# News and announcements

- Report your preferences over papers soon!
  - deadline this Thursday before the class
- Drop deadline Oct 17
- Catalan independence referendum
  - Nov 9, 2014

# Nobel prize in Economics 2013

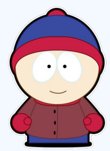
Alvin E. Roth

Lloyd Shapley

- "for the theory of stable allocations and the practice of market design."

# Two-sided one-one matching

Boys

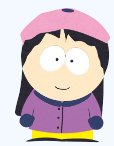

Stan


Kyle


Kenny


Eric

Girls


Wendy


Rebecca

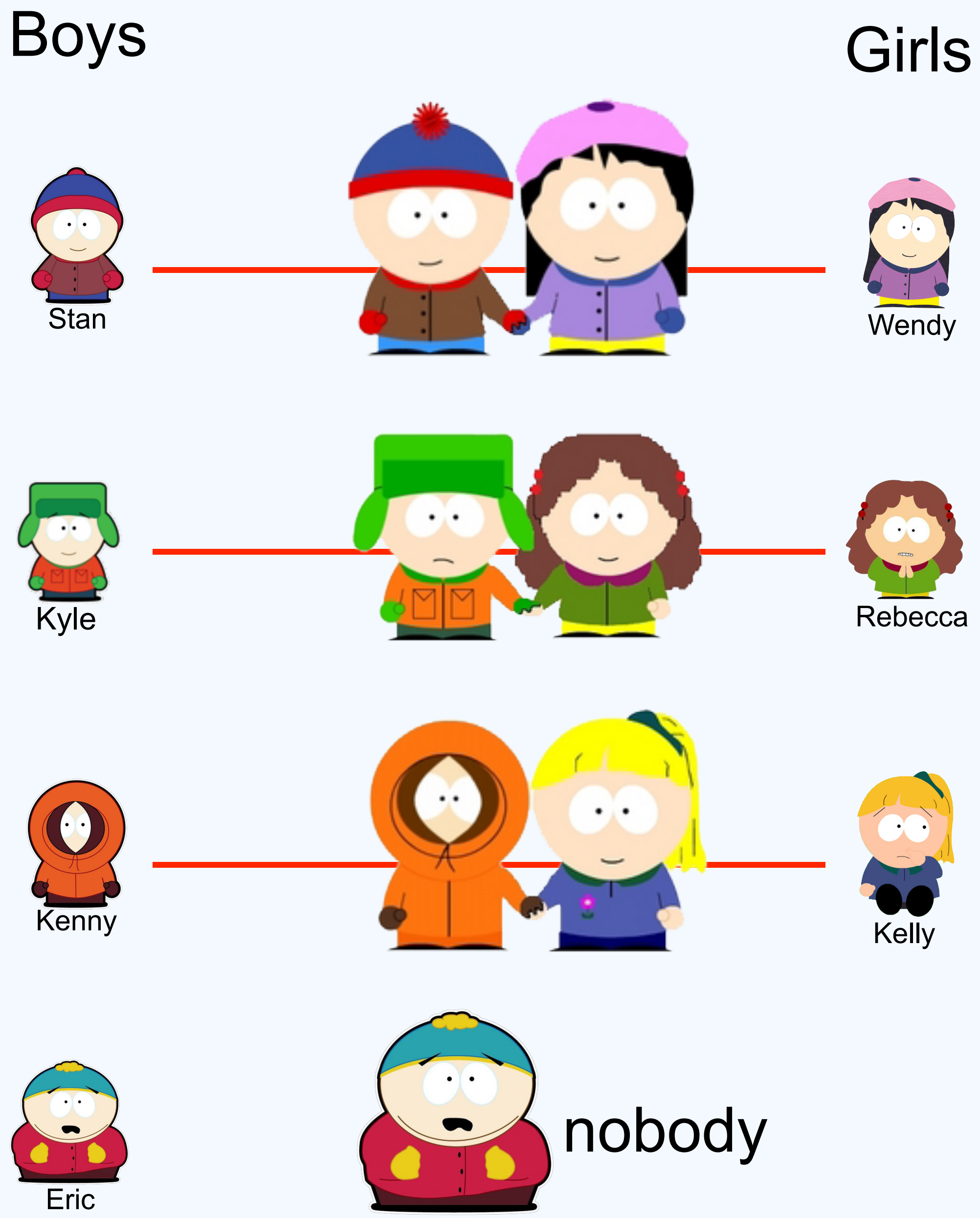

Kelly

Applications: student/hospital, National Resident Matching Program

# Formal setting

- Two groups: $B$ and $G$
- Preferences:
  - members in $B$: full ranking over $G \cup$ {nobody}
  - members in $G$: full ranking over $B \cup$ {nobody}
- Outcomes: a matching M: $B \cup G \rightarrow B \cup G \cup$ {nobody}
  - M($B$) $\subseteq G \cup$ {nobody}
  - M($G$) $\subseteq B \cup$ {nobody}
  - [M($a$)=M($b$)≠nobody] $\Rightarrow$ [$a$=$b$]
  - [M($a$)=$b$] $\Rightarrow$ [M($b$)=$a$]

# Example of a matching



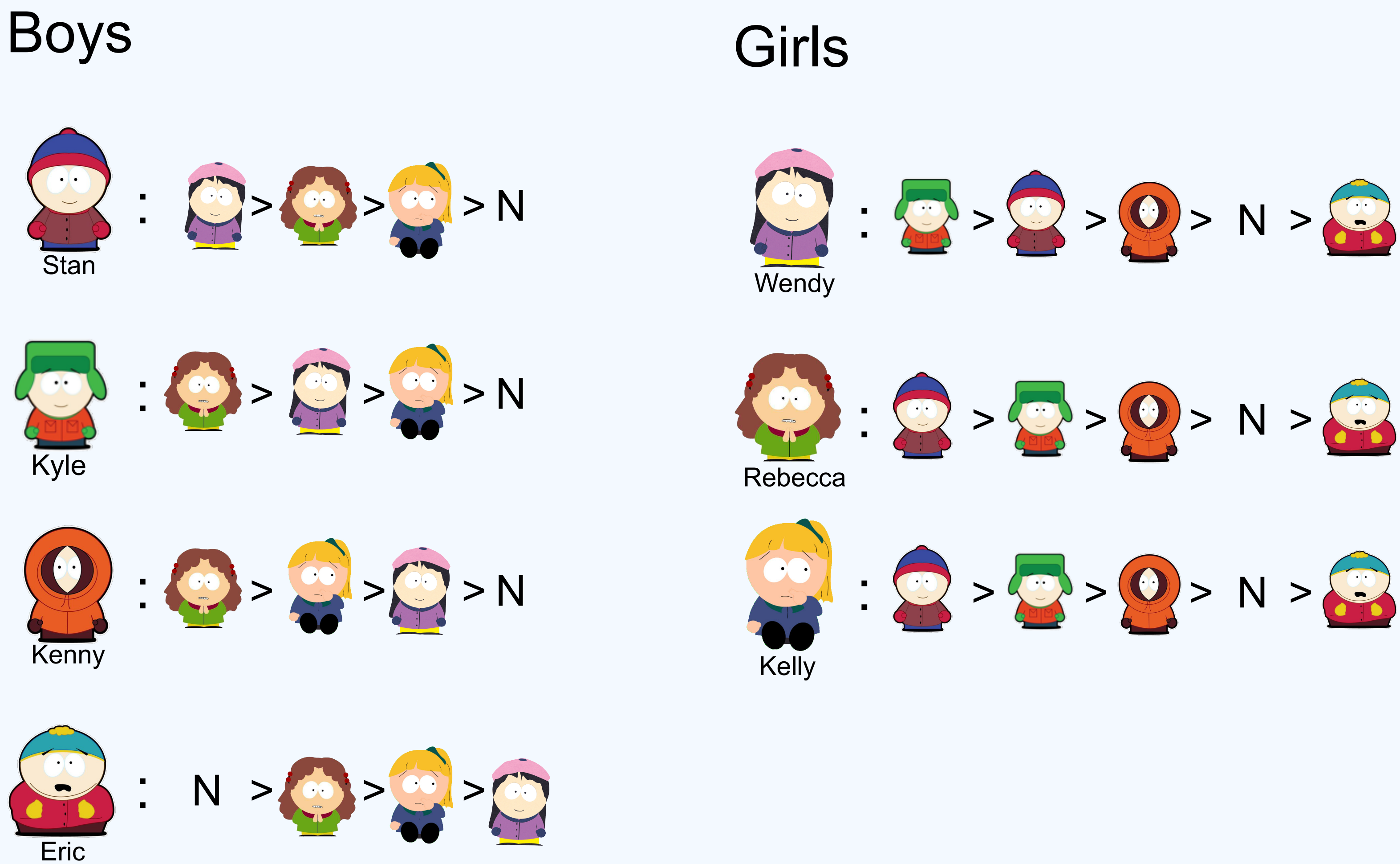
# Good matching?

- Does a matching always exist?
  - apparently yes
- Which matching is the best?
  - utilitarian: maximizes “total satisfaction”
  - egalitarian: maximizes minimum satisfaction
  - but how to define utility?

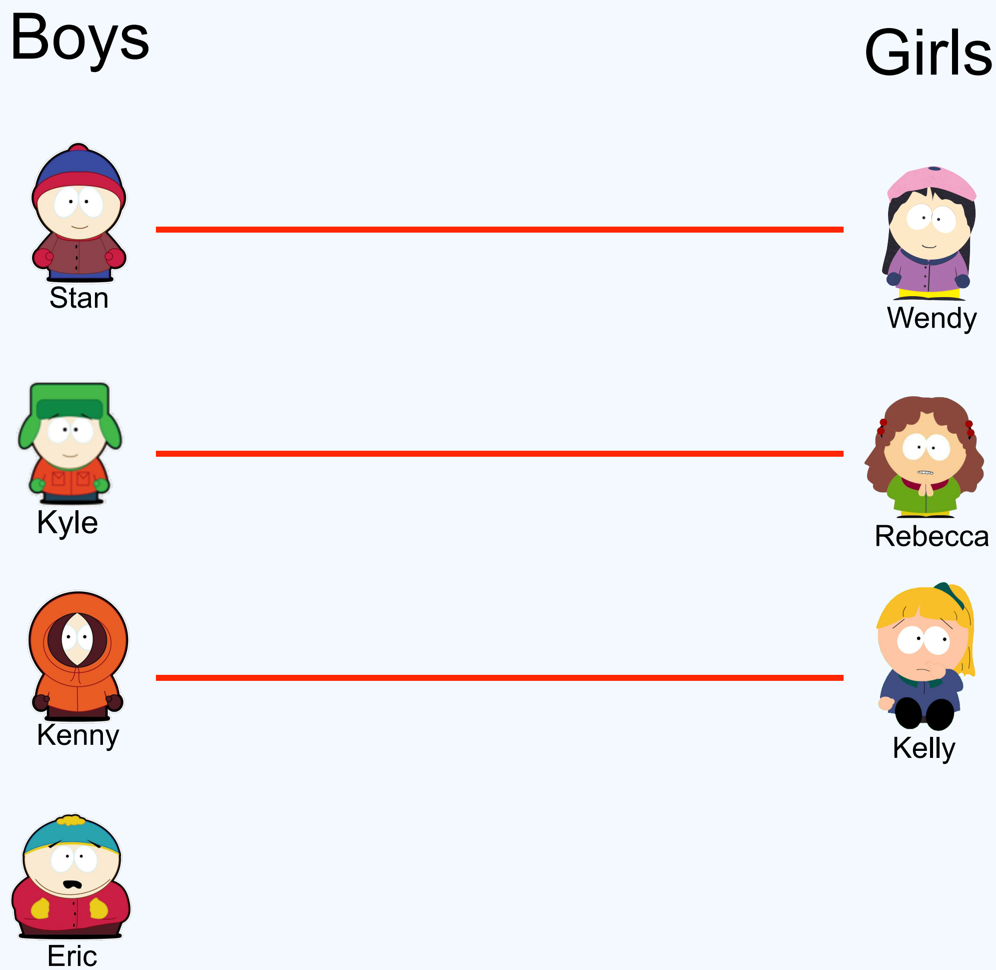
# Stable matchings

- Given a matching M, $(b,g)$ is a blocking pair if
  - $g >_b M(b)$
  - $b >_g M(g)$
  - ignore the condition for nobody
- A matching is stable, if there is no blocking pair
  - no (boy,girl) pair wants to deviate from their currently matches

# Example

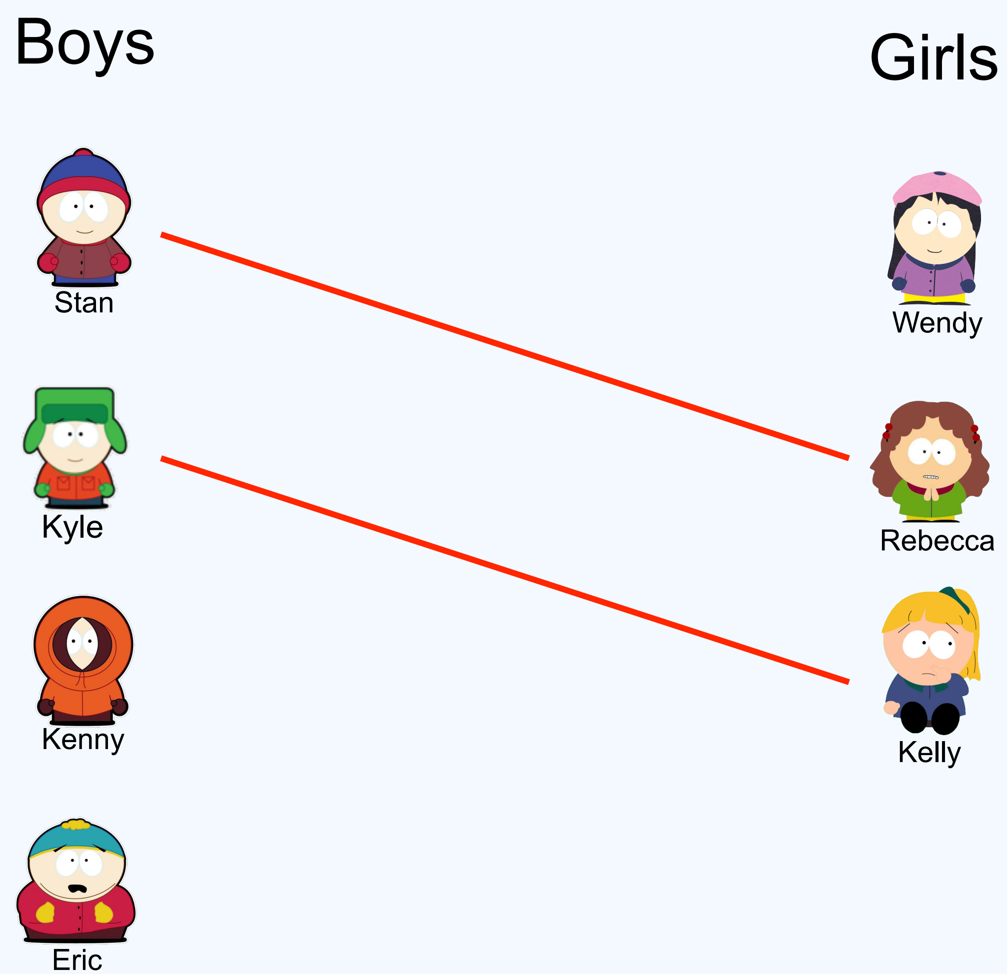

Boys
Stan : > > > N
Kyle : > > > N
Kenny : > > > N
Eric : N > > >
Girls
Wendy : > > > N >
Rebecca : > > > N >
Kelly : > > > N >

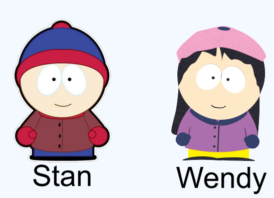
# A stable matching




no link = matched to “nobody”

# An unstable matching

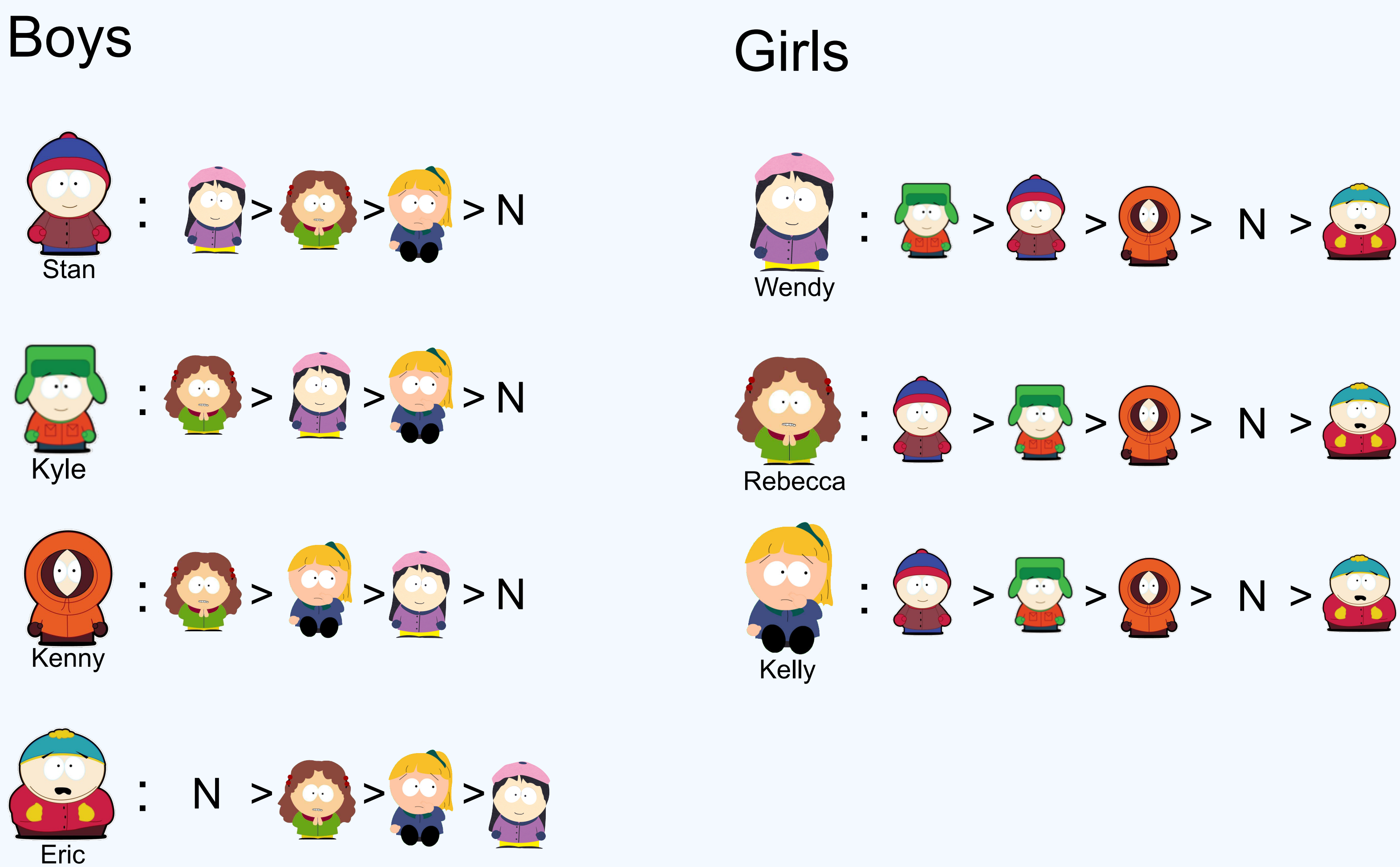



Blocking pair: (  )

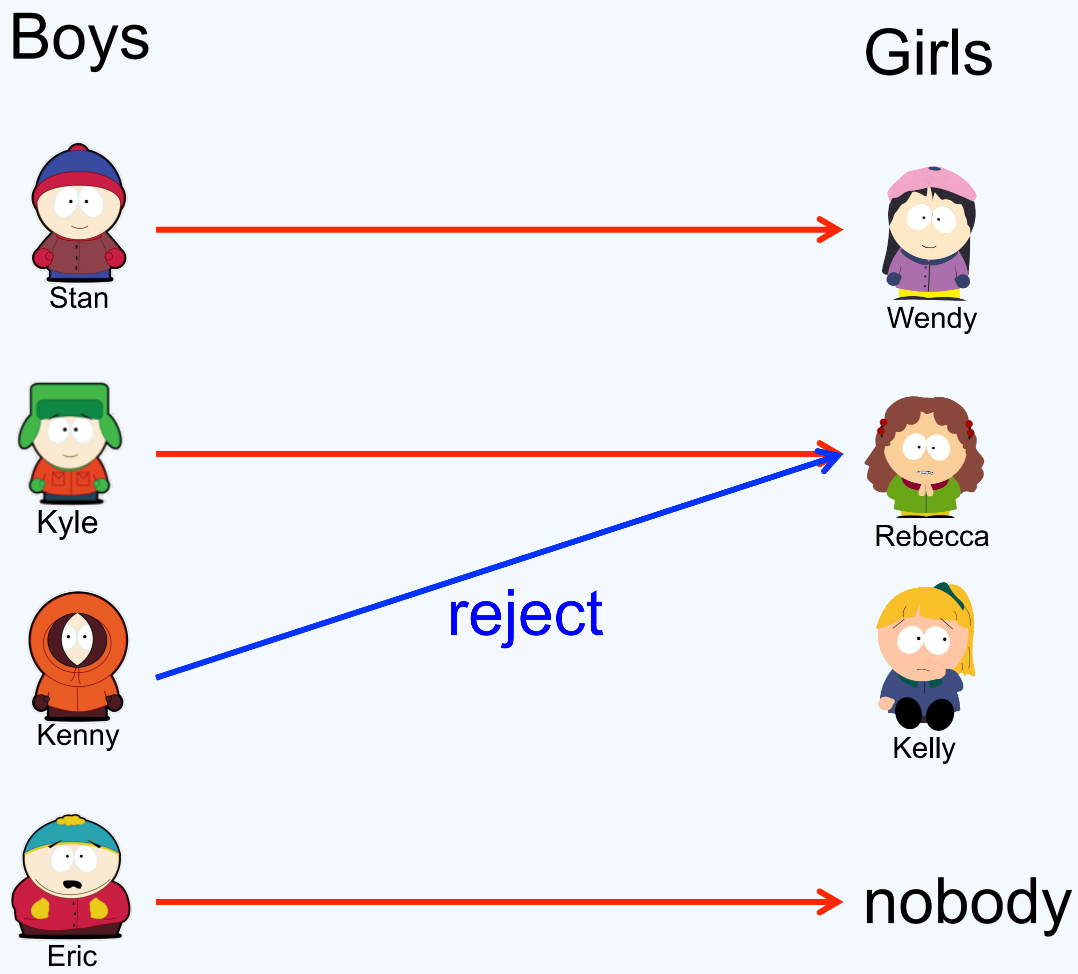
# Does a stable matching always exist?

- Yes: Gale-Shapley's deferred acceptance algorithm (DA)
- Men-proposing DA: each girl starts with being matched to "nobody"
  - each boy proposes to his top-ranked girl (or "nobody") who has not rejected him before
  - each girl rejects all but her most-preferred proposal
  - until no boy can make more proposals
- In the algorithm
  - Boys are getting worse
  - Girls are getting better

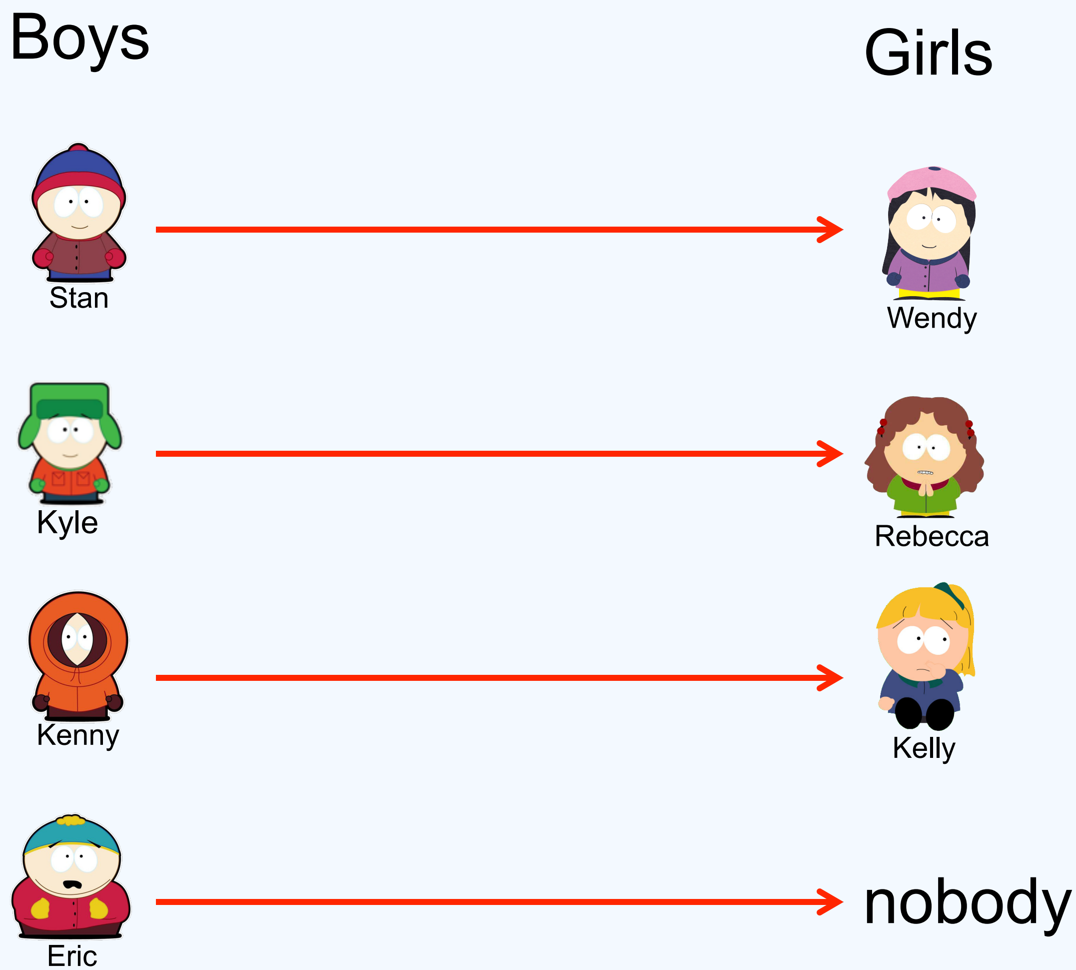
# Men-proposing DA (on blackboard)



# Round 1

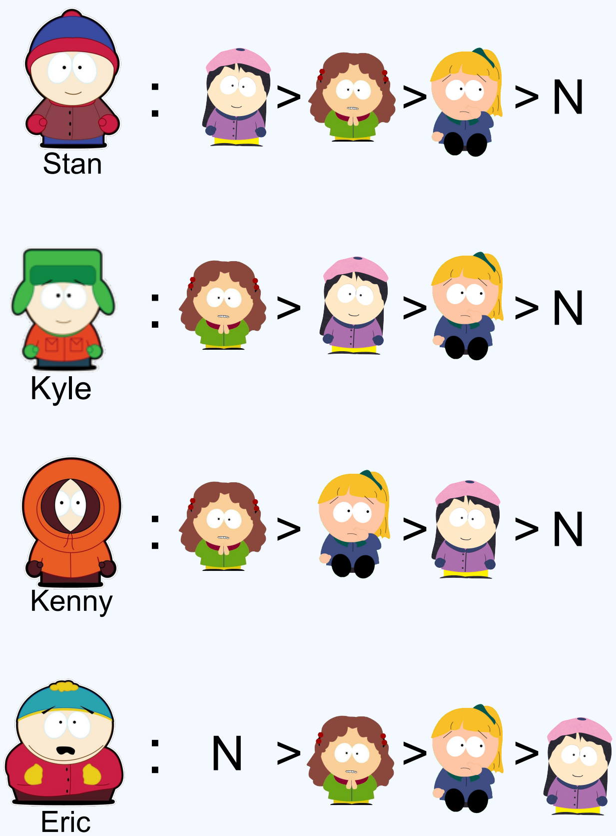

Boys
Girls
Stan
Wendy
Kyle
Rebecca
Kenny
reject
Kelly
Eric
nobody

# Round 2

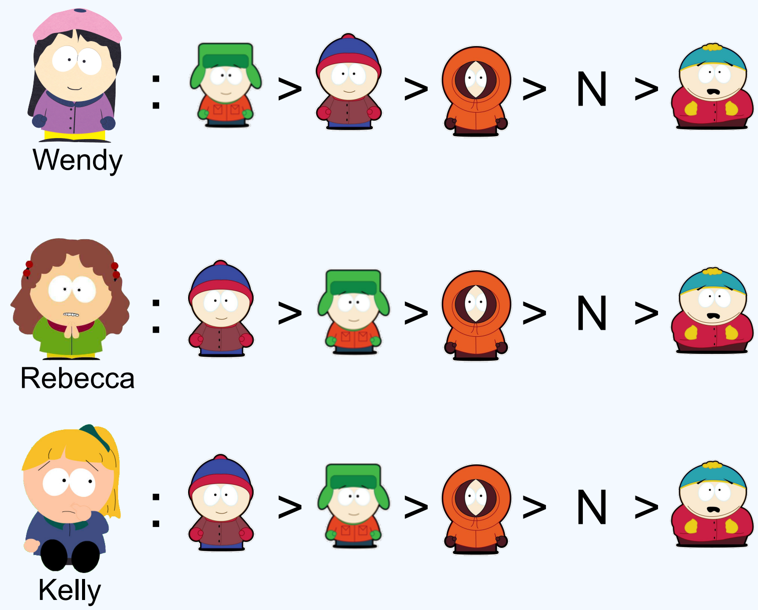


# Women-proposing DA (on blackboard)

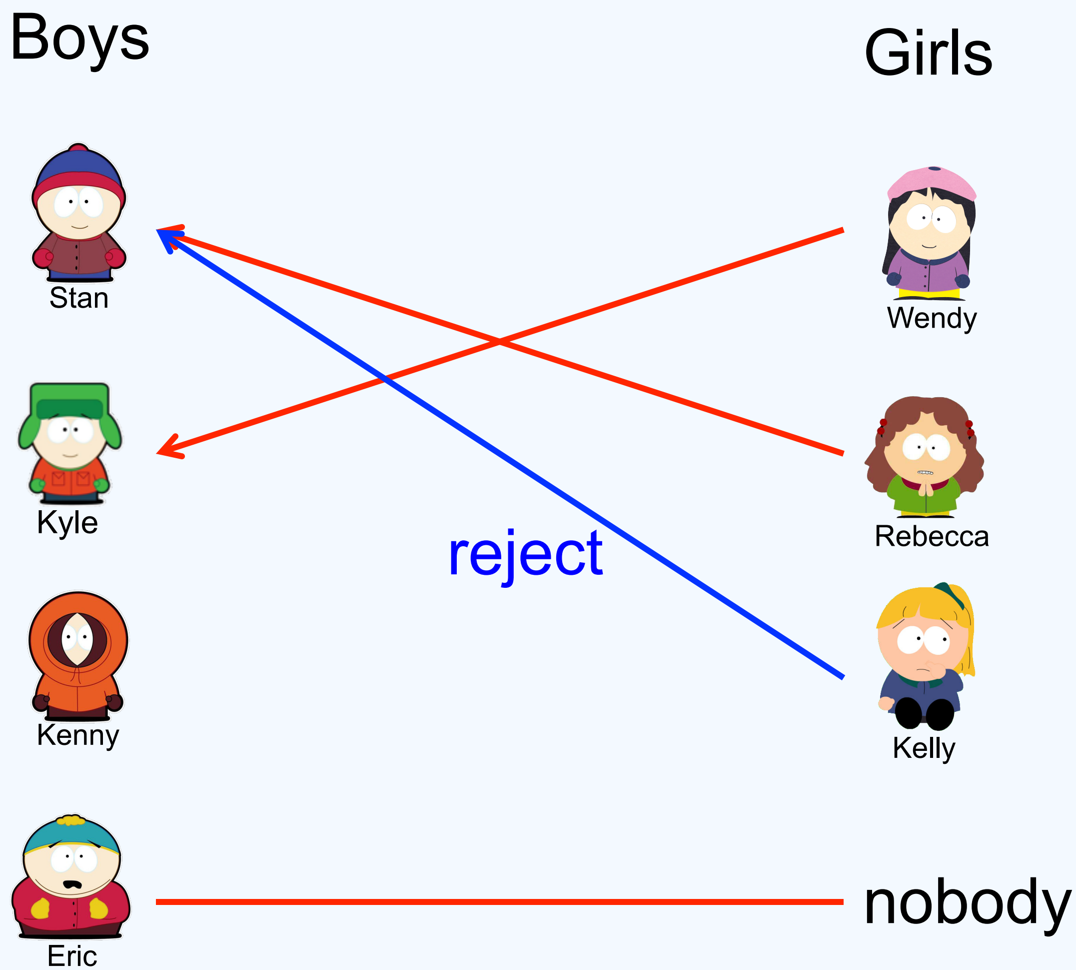
Boys



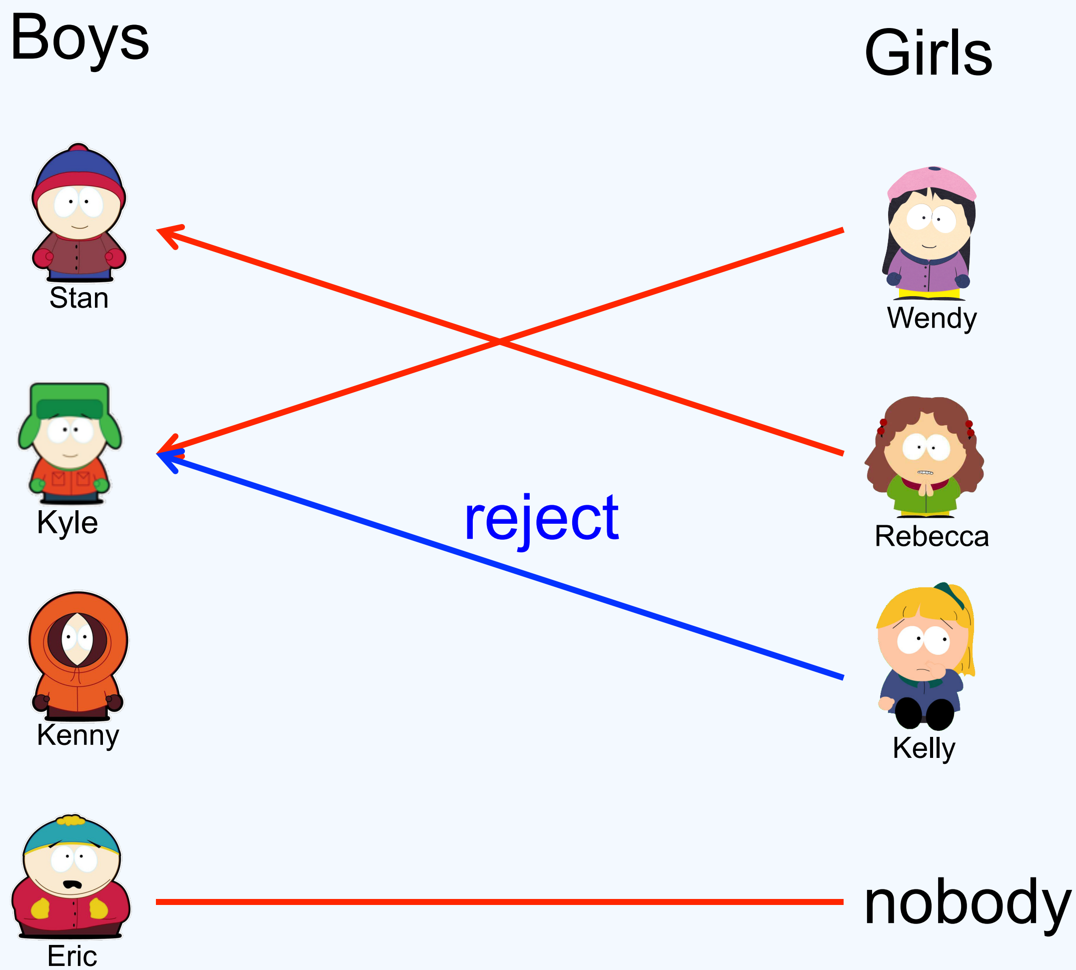

Girls



# Round 1

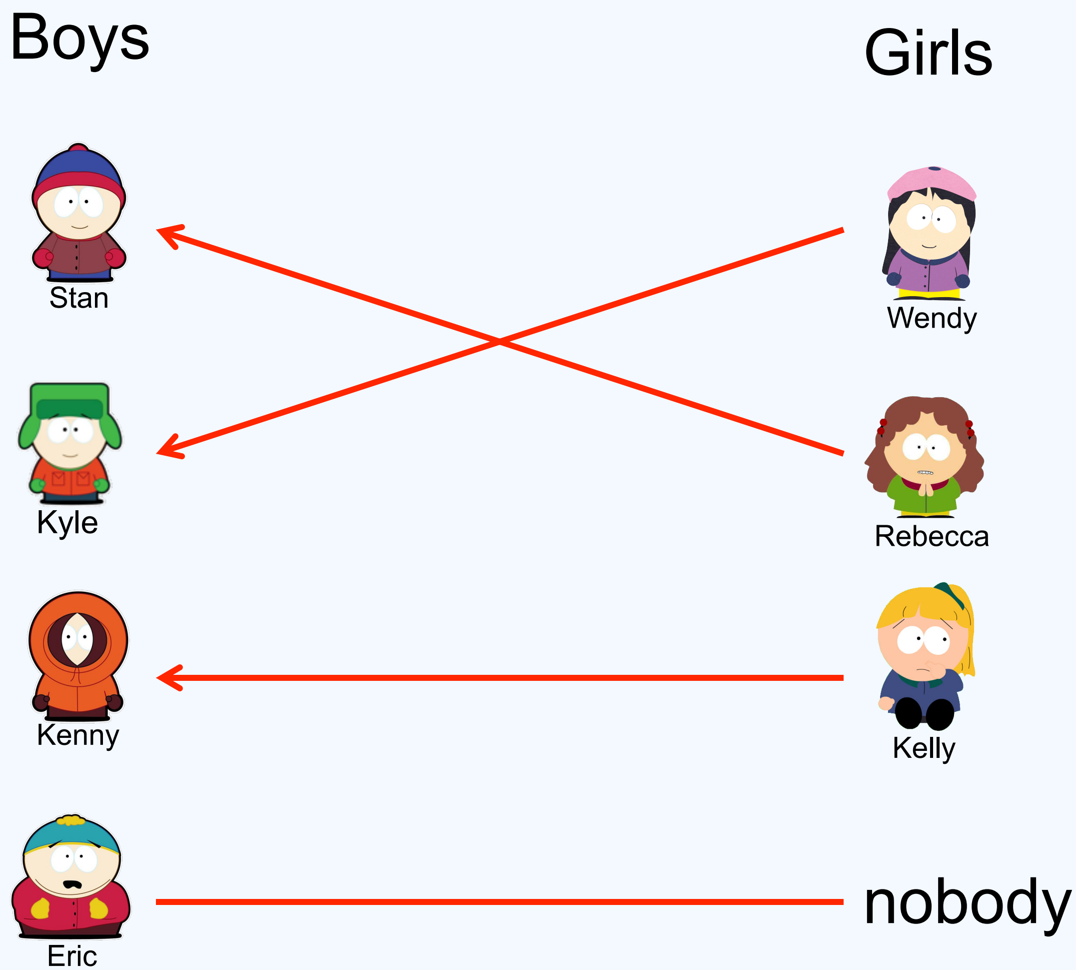

Boys
Girls
Stan
Wendy
Kyle
Rebecca
reject
Kenny
Kelly
Eric
nobody

# Round 2

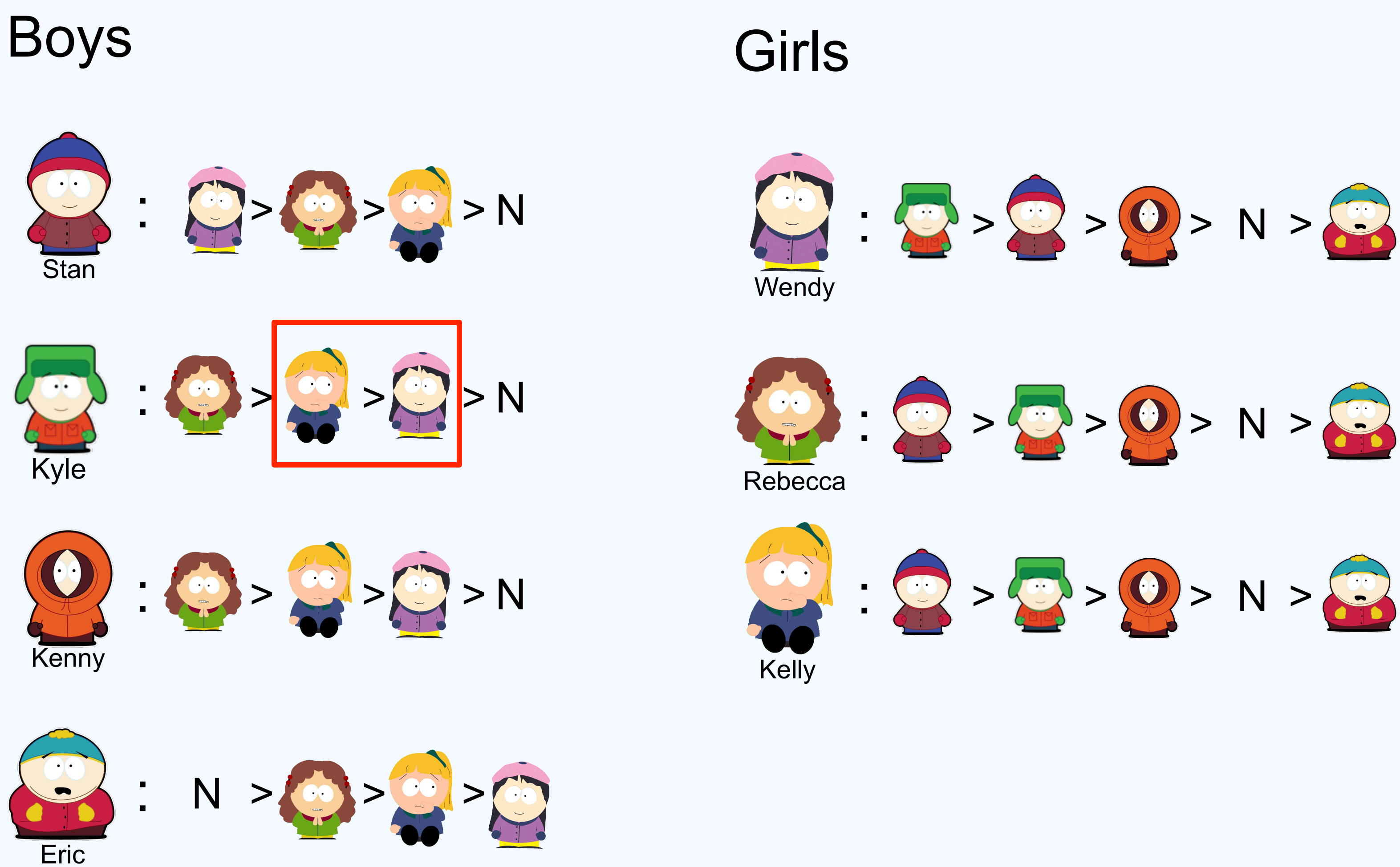


Boys
Girls
Stan
Wendy
Kyle
Rebecca
reject
Kenny
Kelly
Eric
nobody

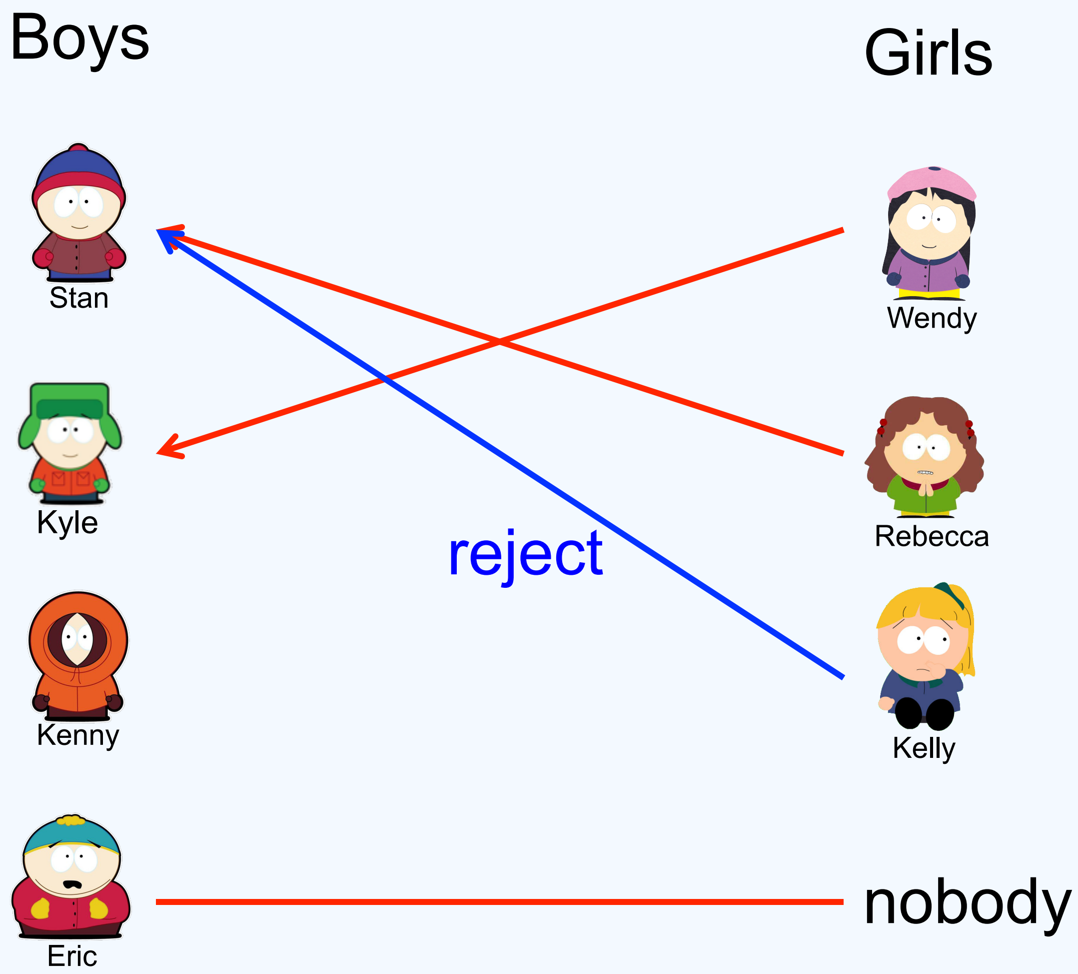
# Round 3



# Women-proposing DA with slightly different preferences

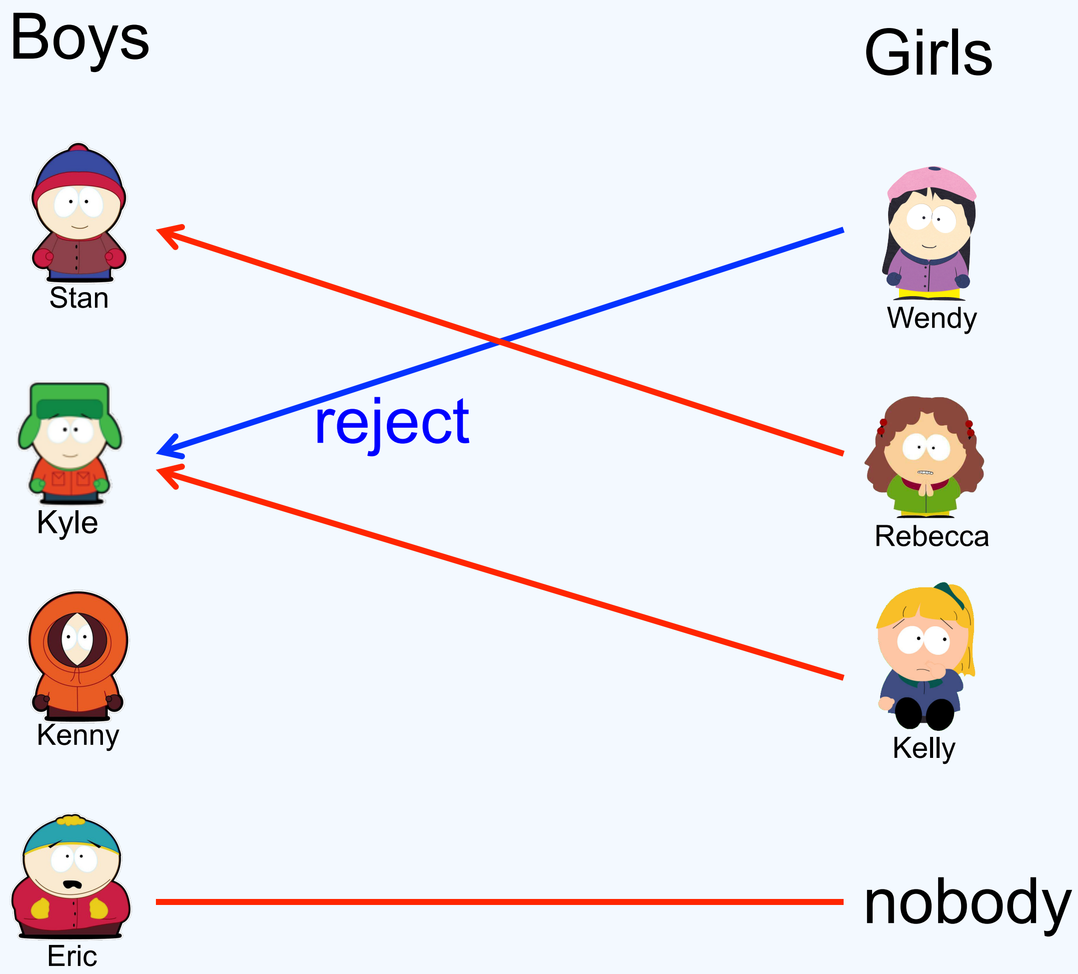


# Round 1



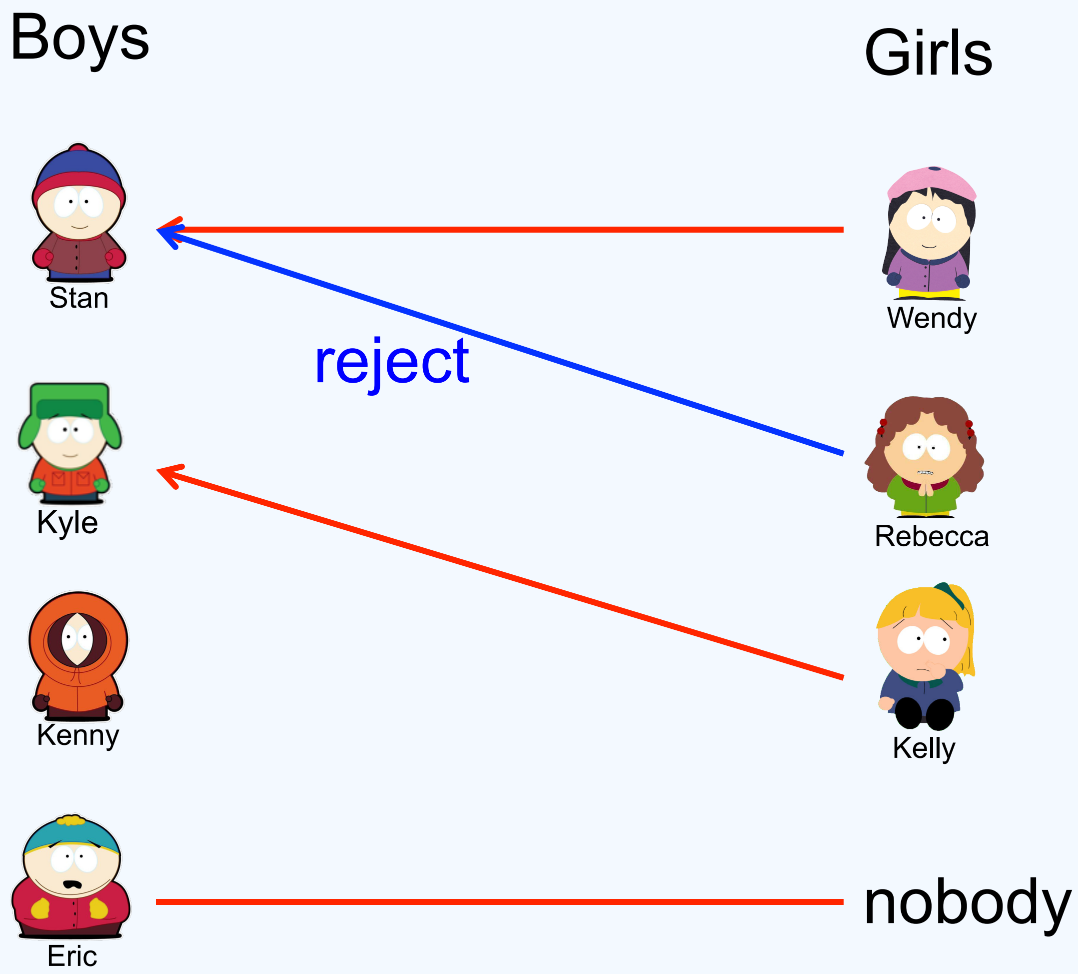
Boys
Girls
Stan
Kyle
Kenny
Eric
Wendy
Rebecca
Kelly
reject
nobody

# Round 2

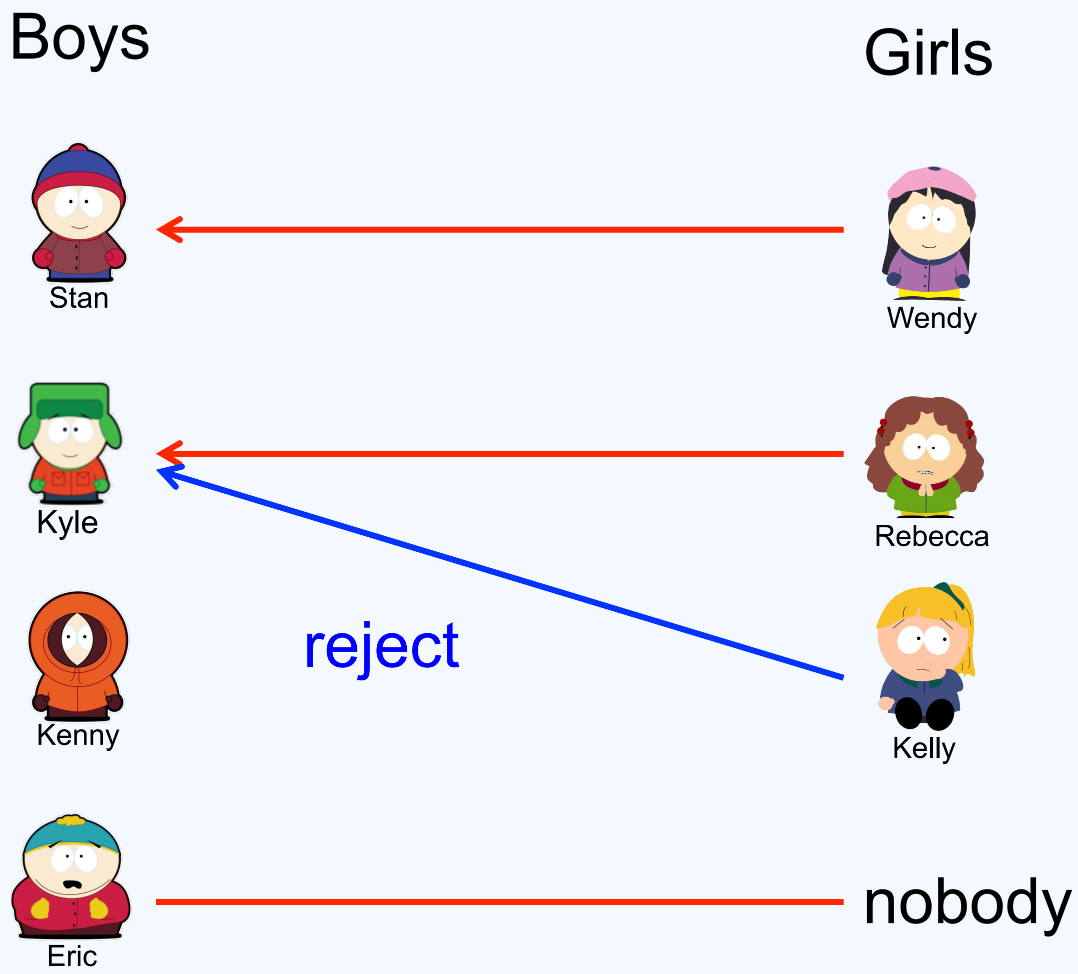


Boys
Girls
Stan
Wendy
reject
Kyle
Rebecca
Kenny
Kelly
Eric
nobody

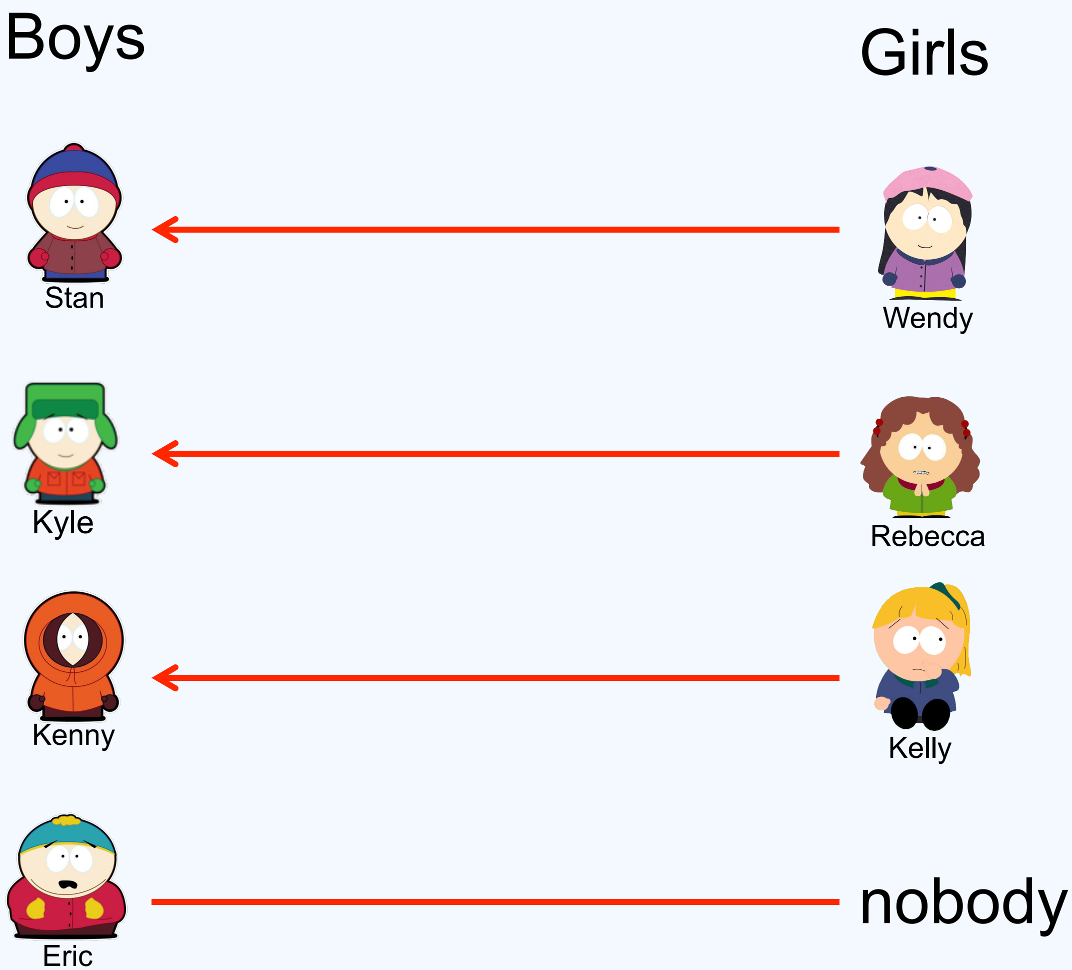
# Round 3


Boys
Girls
Stan
Wendy
reject
Kyle
Rebecca
Kenny
Kelly
Eric
nobody

# Round 4


Boys
Girls
Stan
Wendy
Kyle
Rebecca
Kenny
reject
Kelly
Eric
nobody

# Round 5



# Properties of men-proposing DA

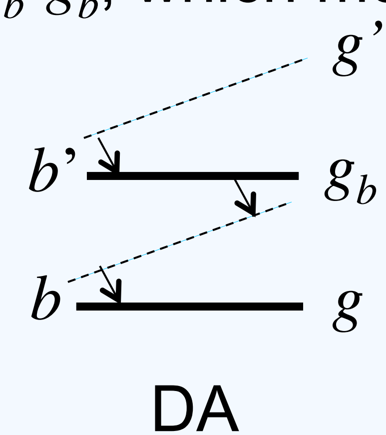
- Can be computed efficiently
- Outputs a stable matching
  - The "best" stable matching for boys, called men-optimal matching
  - and the worst stable matching for girls
- Strategy-proof for boys

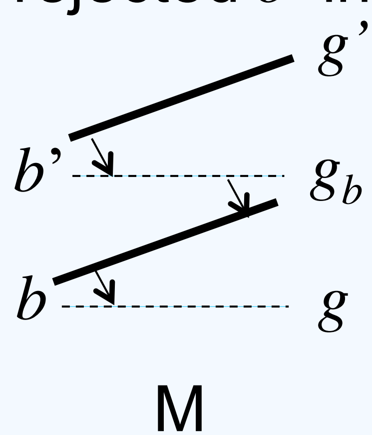
# The men-optimal matching

- For each boy $b$, let $g_b$ denote his most favorable girl matched to him in any stable matching
- A matching is men-optimal if each boy $b$ is matched to $g_b$
- Seems too strong, but…

# Men-proposing DA is men-optimal

- Theorem. The output of men-proposing DA is men-optimal
- Proof: by contradiction
  - suppose $b$ is the first boy not matched to $g \neq g_b$ in the execution of DA,
  - let M be an arbitrary matching where $b$ is matched to $g_b$
  - Suppose $b$' is the boy whom $g_b$ chose to reject $b$, and M($b$')=$g$'
  - $g' >_{b'} g_b$, which means that $g$' rejected $b$' in a previous round



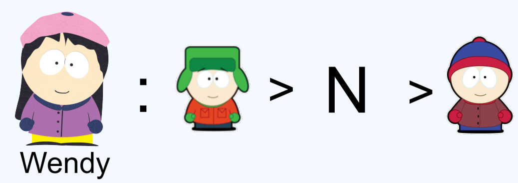



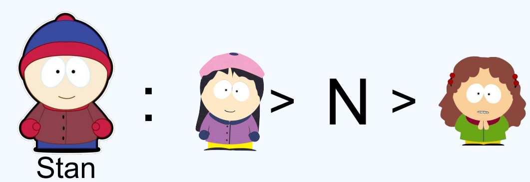
# Strategy-proofness for boys

- Theorem. Truth-reporting is a dominant strategy for boys in men-proposing DA

# No matching mechanism is strategy-proof and stable

- Proof.


- If (S,W) and (K,R) then 

- If (S,R) and (K,W) then 

# Recap: two-sided 1-1 matching

- Men-proposing deferred acceptance algorithm (DA)
  - outputs the men-optimal stable matching
  - runs in polynomial time
  - strategy-proof on men's side

# Next class: Fair division

- Indivisible goods: one-sided 1-1 or 1-many matching (papers, apartments, etc.)
- Divisible goods: cake cutting