a
constraint
surface
(dung beetle,
Natl Geo)

Why Nonprehensile Manipulation?

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- Given a robot, increase the set of solvable tasks
- Given a task, use cheaper, simpler robots (automation)
- Most manipulation is nonprehensile! (pushing, throwing, tapping, sliding, rolling, batting, kicking, …)

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Research Topics

- sensing/observability/uncertainty
- mechanics and modeling
- motion planning
- feedback control
- understanding what tasks are solvable (e.g., accessibility, controllability)

Outline

a nonprehensile primitive: vibratory sliding

- asymptotic velocity fields
- velocity fields for rigid bodies
- feasible velocity fields for point parts

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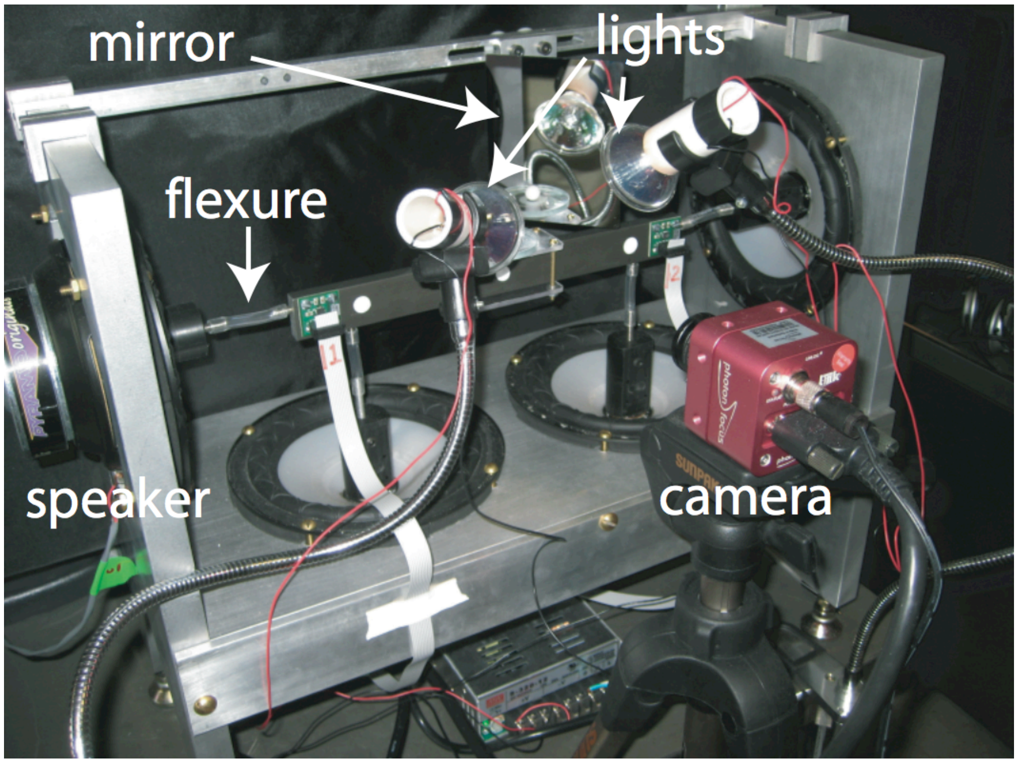
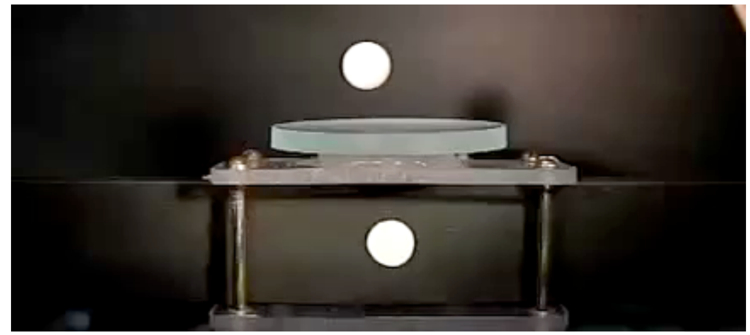
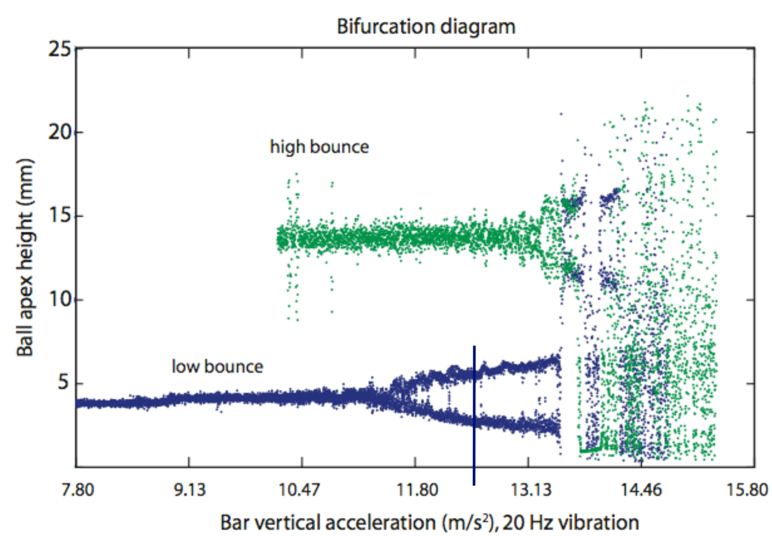
Batting and Sliding



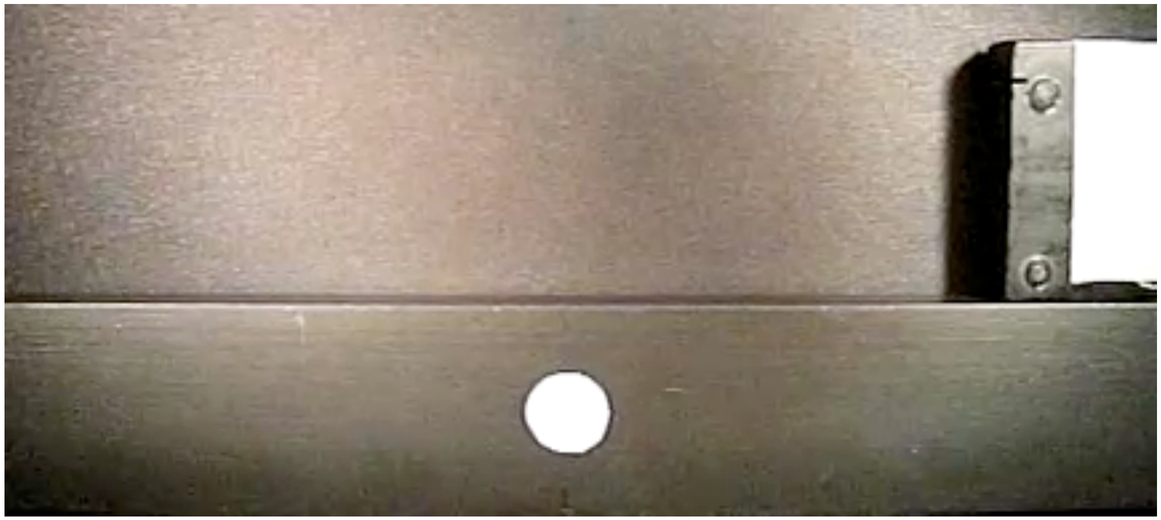
3-DOF “VPOD” vibratory vertical plane manipulator
with 3D high-speed vision





Sliding Manipulation

size scale



15 Hz vibration, 20x slow motion

$$f_{\text{fric}} = \mu\, f_{\text{normal}} \frac{\mathbf{v}_{\text{rel}}}{\left\|\mathbf{v}_{\text{rel}}\right\|}$$

friction force

friction coefficient

slipping direction

Sliding Manipulation

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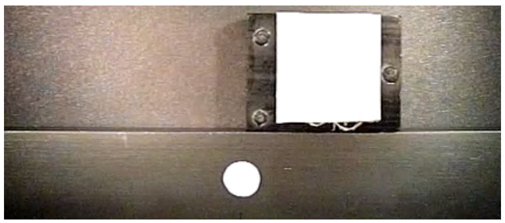
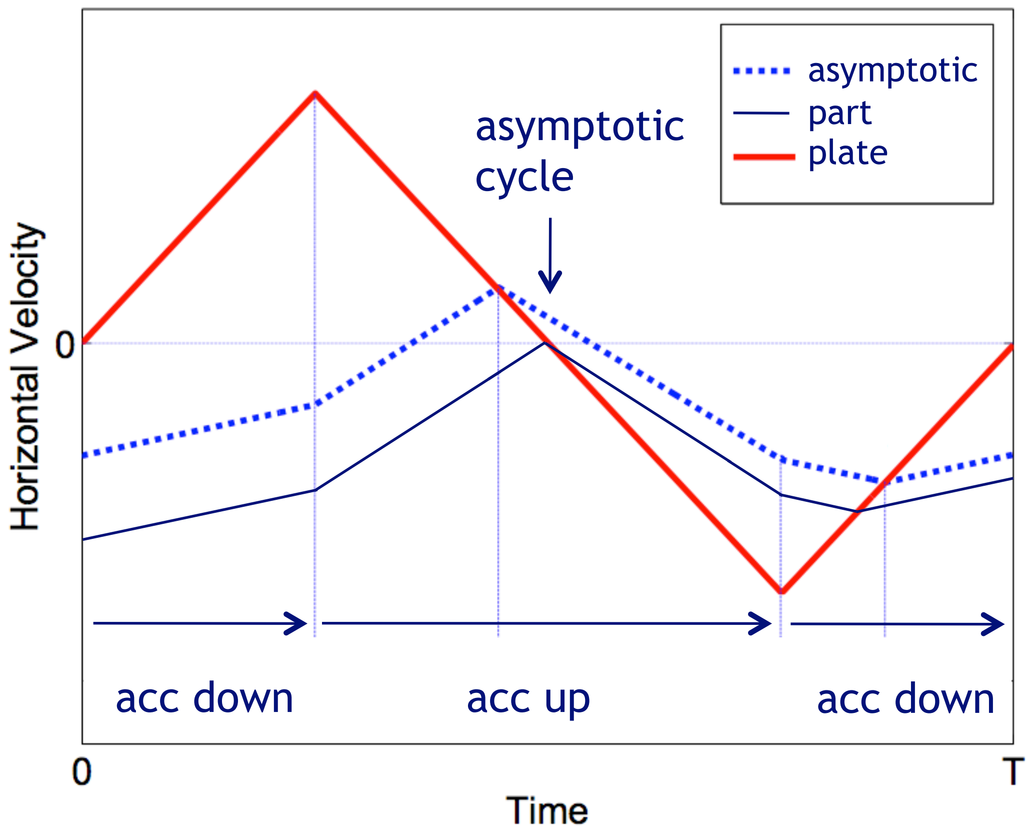


square wave vertical and horizontal acceleration



Sliding Manipulation

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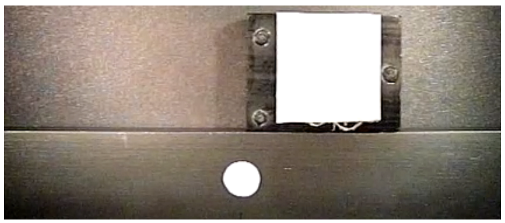

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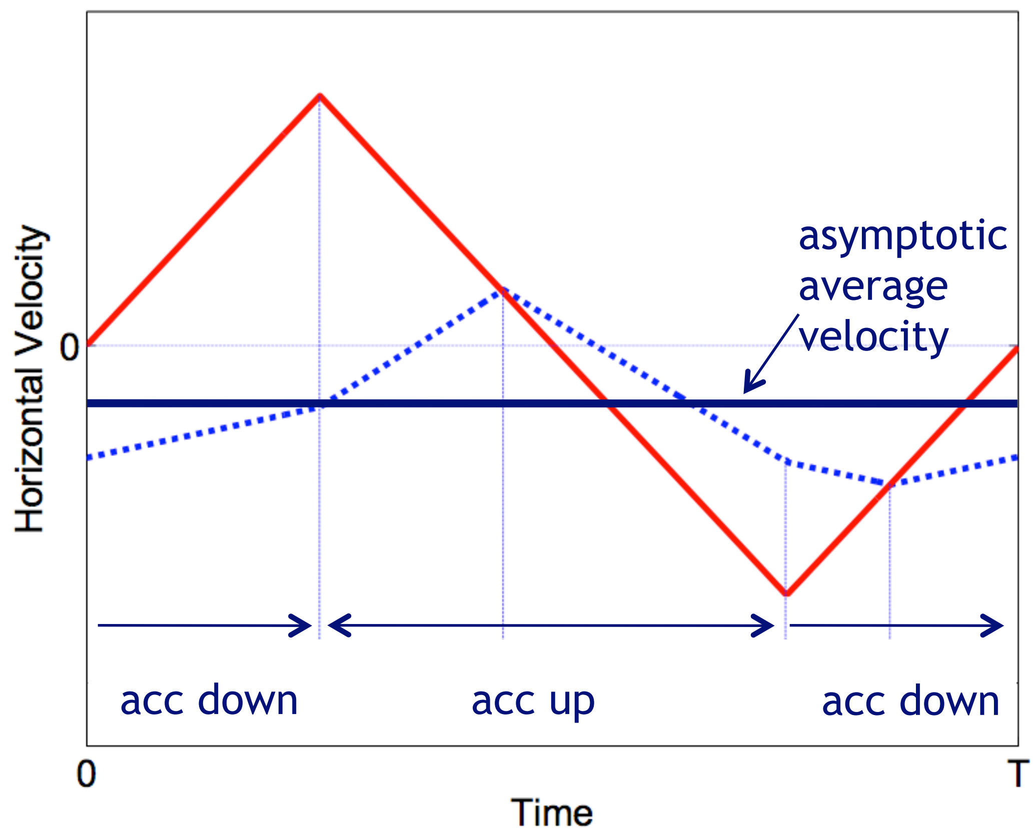
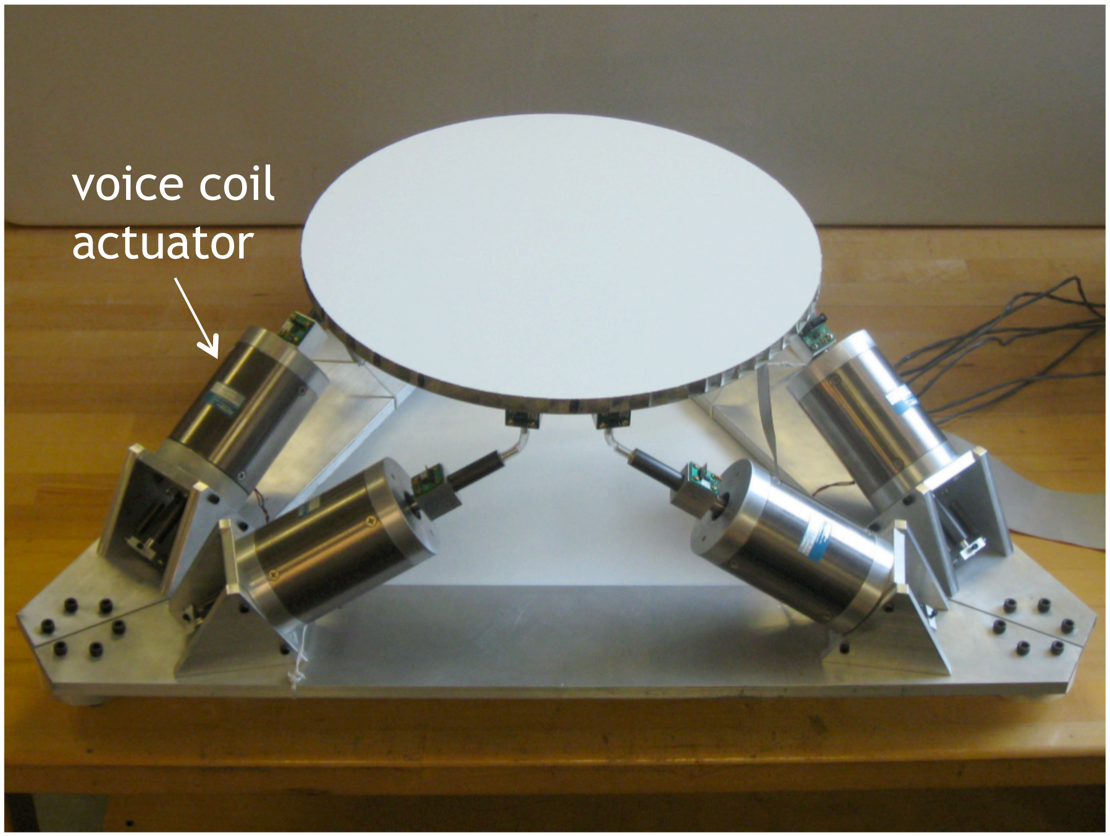
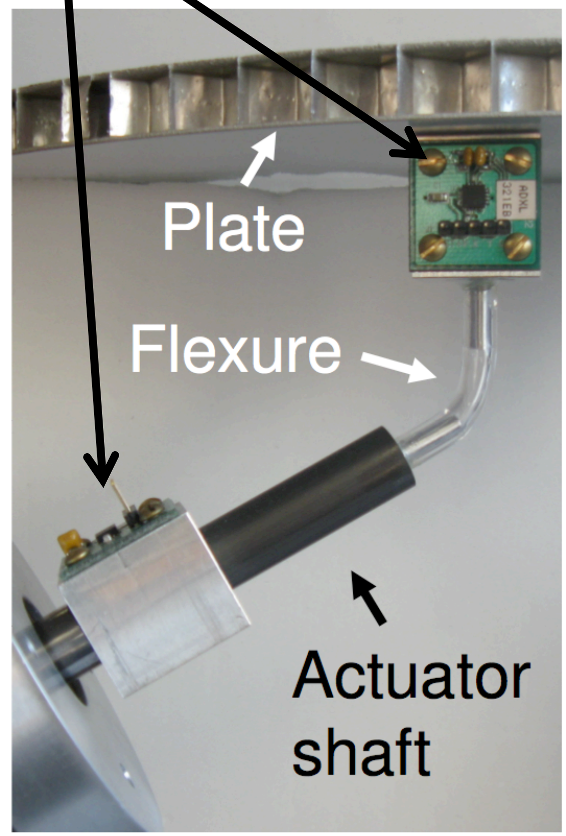


The 6-DOF PPOD
(Programmable Parts-feeding Oscillatory Device)





PPOD2: flexure-based Stewart platform

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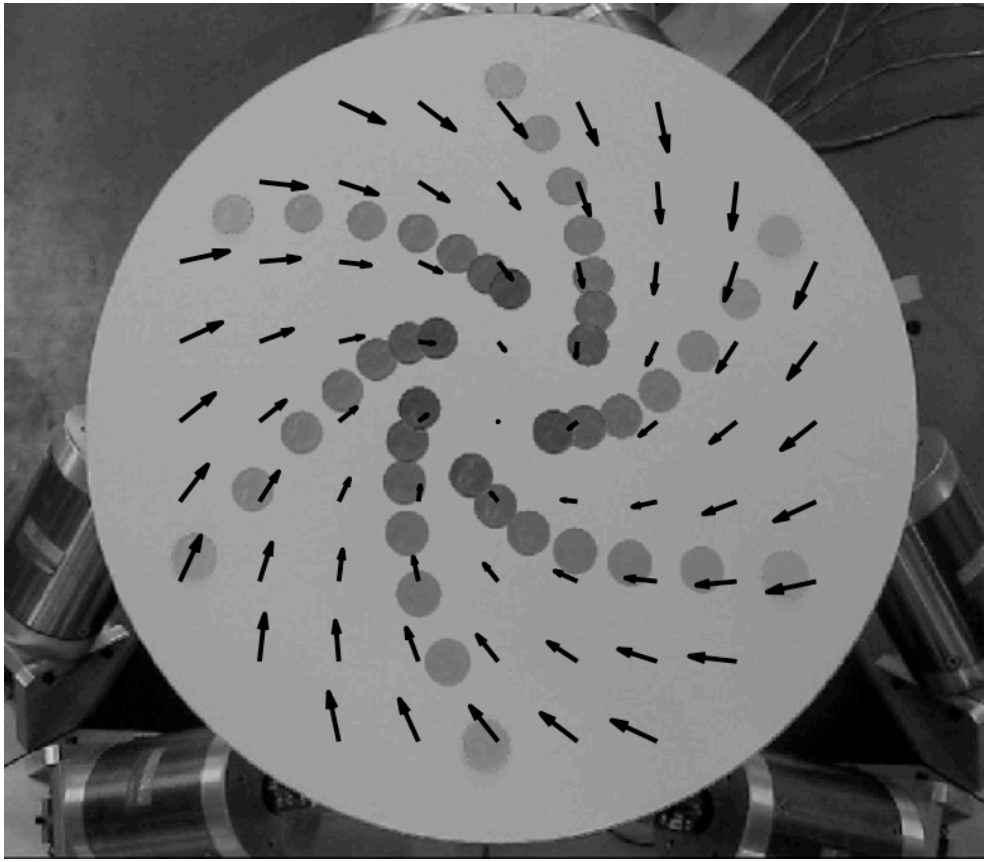

asymptotic average velocity field

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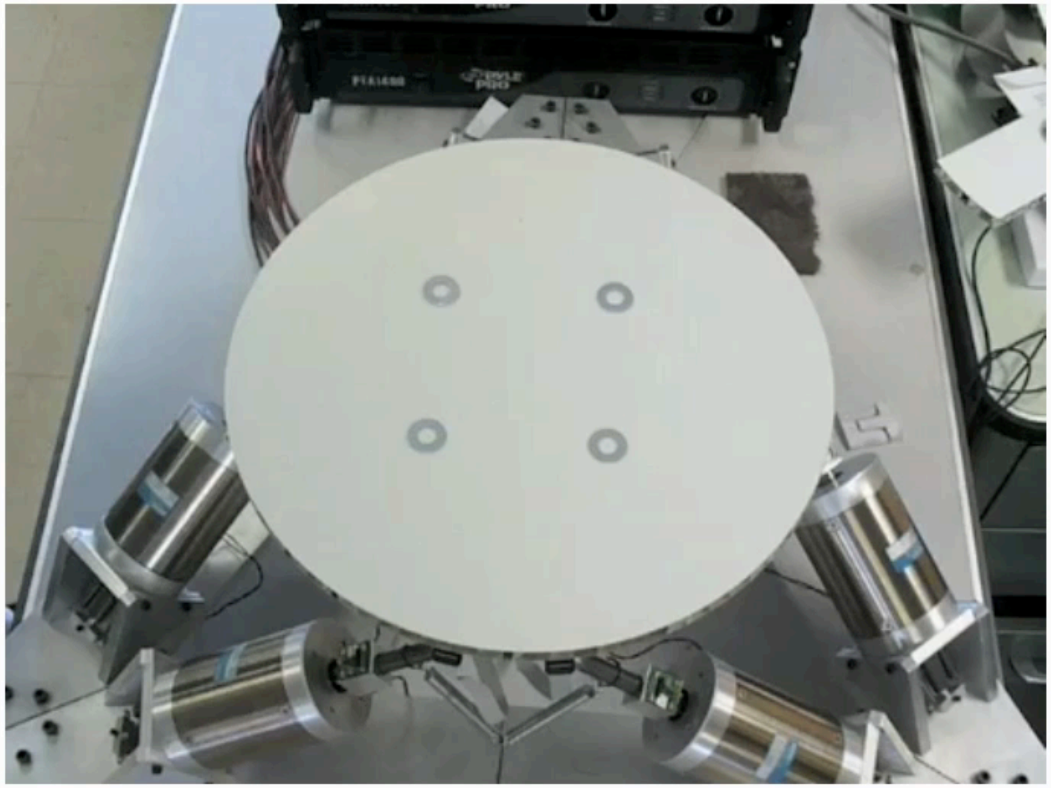
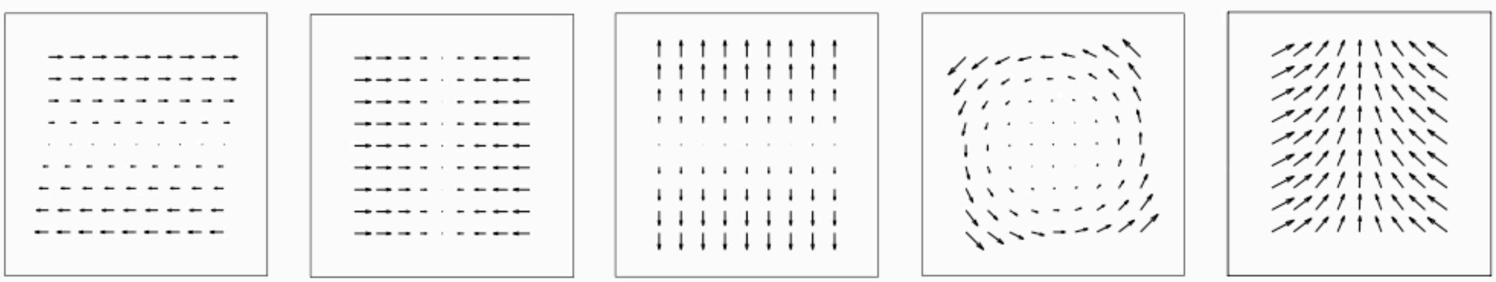
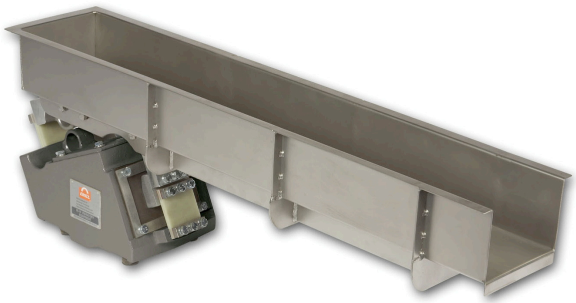


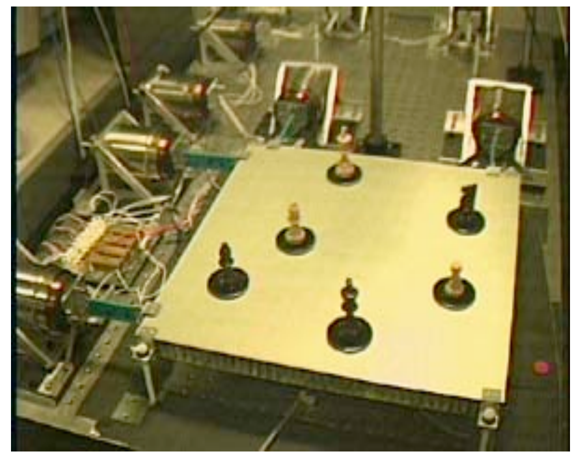


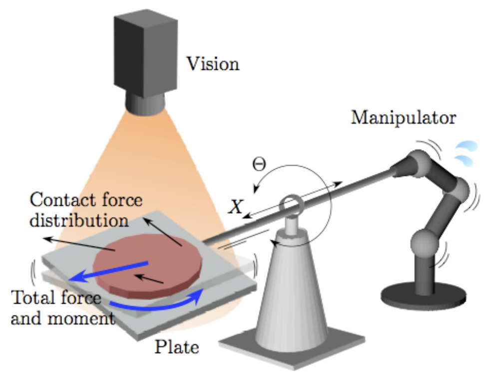
Related Work

vibratory linear conveyors

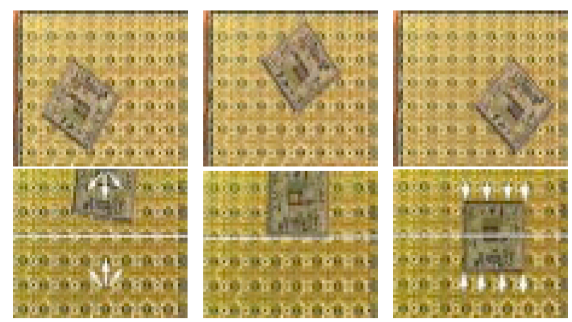


horizontally-vibrating plate
[Reznik, Canny
Bohringer, Goldberg, et al.]





pizza manipulation
[Higashimori, Utsumi, Kaneko]



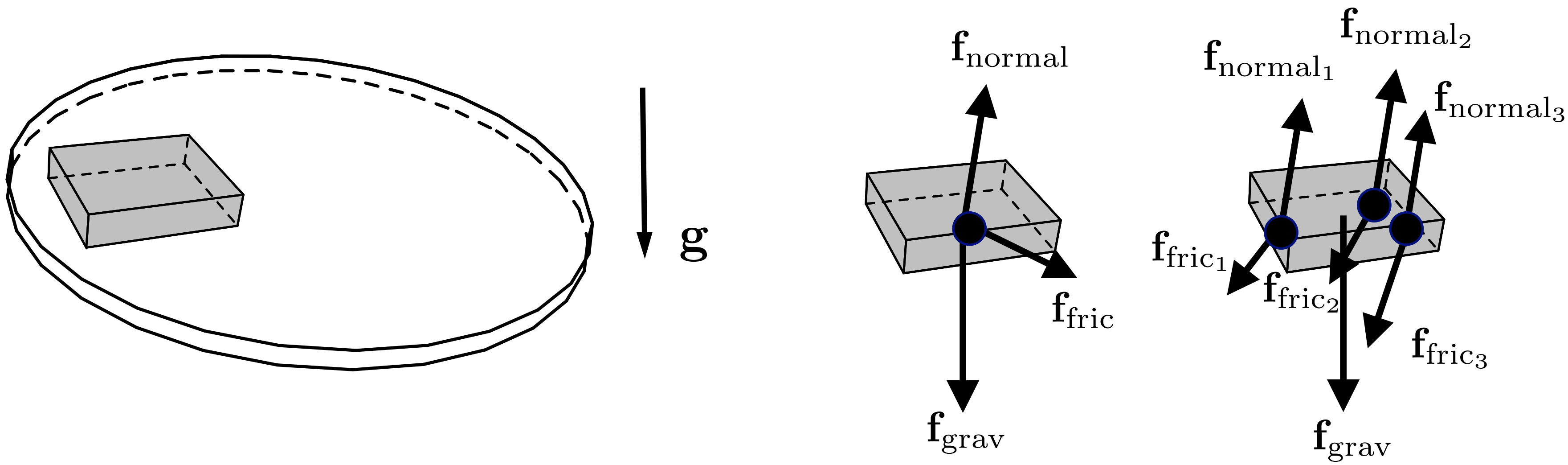
arrays of vibrating plates, MEMS, airjets, wheels
[Frei et al., Bohringer and Donald, Luntz et al.,
Murphey and Burdick, Kavraki, Goldberg et al.]

Outline

a nonprehensile primitive: vibratory sliding

- asymptotic velocity fields
- velocity fields for rigid bodies
- feasible velocity fields for point parts

Part Dynamics



$$\mathbf{f}_{\mathrm{fric}_i} = \mu_i N_i \frac{\mathbf{v}_{\mathrm{rel}_i}}{\left\|\mathbf{v}_{\mathrm{rel}_i}\right\|}$$

- direction of **relative velocity** between part and plate determines **direction** of friction force
- vertical **acceleration** of plate determines normal force and therefore **magnitude** of friction force
- by exploiting full 6-DOF motion, the direction and magnitude of the friction forces on a part can be made **configuration-dependent**

Part Dynamics

Given:

1. Periodic control signal (plate acceleration)
2. Part parameters (inertia, contact locations)
3. Friction parameters (friction coefficients)

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$$\dot{x} = f(x, u)$$

$$x = (q_{\text{plate}}, q_{\text{part}}, v_{\text{plate}}, v_{\text{part}})$$

$$u = \dot{v}_{\text{plate}}$$

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Simplified dynamics:

1. Sliding at all contacts
2. Fixed plate and part configurations
3. No Coriolis or centripetal effects

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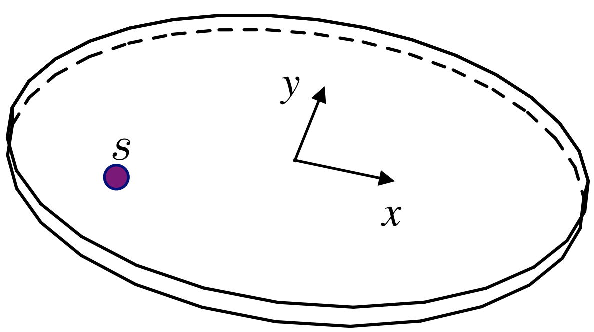
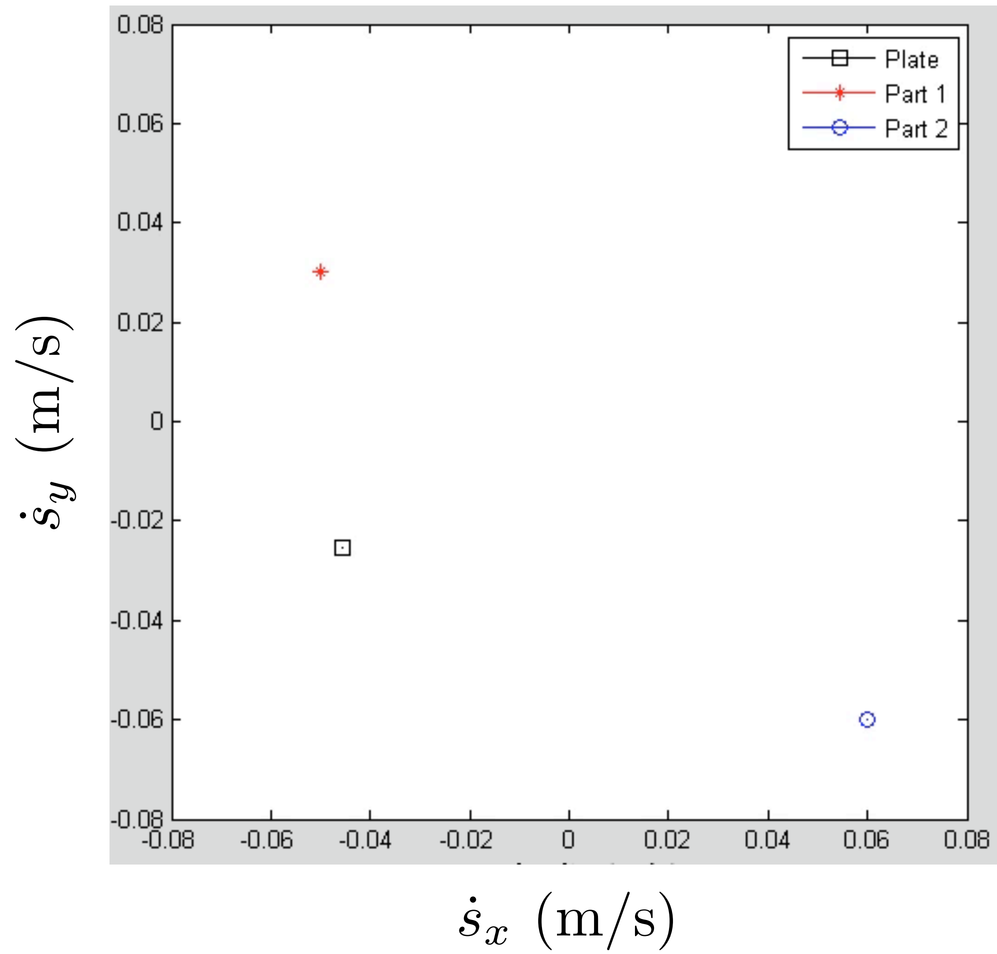
Natural representation of simplified part dynamics:

velocity field on part's **configuration space**
(not force field on plate surface)

Asymptotic Behavior (Point Part)

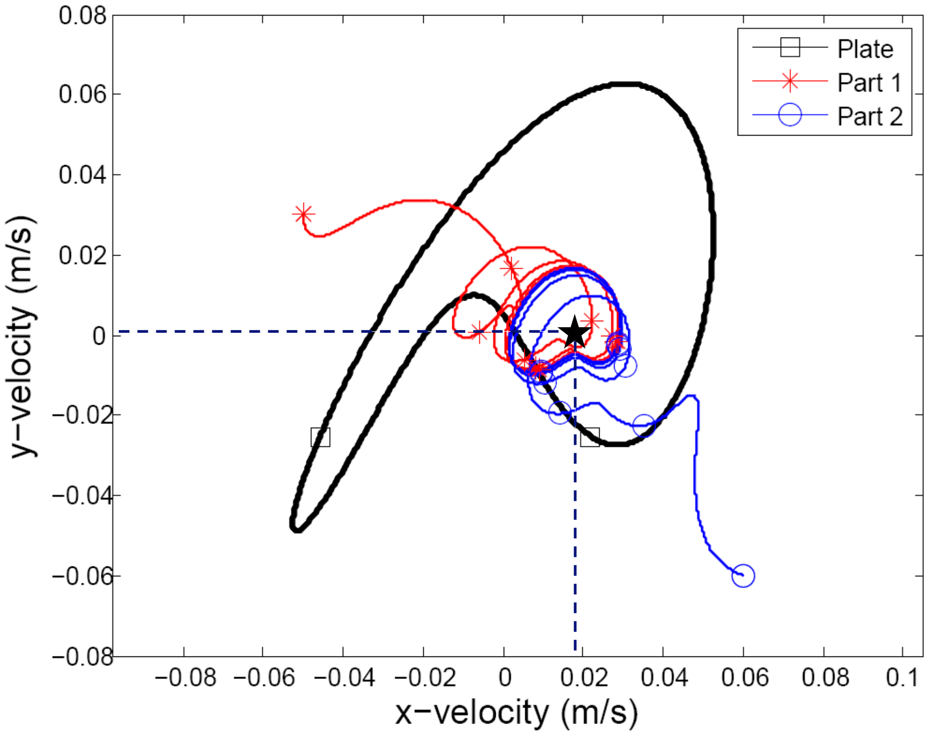
Two velocity trajectories (red and blue) for the purple part shown at left, assuming its configuration does not change

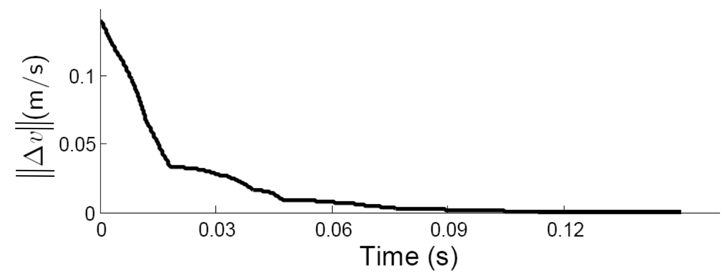


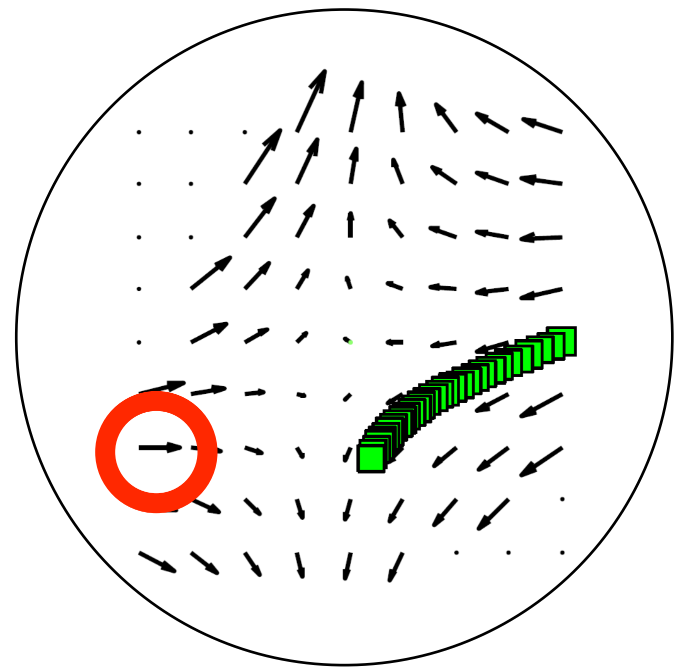


$$\mathbf{f}_{\mathrm{fric}} = \mu N \frac{\mathbf{v}_{\mathrm{rel}}}{\|\mathbf{v}_{\mathrm{rel}}\|}$$

Asymptotic Velocity (Point Part)







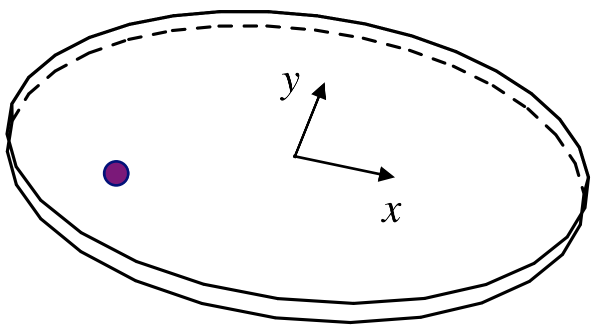
Asymptotic velocity at configuration (x,y):

$$\mathbf{v}_a(x,y) = \frac{1}{T}\int_0^T \mathbf{v}'(t)\,dt$$

where $\mathbf{v}'(t)$ is the limit cycle.

asymptotic velocity field for a point part

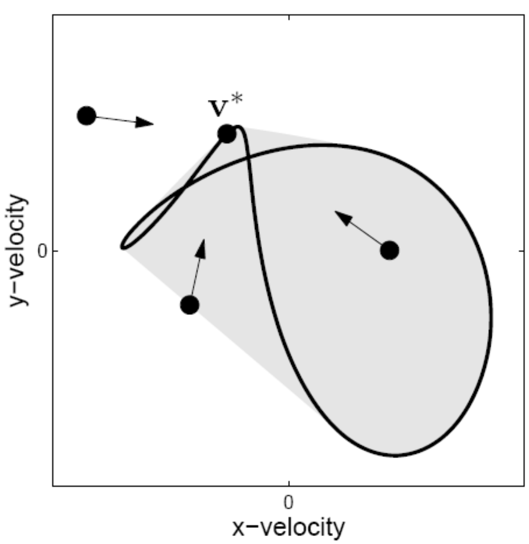
Asymptotic Velocity (Point Part)



<u>Theorem</u>

Given:

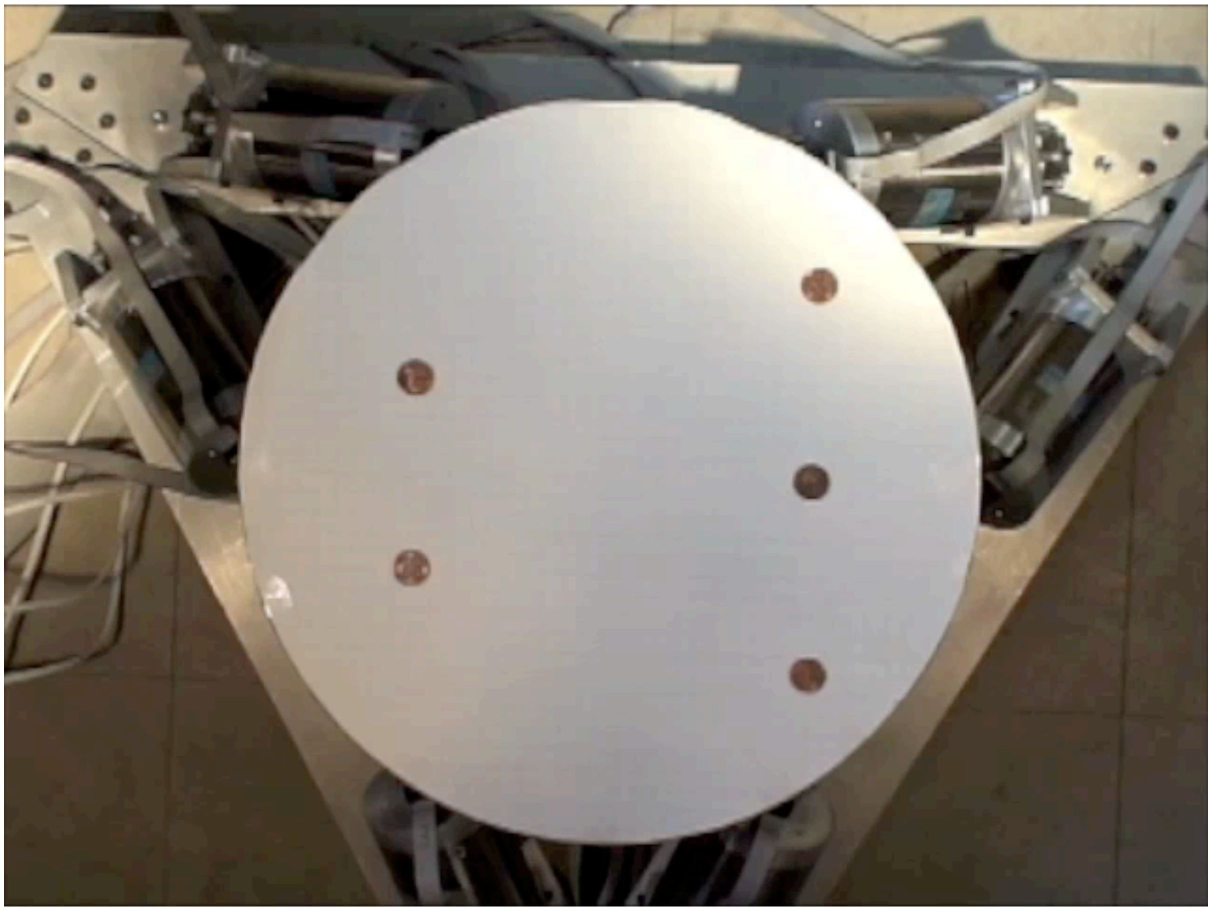
- simplified dynamics
- plate oscillation of period T

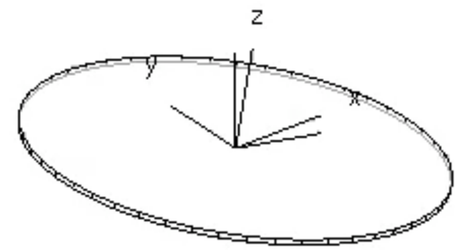


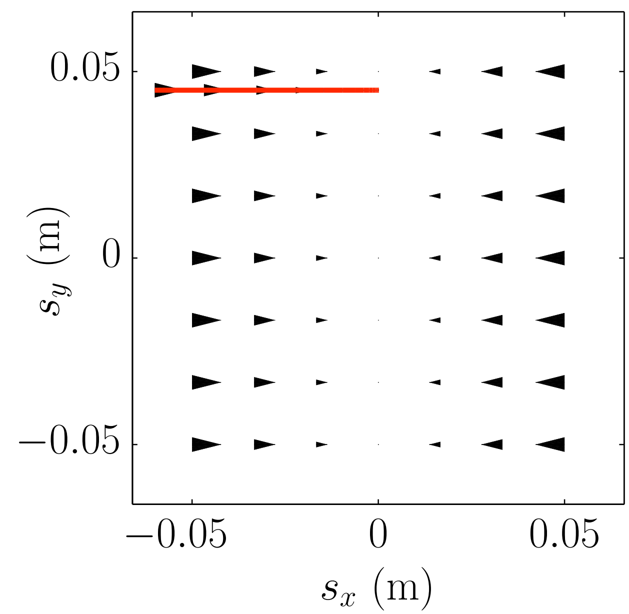
The part asymptotically converges to a unique velocity limit cycle of period T on or inside the convex hull of the plate's velocity.

$$\mathbf{f}_{\mathrm{fric}} = \mu N \frac{\mathbf{v}_{\mathrm{rel}}}{\|\mathbf{v}_{\mathrm{rel}}\|}$$

Example: LineSink

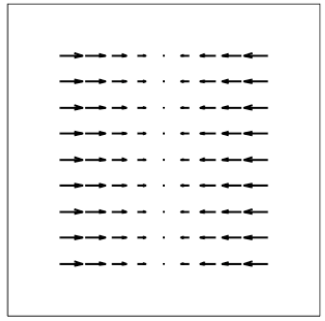


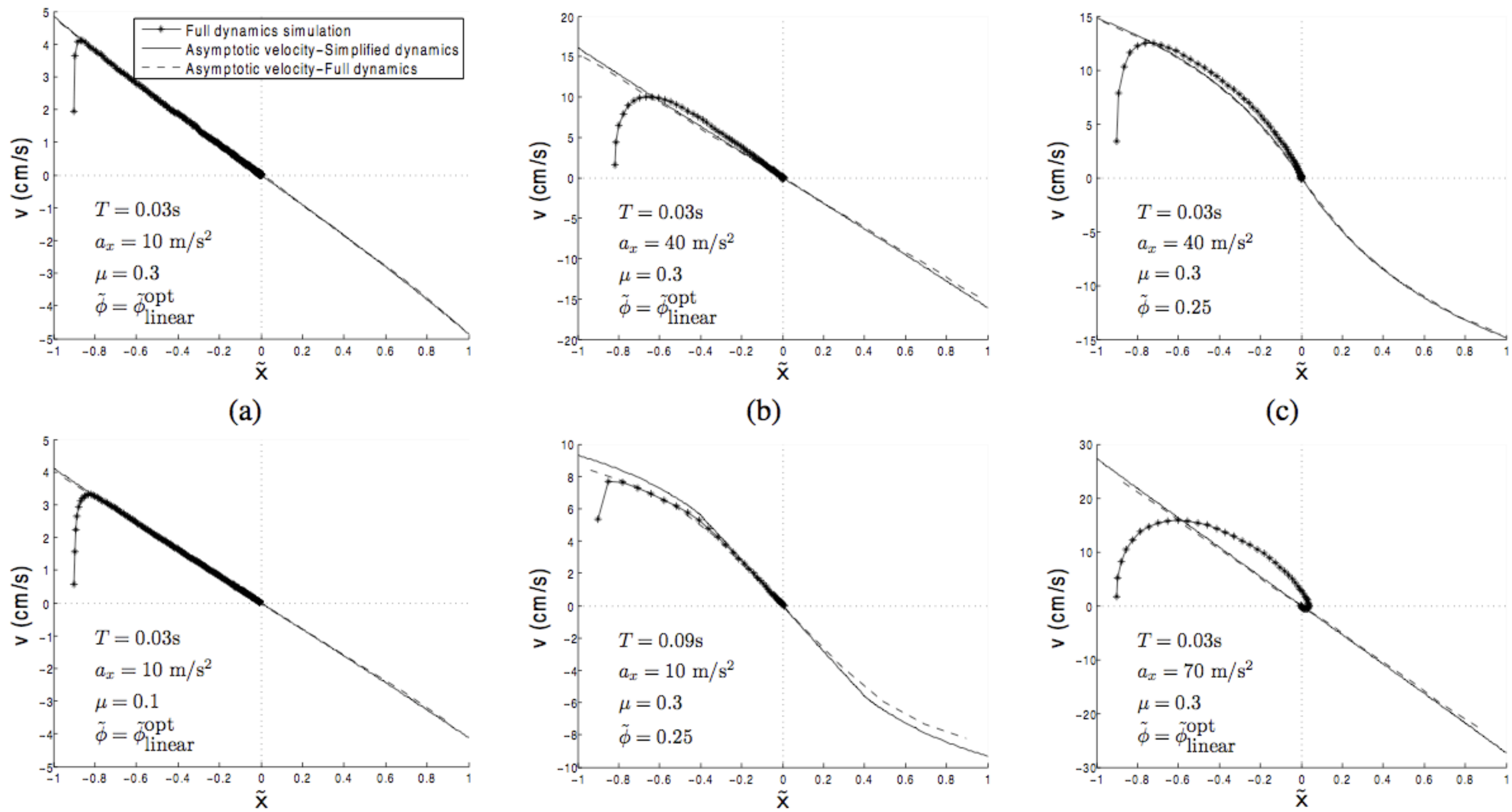




Asymptotics vs. Full Simulation

LineSink





(a) (b) (c)

(d) (e) (f)

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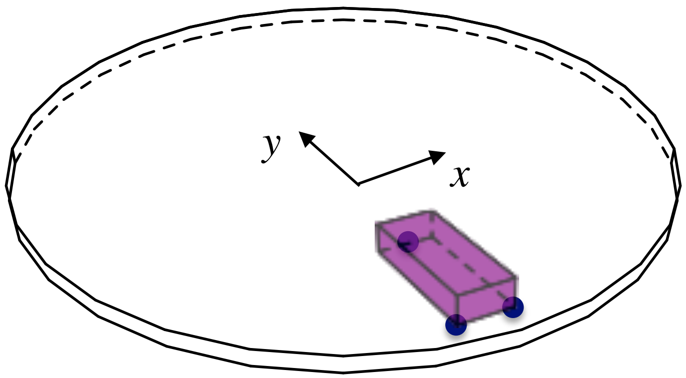
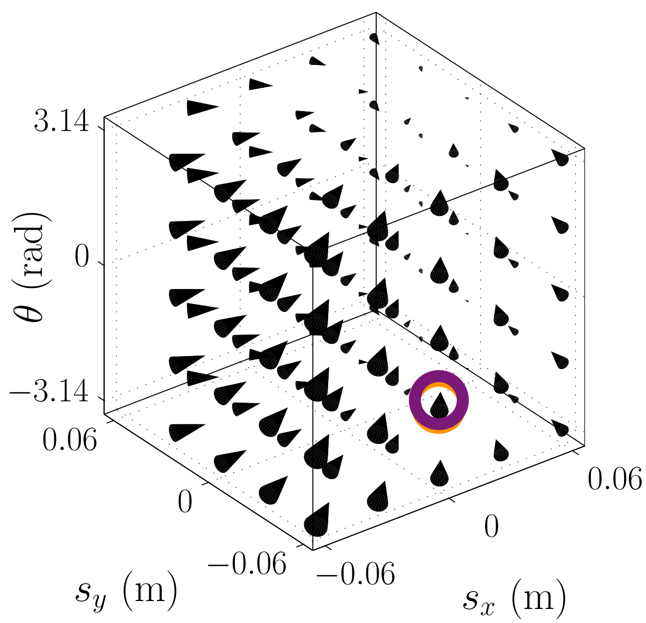
Asymptotic Velocity (Rigid Part)



$$\mathbf{v}_a : SE(2) \rightarrow \mathbb{R}^3$$



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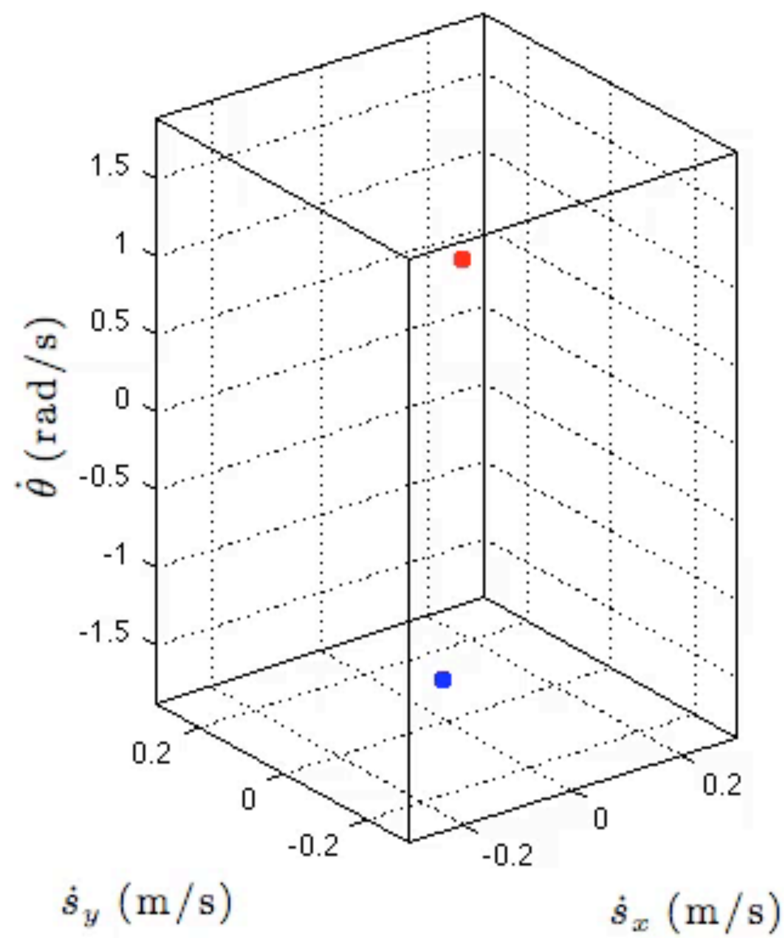

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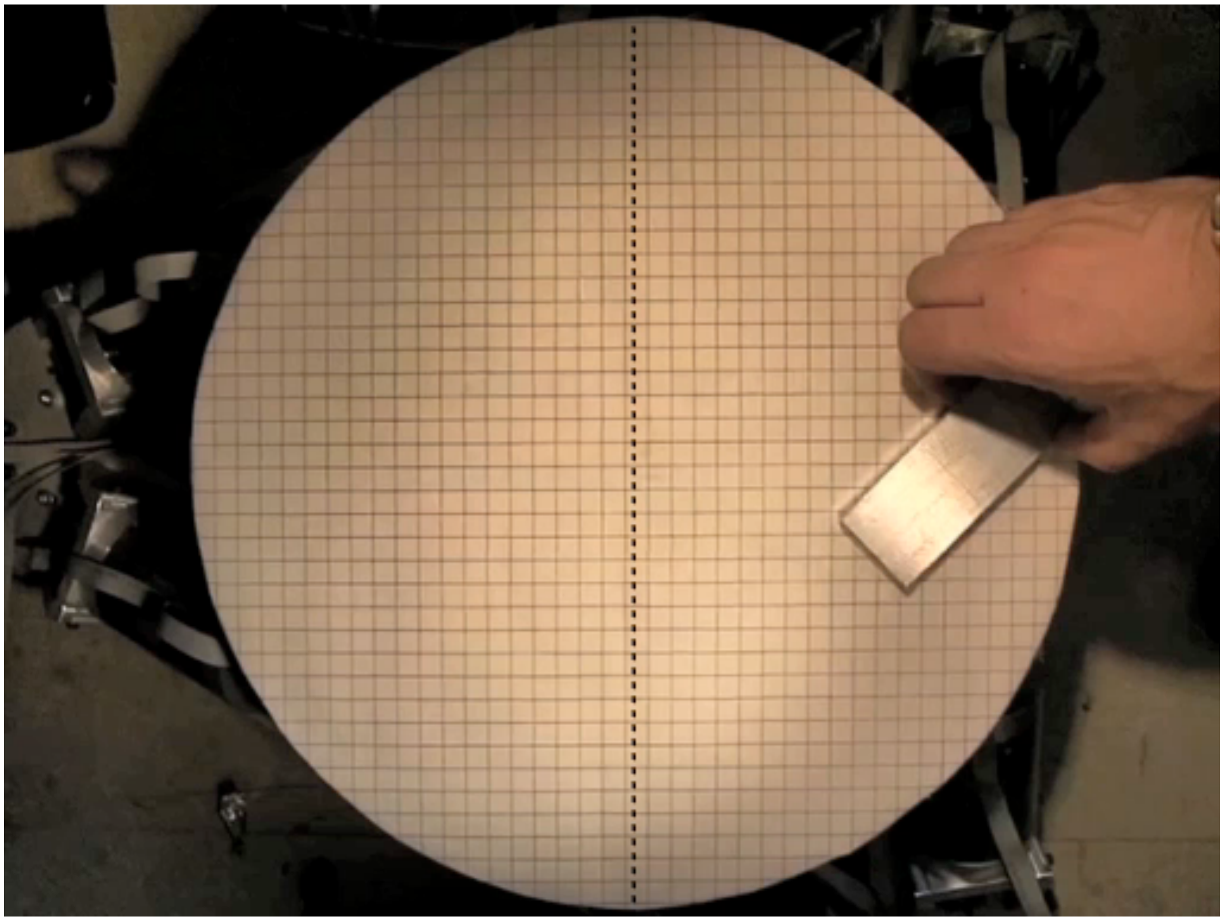
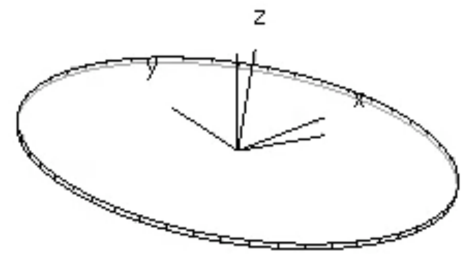
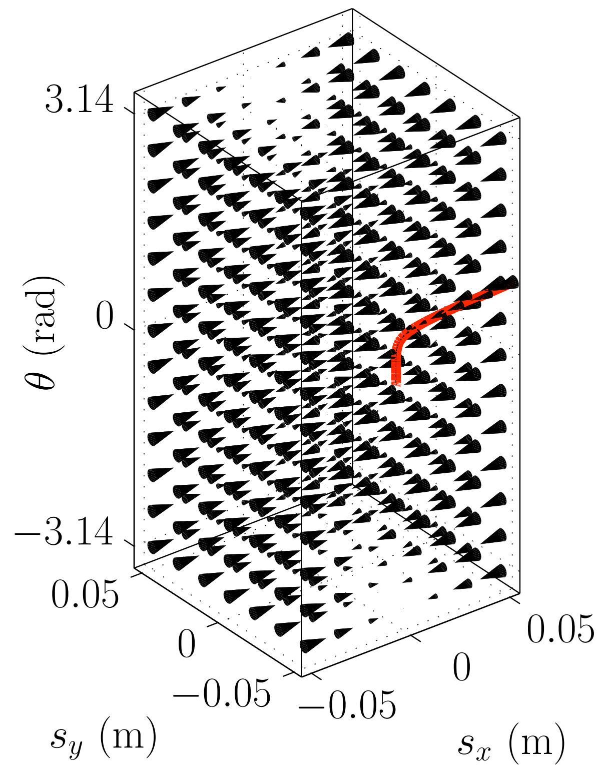
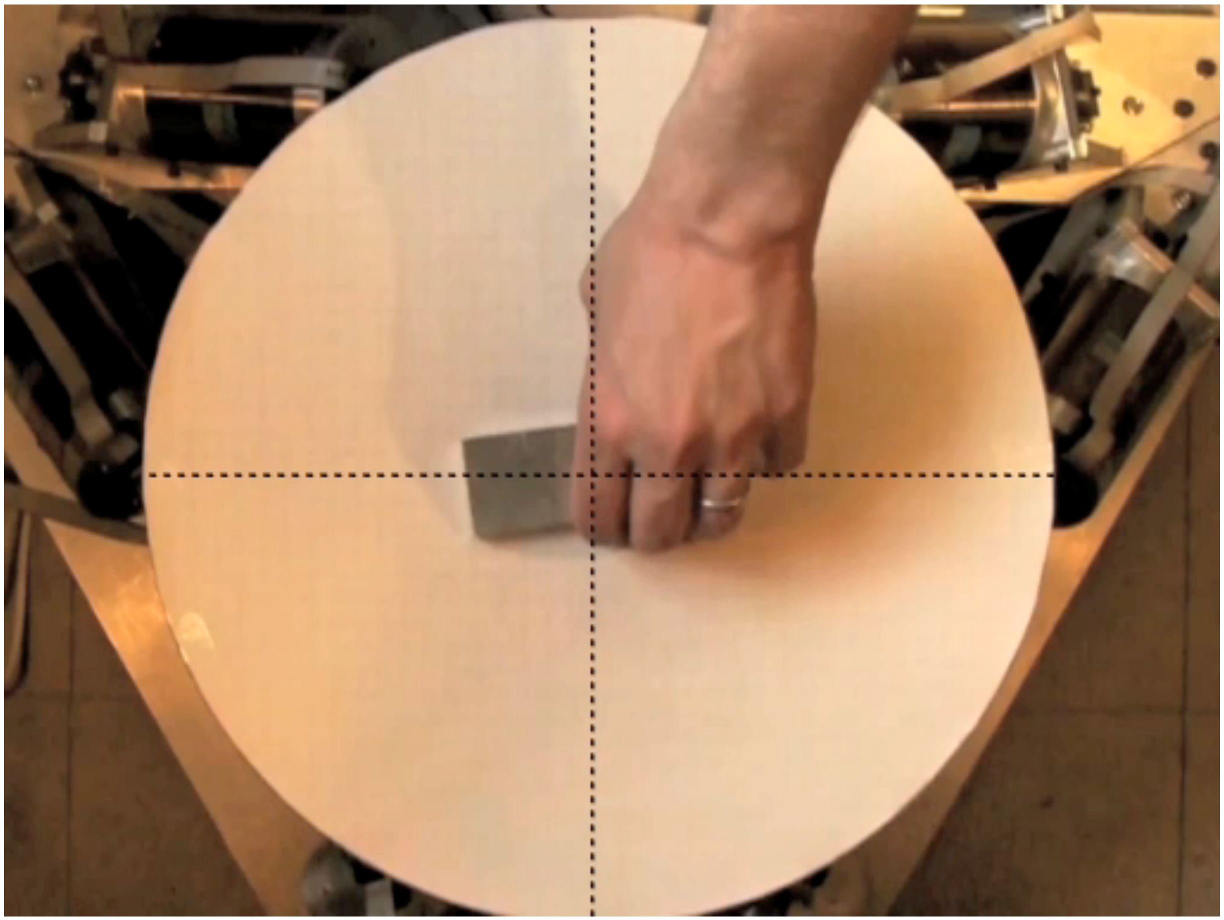
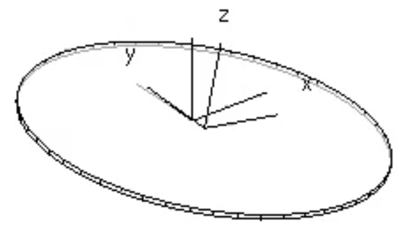


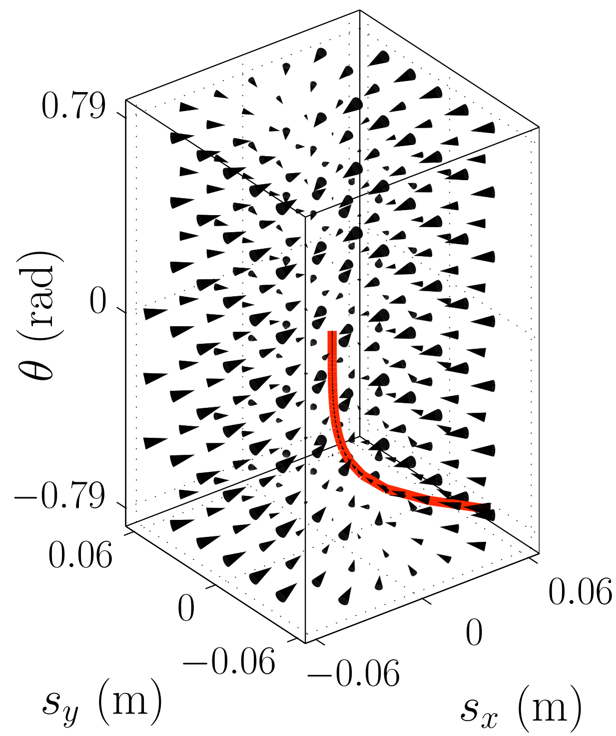




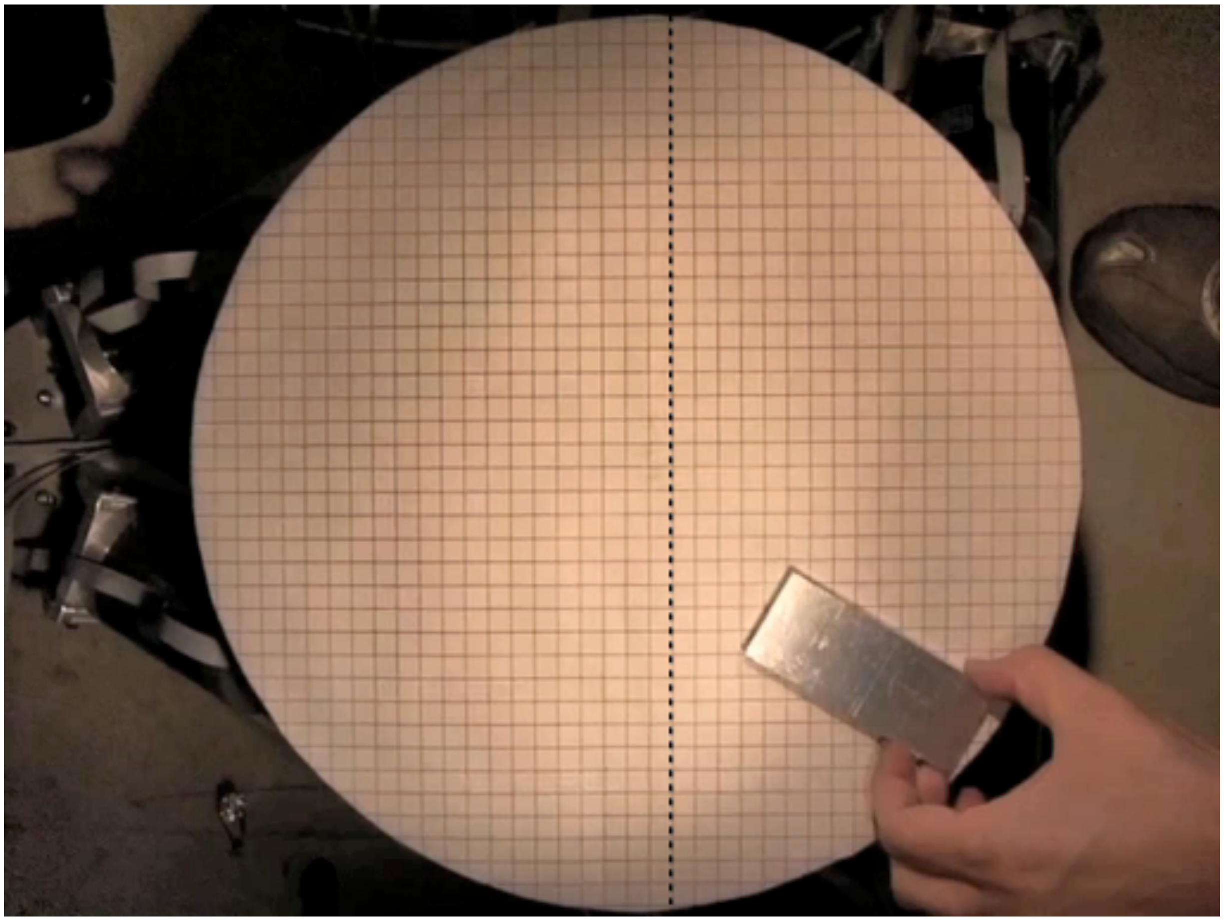
Sensorless Positioning and Orienting

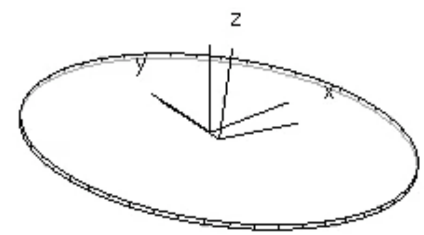


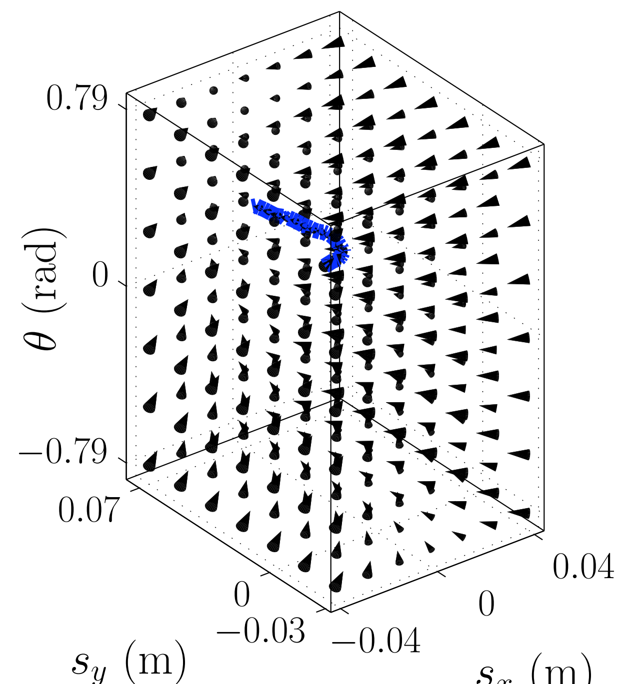




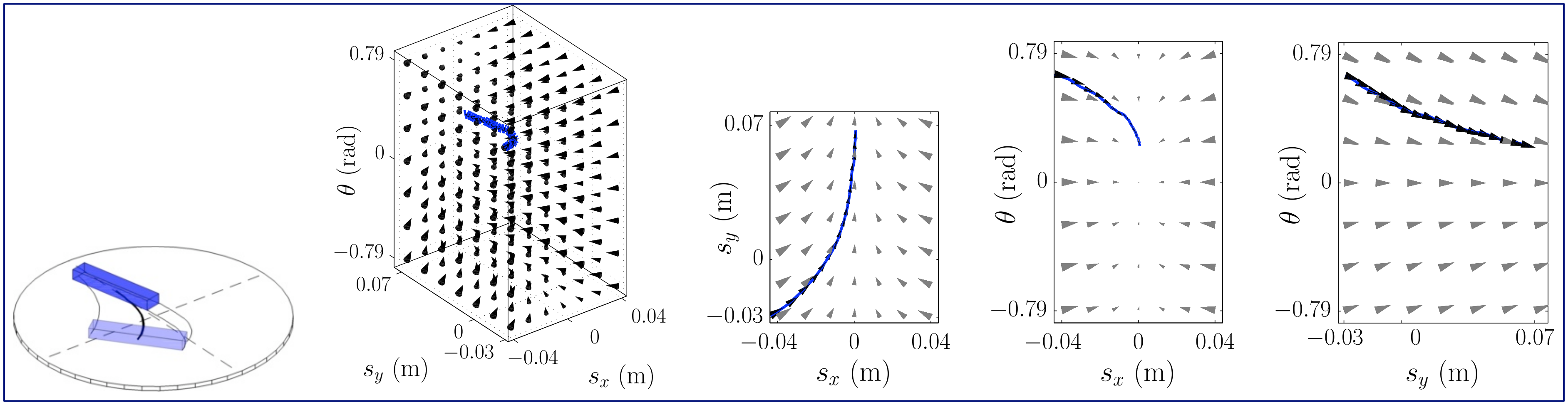
Sensorless Orienting and Transport

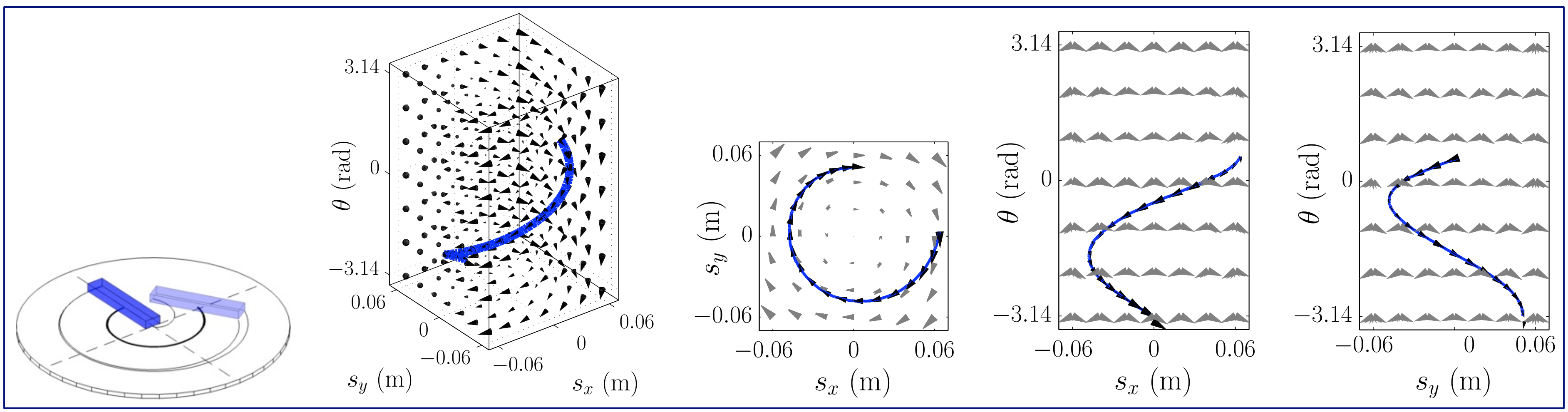






Asymptotics vs. Experimental Results





Outline

a nonprehensile primitive: vibratory sliding

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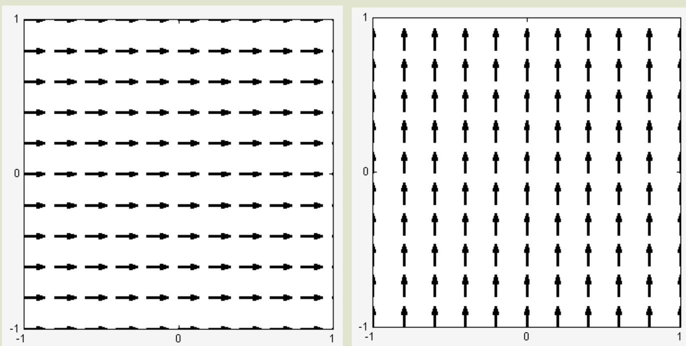
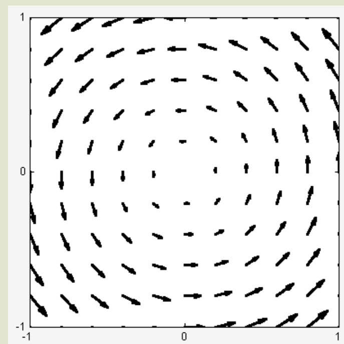
Basis Fields

(1 in-plane acceleration + 1 out-of-plane acceleration at the same frequency)

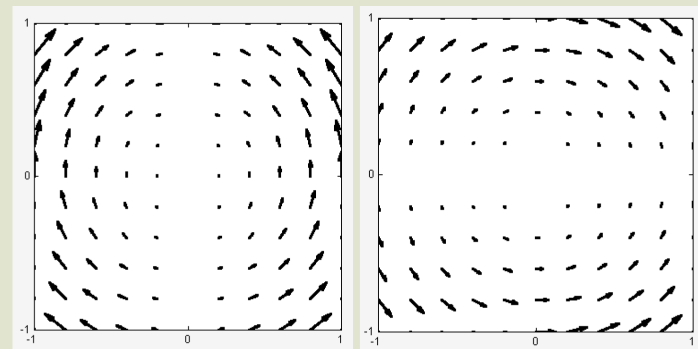
Translational



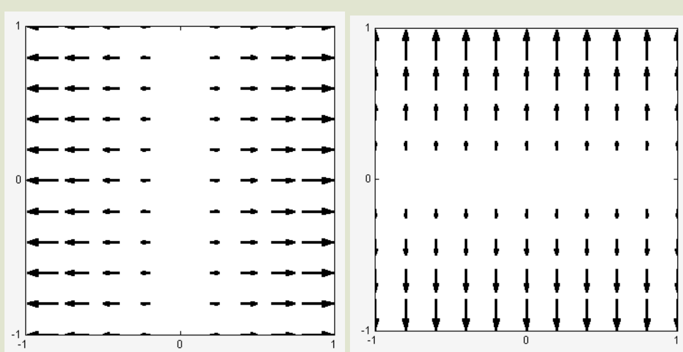
Circular



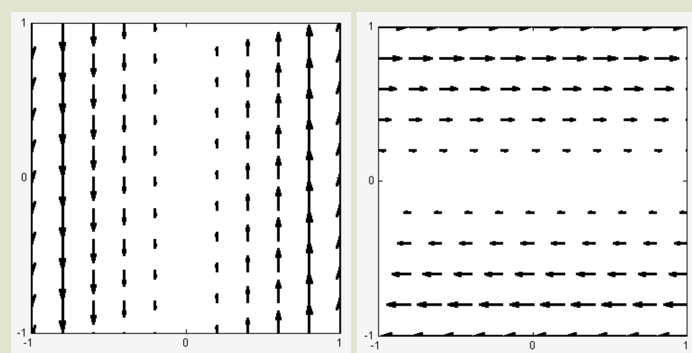
Divergent Circular



Line Source / Line Sink

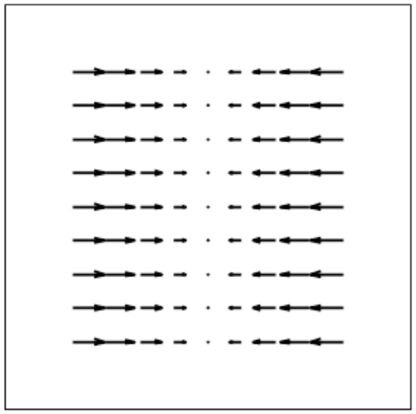


Shear



Dynamics Are Nonlinear

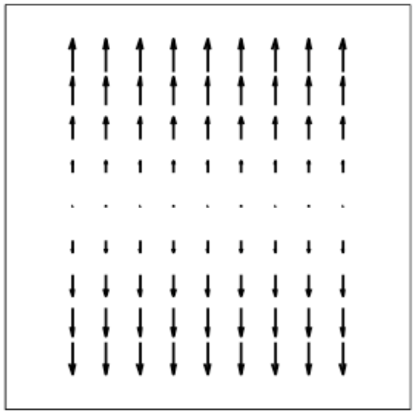
(a) `LineSinkX`



$$\ddot{p}_x = 10\sin(60\pi t)$$
$$\ddot{p}_y = 0$$
$$\alpha_x = 0$$
$$\alpha_y = 100\sin(60\pi t + \tfrac{3}{2}\pi)$$

$$\mathbf{v}_a \approx \begin{bmatrix} -0.27x \\ 0 \end{bmatrix}$$

(b) `LineSourceY`

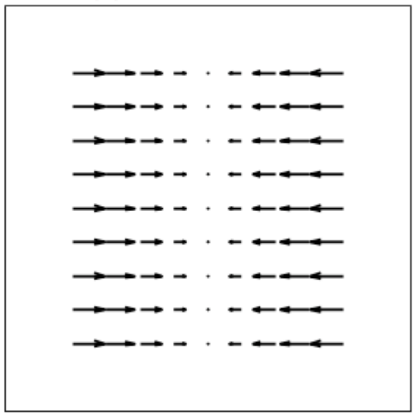


$$\ddot{p}_x = 0$$
$$\ddot{p}_y = 10\sin(60\pi t)$$
$$\alpha_x = 100\sin(60\pi t + \tfrac{3}{2}\pi)$$
$$\alpha_y = 0$$

$$\mathbf{v}_a \approx \begin{bmatrix} 0 \\ 0.27y \end{bmatrix}$$

Dynamics Are Nonlinear

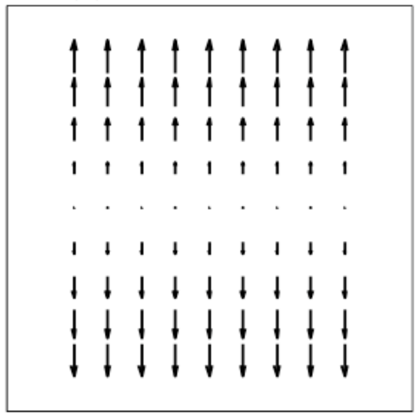
(a) `LineSinkX`



$$\ddot{p}_x = 10\sin(60\pi t)$$
$$\ddot{p}_y = 0$$
$$\alpha_x = 0$$
$$\alpha_y = 100\sin(60\pi t + \tfrac{3}{2}\pi)$$

$$\mathbf{v}_{\rm a} \approx \begin{bmatrix} -0.27x \\ 0 \end{bmatrix}$$

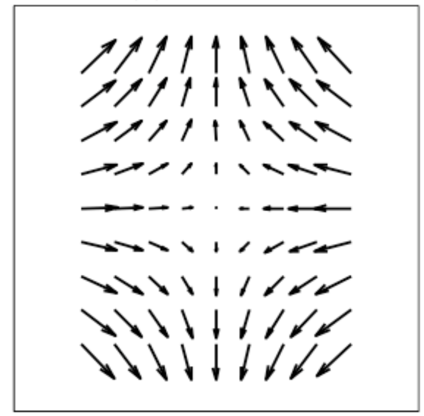
(b) `LineSourceY`



$$\ddot{p}_x = 0$$
$$\ddot{p}_y = 10\sin(60\pi t)$$
$$\alpha_x = 100\sin(60\pi t + \tfrac{3}{2}\pi)$$
$$\alpha_y = 0$$

$$\mathbf{v}_{\rm a} \approx \begin{bmatrix} 0 \\ 0.27y \end{bmatrix}$$

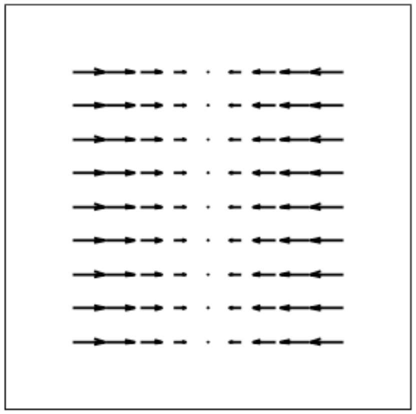
(d) `Saddle`



$$\mathbf{v}_{\rm a} \approx \begin{bmatrix} -0.27x \\ 0.27y \end{bmatrix}$$

Dynamics Are Nonlinear

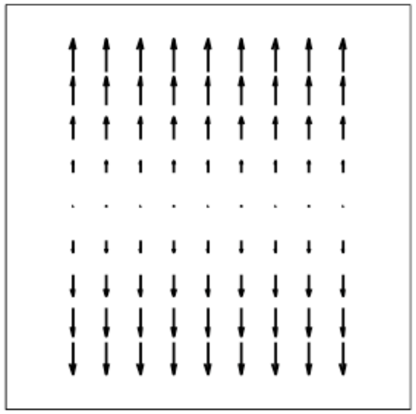
(a) `LineSinkX`



$$\ddot{p}_x = 10\sin(60\pi t)$$
$$\ddot{p}_y = 0$$
$$\alpha_x = 0$$
$$\alpha_y = 100\sin(60\pi t + \tfrac{3}{2}\pi)$$

$$\mathbf{v}_a \approx \begin{bmatrix} -0.27x \\ 0 \end{bmatrix}$$

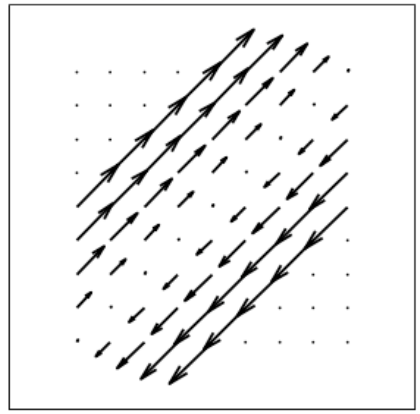
(b) `LineSourceY`



$$\ddot{p}_x = 0$$
$$\ddot{p}_y = 10\sin(60\pi t)$$
$$\alpha_x = 100\sin(60\pi t + \tfrac{3}{2}\pi)$$
$$\alpha_y = 0$$

$$\mathbf{v}_a \approx \begin{bmatrix} 0 \\ 0.27y \end{bmatrix}$$

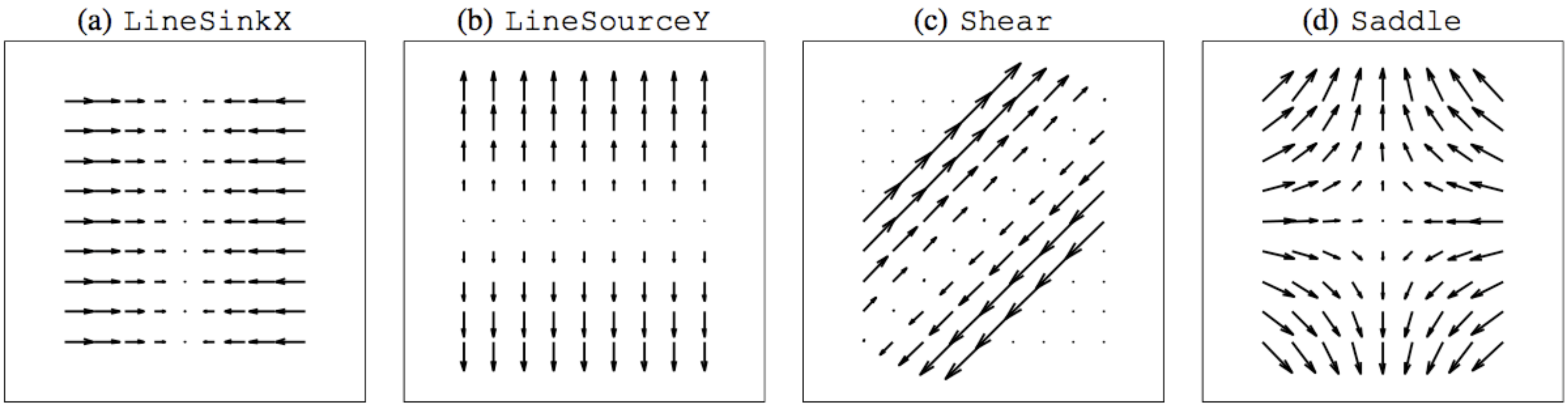
(c) `Shear`



$$\ddot{p}_x = 10\sin(60\pi t)$$
$$\ddot{p}_y = 10\sin(60\pi t)$$
$$\alpha_x = 100\sin(60\pi t + \tfrac{3}{2}\pi)$$
$$\alpha_y = 100\sin(60\pi t + \tfrac{3}{2}\pi)$$

$$\mathbf{v}_a \approx \begin{bmatrix} -0.35x + 0.35y \\ -0.34x + 0.35y \end{bmatrix}$$

Dynamics Are Nonlinear



(a) LineSinkX

$$\ddot{p}_x = 10\sin(60\pi t)$$
$$\ddot{p}_y = 0$$
$$\alpha_x = 0$$
$$\alpha_y = 100\sin(60\pi t + \tfrac{3}{2}\pi)$$

$$\mathbf{v}_{\mathrm{a}} \approx \begin{bmatrix} -0.27x \\ 0 \end{bmatrix}$$

(b) LineSourceY

$$\ddot{p}_x = 0$$
$$\ddot{p}_y = 10\sin(60\pi t)$$
$$\alpha_x = 100\sin(60\pi t + \tfrac{3}{2}\pi)$$
$$\alpha_y = 0$$

$$\mathbf{v}_{\mathrm{a}} \approx \begin{bmatrix} 0 \\ 0.27y \end{bmatrix}$$

(c) Shear

$$\ddot{p}_x = 10\sin(60\pi t)$$
$$\ddot{p}_y = 10\sin(60\pi t)$$
$$\alpha_x = 100\sin(60\pi t + \tfrac{3}{2}\pi)$$
$$\alpha_y = 100\sin(60\pi t + \tfrac{3}{2}\pi)$$

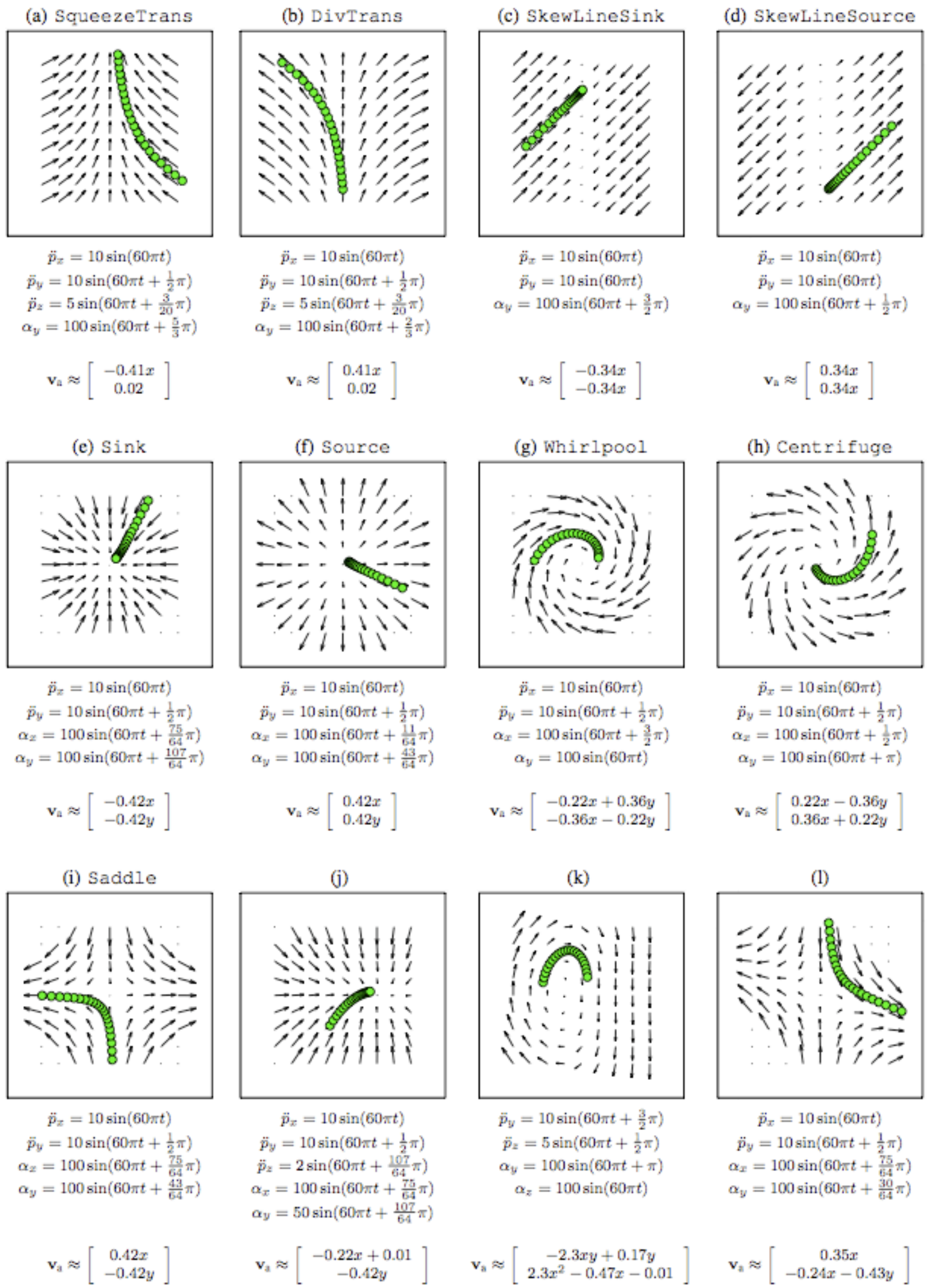
$$\mathbf{v}_{\mathrm{a}} \approx \begin{bmatrix} -0.35x + 0.35y \\ -0.34x + 0.35y \end{bmatrix}$$

(d) Saddle

$$\ddot{p}_x = 10\sin(60\pi t)$$
$$\ddot{p}_y = 10\sin(60\pi t + \tfrac{1}{2}\pi)$$
$$\alpha_x = 57\sin(60\pi t + \tfrac{5}{32}\pi)$$
$$\alpha_y = 57\sin(60\pi t + \tfrac{53}{32}\pi)$$

$$\mathbf{v}_{\mathrm{a}} \approx \begin{bmatrix} -0.27x \\ 0.27y \end{bmatrix}$$

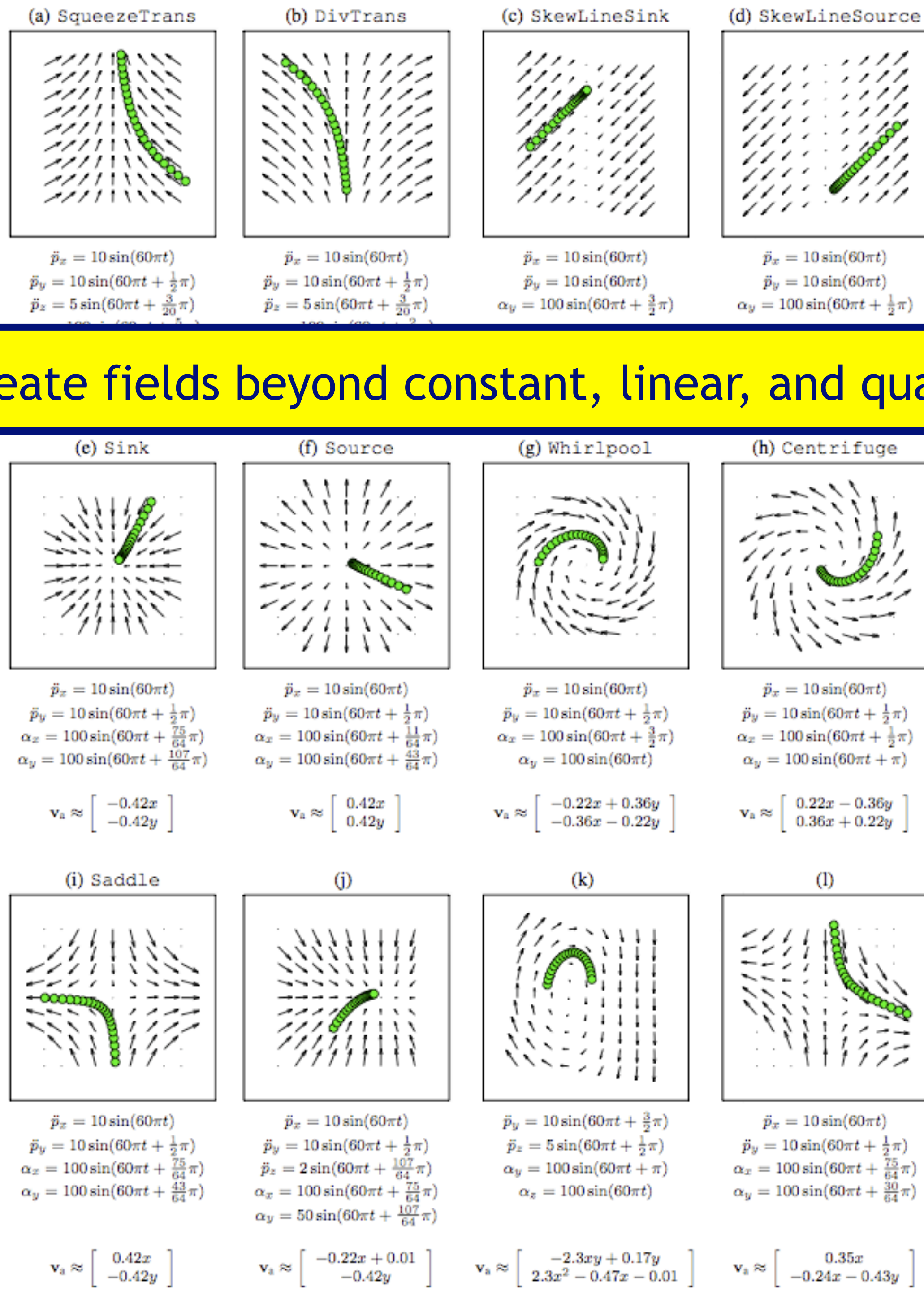
design by nonlinear optimization,
initial guess from linear superposition of “basis” fields



$A\sin(2\pi f t + \phi)$

- single vibration frequency
- six amplitudes
- five phases

11 control inputs ⟶ fields that are constant, linear, and quadratic as a function of position



harmonics create fields beyond constant, linear, and quadratic

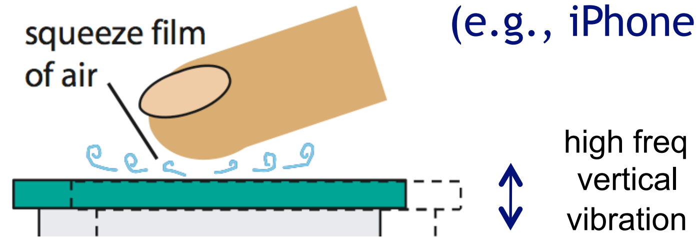
$$A\sin(2\pi f t + \phi)$$

- single vibration frequency
- six amplitudes
- five phases

11 control inputs →
fields that are constant, linear, and quadratic as a function of position

Extensions

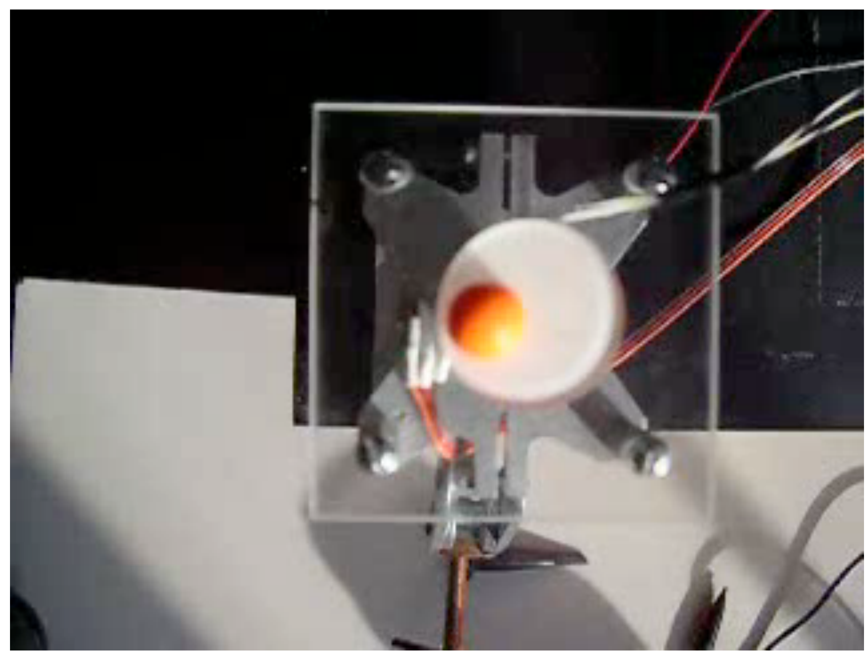
glass haptic display
(e.g., iPhone)



- controlling friction

$$\mathbf{f}_{\text{fric}} = \boxed{\mu} N \frac{\mathbf{v}_{\text{rel}}}{\|\mathbf{v}_{\text{rel}}\|}$$

- part interaction, assembly

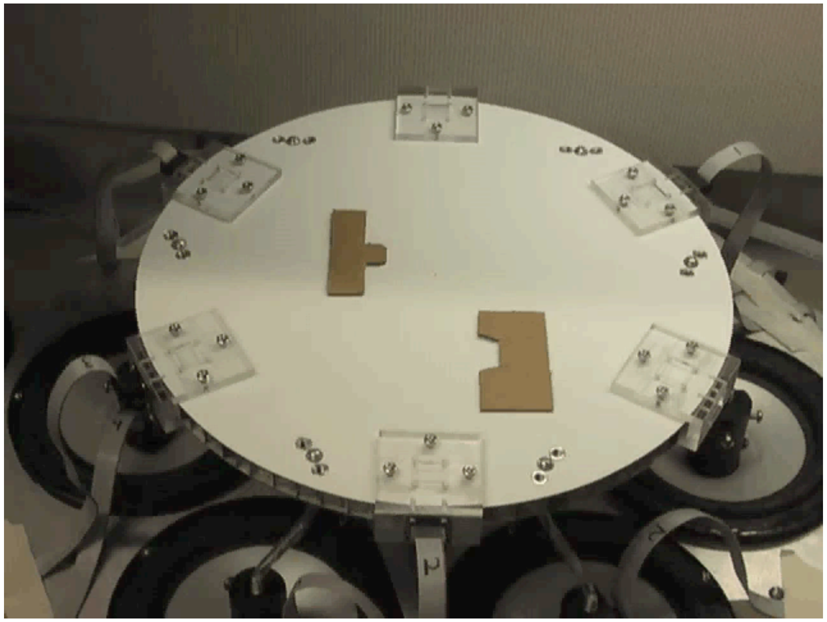


Colgate, Peshkin, et al.

Extensions

- controlling friction

- part interaction, assembly

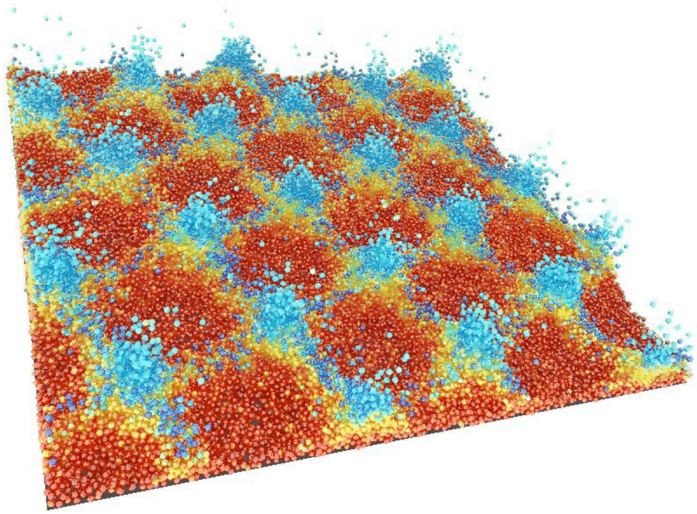


world's worst peg-in-hole

Extensions

- controlling friction

- part interaction, assembly



Umbanhowar, Swinney, et al.