CSCI-4979/6976: Computational Social Choice, 2014 Fall
Homework 1

Due on Sep. 11 before the class. You can either send it through email or hand in an hard copy before the class (electronic version is strongly preferred). If you have questions about the statements please post on Piazza.

In all proofs you can assume that there no ties.

Problem 1. (3pt) Let the voting rule be STV.

(a) Consider the following profile:

$$27@[a \succ b \succ c] \qquad 42@[c \succ a \succ b] \qquad 24@[b \succ c \succ a]$$

What happens when four votes switch from $a \succ b \succ c$ to $c \succ a \succ b$, and what axiomatic property does this violate?

(b) For the same profile in (a), what paradoxical outcome occurs when four voters with $a \succ b \succ c$ don't vote?

(c) Prove that STV does not satisfy consistency.

Problem 2. (3pt) Prove that all positional-scoring rules satisfy anonymity, neutrality, and consistency. **Remember that we can assume that there are no ties.**

Problem 3. (2pt) Prove that for any profile, weights on the edges of WMG have the same parity.

Problem 4. (5pt) Let $\vec{s}_B = (m-1, \ldots, 0)$ denote the scoring vector for Borda.

(a) Prove that for any $p > 0, q \in \mathbb{R}$, the positional scoring rule r with the scoring vector $p \cdot \vec{s}_B + q = (p(m-1)+q, p(m-2)+q, \ldots, q)$ is equivalent to Borda. That is, for any profile P, $r(P) = \text{Borda}(P)$.

(b) **(a little hard)** Prove the reverse of (a). That is, prove that a position scoring rule r with scoring vector $\vec{s} = (s_1, \ldots, s_m)$ is equivalent to Borda only if there exist $p > 0, q \in \mathbb{R}$ such that $\vec{s} = p \cdot \vec{s}_B + q$.
Hint: show that $s_1 - s_2 = s_2 - s_3 = \cdots = s_{m-1} - s_m$.

4′ Bonus question (hard): (3pt) Can you find a positional scoring rule different from Borda that is also based on weighted majority graph? (Note that the positional scoring rule with scoring vector $\vec{s} = p \cdot \vec{s}_B + q$ where $p > 0$ is NOT considered different from Borda.)

Problem 5. (5pt) Prove or disprove the following conjecture.

For any $m \geq 3$, there exists a profile P such that for different $k \leq m-1$, k-approval selects a different winner.

5′ Bonus question (very hard): (3pt)

- If you prove the conjecture, try to prove or disprove a stronger conjecture: for any $m \geq 3$, there exists a profile P such that Borda and all k-approval's (for $k \leq m-1$) choose different winners. Notice that there are m voting rules and m alternatives.

- If you disprove the conjecture, try to find the largest $m' < m$ such that there exists a profile P such that Borda and k-approval (for $k \leq m'-1$) choose different winners. Notice that in this case m' is a function of m.