CSCI-4979/6976: Computational Social Choice, 2014 Fall
Homework 2

Due on Sep. 25 before the class. You can either send it through email or hand in an hard copy before the class. If you have questions about the statements please post on Piazza.

1 Comprehension questions.

Recall the "what why how" questions.

- What is the problem? (Describe the problem, but don't copy the definitions.)
- Why we want to study this problem? How general it is? (You need to argue why there does not exist a trivial solution. For generality, you can find a few applications covered/not covered by the problem.)
- How was problem addressed? (Described the solutions you learned in the class. Again, don't copy the definitions.)

You will need to answer these questions for all lines of research covered in the class, as well as research papers you will read and present in the second part of the class. Write down all your thoughts.

You will always have opportunities to earn bonus points by answering the "appreciate the work" and "critical thinking" questions.

- Appreciate the work. (Why it is an interesting and non-trivial approach? What is the thing you like most? This somehow overlaps the "why" question.)
- Critical thinking? (Anything you are not happy about the work? Can you think of a way to overcome the limitation?)

Problem 1. (3pt) Answer the "what why how" questions for Game Theory.
1' Bonus question: 1 pt Answer the "appreciate the work" and "critical thinking" questions.

Problem 2. (3pt) Answer the "what why how" questions for Mechanism Design.
2' Bonus question: 1 pt Answer the "appreciate the work" and "critical thinking" questions.

Problem 3. (3pt) Answer the "what why how" questions for (Integer/Mixed Integer) Linear Programming.
3' Bonus question: 1 pt Answer the "appreciate the work" and "critical thinking" questions.

2 Technical questions

Problem 4. (6pts) Consider the extensive-form game illustrated in Fig-

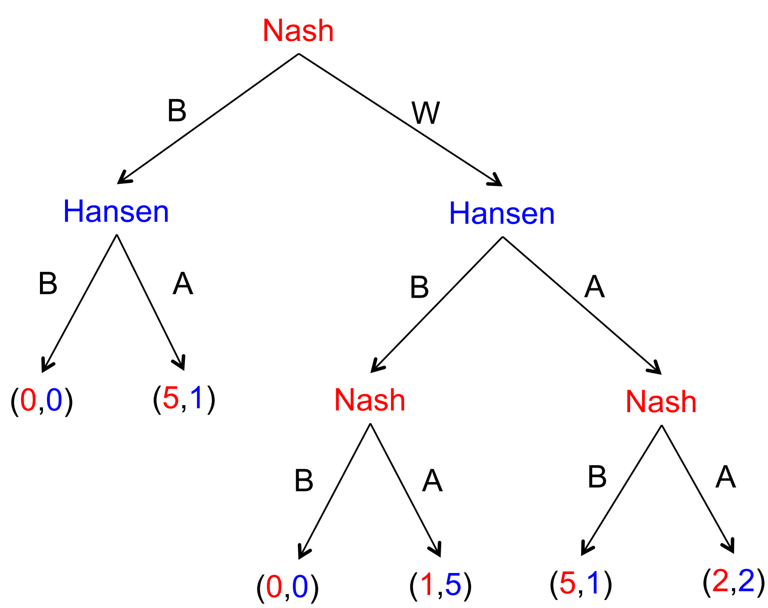

Figure 1: An extensive-form game.

ure 1. In the first round Nash can either choose to go for the Blond (B) or wait for Hanson's move (W).

(a) **(2pt)** Find the SPNE using backward induction.

(b) **(2pt)** Convert the game to a normal form game.

(c) **(2pt)** find all pure-strategy NE of the normal form game.

Problem 5. (2pts) The **Allais paradox** arises when people are tested in the two experiments illustrated in Table 1. In each experiment, a person can choose one of the two options (A or B).

Suppose a person chooses Lottery A for Experiment 1 and Lottery B for Experiment 2 (in fact many people do so). Show that this is a failure of utility theory.

Experiment 1		Experiment 2	
Lottery A	Lottery B	Lottery A	Lottery B
\$1M@100%	\$1M@89% + \$5M@10% + 0@1%	\$1M@11%+0@89%	\$0M@90% + \$5M@10%

Table 1: Allais paradox.

Problem 6. (6pts) In an *ad auction* there are n bidders bidding for $m < n$ slots. Each bidder is interested in getting only one slot. The slots are ranked from the top (first) to the bottom (last) on the right side of a webpage. The ith slot will get s_i clicks. We can assume that $s_1 > s_2 > \cdots > s_m$.

An outcome consists in two parts: an allocation of m slots to m different bidders, and the **pay-per-click payment** p_i for the ith slot. Each bidder j has a private value v_j^* for each **click**, thus her utility for getting the ith slot is $s_i(v_j^* - p_i)$.

The *generalized second price auctions (GSP)* is a popular mechanism for ad auctions currently used at many companies including Google. It works as follows.

- Rank the bids from high to low. Let $b'_1 > b'_2 > \cdots > b'_n$ denote these bids.
- Allocate the 1st slot to the bidder with the highest bid b'_1, the 2nd slot to the bidder with the second highest bid b'_2, ..., allocate the mth slot to the bidder with bid b'_m.
- For each $i \leq m$, let $p_i = b_{i+1}$.

Questions.

(a) **(1pt)** Is GSP a direct revelation mechanism? Why?

(b) **(2pts)** Suppose $n = 4$ and $m = 3$; $s_1 = 100, s_2 = 60, s_3 = 40$; $v_1^* = 10, v_2^* = 9, v_3^* = 7, v_4^* = 1$. Show that GSP is not truthful in this case. That is, when all bidders report truthfully, at least one of the bidders have incentive to lie. Identify all bidders who have incentive to lie.

(c) **(3pts)** What is the VCG outcome and payments (note: pay-per-click payments, not the total payments) given that all bidders report their true values?

Problem 7. (3pts) Given $m \in \mathbb{N}$ and m positional scoring rules with scoring vectors $(\vec{s^1}, \vec{s^2}, \ldots, \vec{s^m})$, where $\vec{s^i} = (s_1^i, \ldots, s_m^i)$.

Design a mixed integer programming to find a profile P with the smallest number of votes so that all these m positional scoring rules output different winners.

Comment: The MIP should be able to identify "failures", that is, situations where such a profile does not exist. You don't need to write down all constraints explicitly, but make sure that every parameter you use in the MIP is well defined.

hint: use $m!$ variables to represent the number of each ranking in P.

7' Bonus question (3pt): can you use only polynomially many (integer or real) variables and constraints?

hint: Check out "Doubly stochastic matrix" and "Birkhoffvon Neumann theorem" at `http://en.wikipedia.org/wiki/Doubly_stochastic_matrix`

Problem 8. (4pts) Design a mixed integer programming to solve the dominating set problem using polynomially many variables and constraints. An instance of dominating set is (G, k), where $G = (V, E)$ is an undirected graph of n vertices.

hint: Remember that dominating set is a decision problem and MIP is an optimization problem. So you need to find a straightforward way to interpret the outcome of your MIP (existence of solution, max value, etc) to an yes/no answer to the dominating set instance. Then you need prove that 1) a solution to the dominating set instance is an optimal solution to your MIP, and 2) an optimal solution to your MIP is a solution to the dominating set instance.