# Computational Voting Theory:

## Game-Theoretic and Combinatorial Aspects

by

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Curtis R. Taylor

Dissertation submitted in partial fulfillment of the requirements for the degree of
Doctor of Philosophy in the Department of Computer Science
in the Graduate School of Duke University
2011

# Abstract

(Computer Science)

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# Abstract

For at least two thousand years, voting has been used as one of the most effective ways to aggregate people's ordinal preferences. In the last 50 years, the rapid development of Computer Science has revolutionize every aspect of the world, including voting. This motivates us to study (1) **conceptually, how computational thinking changes the traditional theory of voting**, and (2) **methodologically, how to better use voting for preference/information aggregation with the help of Computer Science**.

My Ph.D. work seeks to investigate and foster the interplay between Computer Science and Voting Theory. In this thesis, I will discuss two specific research directions pursued in my Ph.D. work, one for each question asked above. The first focuses on investigating how computational thinking affects the game-theoretic aspects of voting. More precisely, I will discuss the rationale and possibility of using computational complexity to protect voting from a type of strategic behavior of the voters, called *manipulation*. The second studies a voting setting called *Combinatorial Voting*, where the set of alternatives is exponentially large and has a combinatorial structure. I will focus on the design and analysis of novel voting rules for combinatorial voting that balance computational efficiency and the expressivity of the voting language, in light of some recent developments in Artificial Intelligence.

To my dearest wife Jing. The past four years have been a very hard time for both of us. Thank you for loving me, supporting me, encouraging me, and smiling and crying for me since the very beginning. I will never forget the 61 days we spent together in 2008, 108 days in 2009, 45 days in 2010, and 58 days by Aug. 11 in 2011.

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## Symbols

| | |
|---|---|
| $\mathcal{C} = \{c_1, \dots, c_m\}$ | The set of alternatives (candidates). |
| $P = (V_1, \dots, V_n)$ | An $n$-profile. |
| $m$ | Number of alternatives. |
| $n$ | Number of voters. |
| $n'$ | Number of manipulators. |
| $r$ | A voting rule. |
| $r^c$ | A voting correspondence. |

## Additional Symbols for Combinatorial Voting

| | |
|---|---|
| $p$ | Number of issues (characteristics). |
| $X_i$ | The $i$th issue. |
| $D_i$ | The $i$th local domains. |
| $\mathcal{X} = D_1 \times \cdots \times D_p$ | A multi-issue domain composed of $p$ issues. |
| $r_i$ | A local rule over $D_i$. |
| $Seq_{\mathcal{O}}(r_1, \dots, r_p)$ | The sequential composition of local rules $r_1, \dots, r_p$. |
| $\mathcal{L}$ | Admissible conditional preference set (Definition 12.2.1). |
| CPnets($\mathcal{L}$) | The set of all CP-nets consistent with $\mathcal{L}$ (Definition 12.2.2). |
| Pref($\mathcal{L}$) | The set of all linear orders that are consistent with $\mathcal{L}$ (Definition 12.2.2). |
| $LD(\mathcal{L})$ | The lexicographic preference domain (Definition 12.2.2). |

## Abbreviations

| | |
|---|---|
| WCM | Weighted coalitional manipulation. |
| UCM | Unweighted coalitional manipulation. |
| UCO | Unweighted coalitional optimization. |
| COd | Coalitional optimization with divisible votes. |
| GSR | Generalized scoring rules. |
| NE | Nash equilibrium. |
| SPNE | Subgame-perfect Nash equilibrium. |

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