

# News in the world

- Scottish independence referendum
  - 45% yes vs 55% no
- The YouGov survey predicts Scots have rejected independence by a margin of 54% to 46%
  - based on the responses of 1,828 people after they voted, as well as 800 people who had already cast their ballots
  - Peter Kellner of YouGov said: "At risk of looking utterly ridiculous in a few hours time, I would say it's a 99% chance of a No victory."
- Where does this 99% come from?
  - we will learn in the hypothesis testing class

# Last class: combinatorial voting

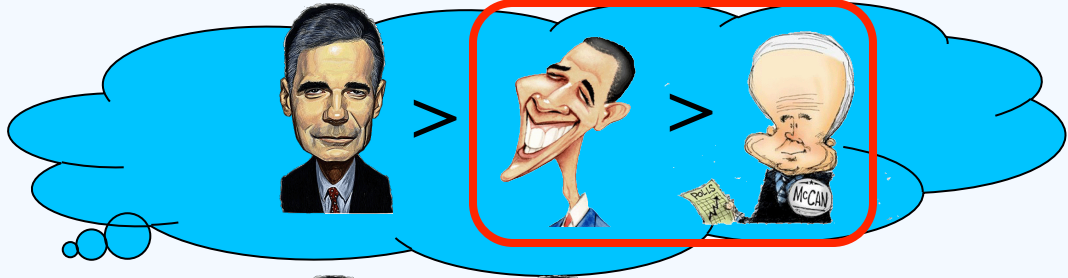


Computational  
efficiency

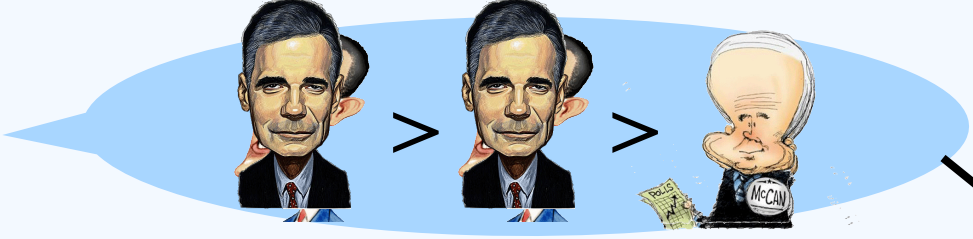
Tradeoff

Expressiveness

# Manipulation under plurality rule (ties are broken in favor of )

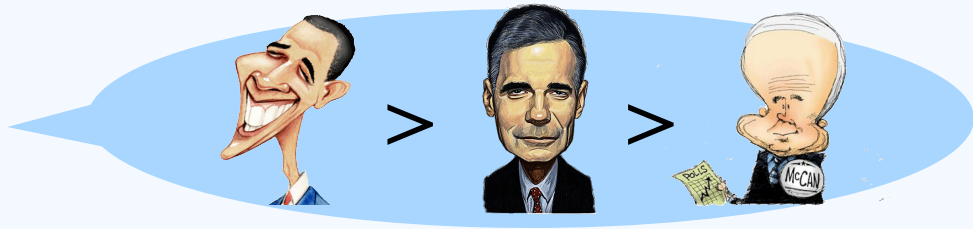


Alice

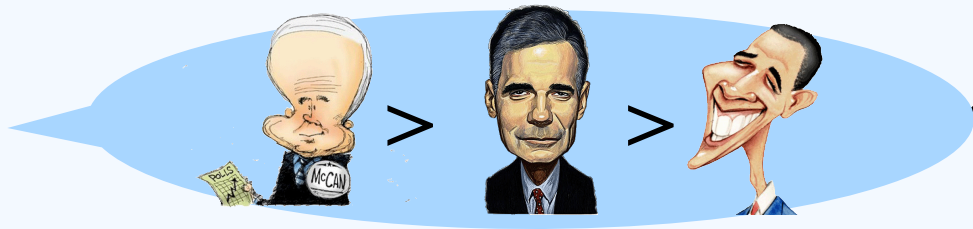


Plurality rule

Bob



Carol



# Strategic behavior (of the agents)

- **Manipulation**: an agent (manipulator) casts a vote that does not represent her true preferences, to make herself better off
- A voting rule is **strategy-proof** if there is never a (beneficial) manipulation under this rule
  - truthful direct revelation mechanism
- Is strategy-proofness compatible with other axioms?

# Any strategy-proof voting rule?



No reasonable voting rule is strategyproof

- [Gibbard-Satterthwaite Theorem](#) [[Gibbard Econometrica-73](#), [Satterthwaite JET-75](#)]: When there are at least three alternatives, no voting rules except dictatorships satisfy
  - **non-imposition**: every alternative wins for some profile
  - **unrestricted domain**: voters can use any linear order as their votes
  - **strategy-proofness**
- Axiomatic characterization for dictatorships!
- Revelation principle: among all voting rules that satisfy non-imposition and unrestricted domain, only dictatorships can be implemented w.r.t. dominant strategy
- Randomized version [[Gibbard Econometrica-77](#)]

# A few ways out

- Relax non-dictatorship: use a dictatorship
- Restrict the number of alternatives to 2
- Relax unrestricted domain: mainly pursued by economists



- **Single-peaked preferences:**
- **Range voting:** A voter submit any natural number between 0 and 10 for each alternative
- **Approval voting:** A voter submit 0 or 1 for each alternative

# Computational thinking

- Use a voting rule that is too complicated so that nobody can easily predict the winner
  - Dodgson
  - Kemeny
  - The randomized voting rule used in Venice Republic for more than 500 years [[Walsh&Xia AAMAS-12](#)]
- We want a voting rule where
  - Winner determination is easy
  - Manipulation is hard
- The **hard-to-manipulate** axiom: manipulation under the given voting rule is NP-hard

# Overview

Manipulation is inevitable  
(Gibbard-Satterthwaite Theorem)

Can we use computational complexity as a barrier?

Yes

Is it a strong barrier?

No

Other barriers?

Limited information  
Limited communication

Why prevent manipulation?

May lead to very  
undesirable outcomes

How often?

Seems not very often



# Manipulation: A computational complexity perspective



If it is **computationally too hard** for a manipulator to compute a manipulation, she is best off voting truthfully

– Similar as in cryptography




For which common voting rules manipulation is computationally hard?

# Unweighted coalitional manipulation (UCM) problem

- Given
  - The voting rule  $r$
  - The non-manipulators' profile  $P^{NM}$
  - The number of manipulators  $n'$
  - The alternative  $c$  preferred by the manipulators
- We are asked whether or not there exists a profile  $P^M$  (of the manipulators) such that  $c$  is the winner of  $P^{NM} \cup P^M$  under  $r$

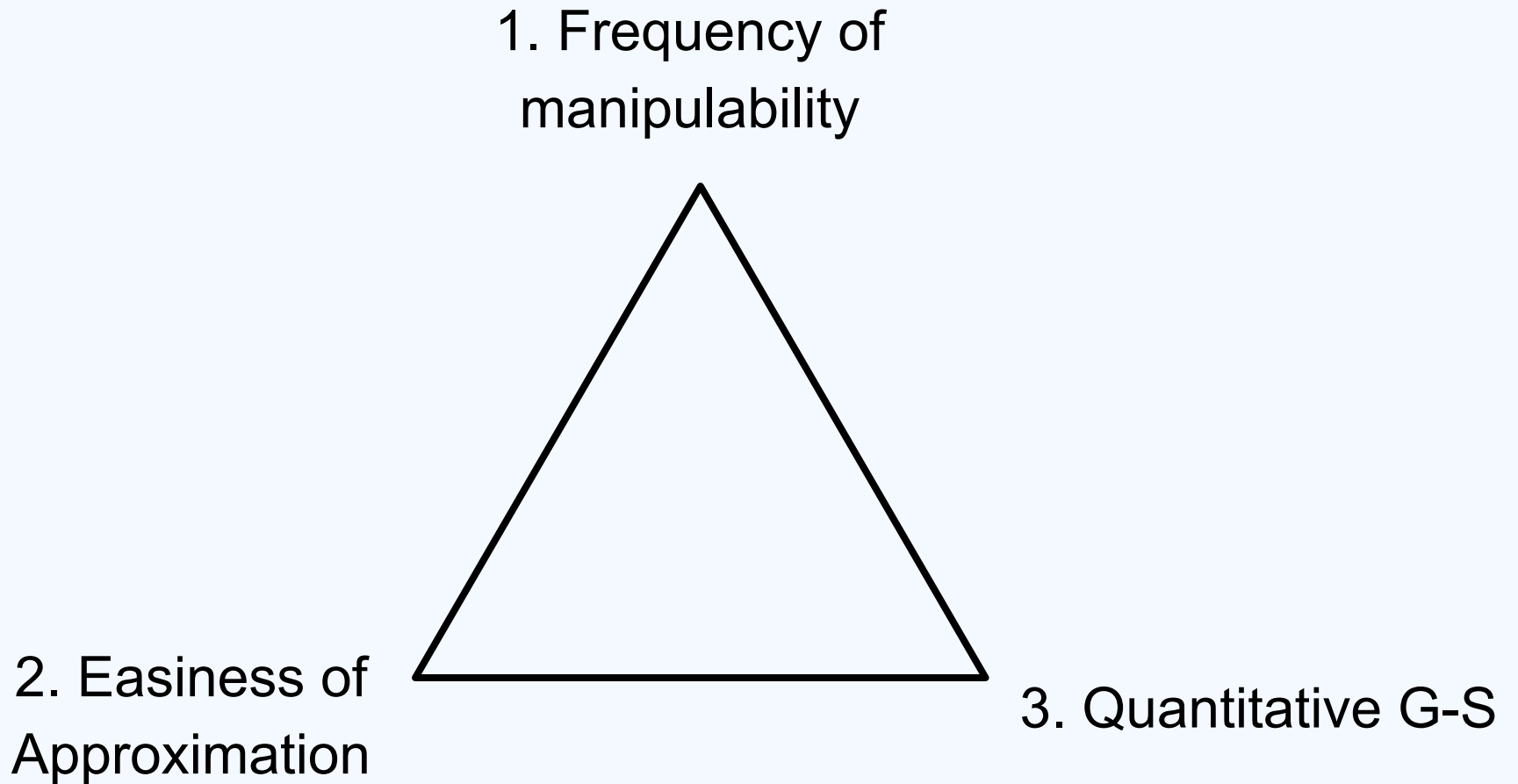
# The stunningly big table for UCM

#manipulators	One manipulator	At least two
Copeland	P [BTT SCW-89b]	<b>NPC</b> [FHS AAMAS-08,10]
STV	<b>NPC</b> [BO SCW-91]	<b>NPC</b> [BO SCW-91]
Veto	P [ZPR AIJ-09]	P [ZPR AIJ-09]
Plurality with runoff	P [ZPR AIJ-09]	P [ZPR AIJ-09]
Cup	P [CSL JACM-07]	P [CSL JACM-07]
Borda	P [BTT SCW-89b]	<b>NPC</b> [DKN+ AAAI-11] [BNW IJCAI-11] 
Maximin	P [BTT SCW-89b]	<b>NPC</b> [XZP+ IJCAI-09]
Ranked pairs	<b>NPC</b> [XZP+ IJCAI-09]	<b>NPC</b> [XZP+ IJCAI-09]
Bucklin	P [XZP+ IJCAI-09]	P [XZP+ IJCAI-09]
Nanson's rule	<b>NPC</b> [NWX AAA-11]	<b>NPC</b> [NWX AAA-11]
Baldwin's rule	<b>NPC</b> [NWX AAA-11]	<b>NPC</b> [NWX AAA-11]

# What can we conclude?

- For some common voting rules, computational complexity provides some protection against manipulation
- Is computational complexity a strong barrier?
  - NP-hardness is a worst-case concept

# Probably NOT a strong barrier



# A first angle: frequency of manipulability

- Non-manipulators' votes are drawn i.i.d.
  - E.g. i.i.d. uniformly over all linear orders (the **impartial culture** assumption)
- How often can the manipulators make  $c$  win?
  - Specific voting rules [Peleg T&D-79, Baharad&Neeman RED-02, Slinko T&D-02, Slinko MSS-04, Procaccia and Rosenschein AAMAS-07]

# A general result [Xia&Conitzer EC-08a]

- **Theorem.** For any **generalized scoring rule**
  - Including many common voting rules

# manipulators	All-powerful	$\Theta(\sqrt{n})$
	No power	

- Computational complexity is **not** a strong barrier against manipulation
  - UCM as a decision problem is **easy to compute** in most cases
  - The case of  $\Theta(\sqrt{n})$  has been studied experimentally in [Walsh IJCAI-09]

# A second angle: approximation

- **Unweighted coalitional optimization (UCO)**: compute the smallest number of manipulators that can make  $c$  win
  - A greedy algorithm has additive error no more than 1 for Borda [Zuckerman, Procaccia, & Rosenschein AIJ-09]



# An approximation algorithm for positional scoring rules

[Xia, Conitzer, & Procaccia  
EC-10]

- A polynomial-time approximation algorithm that works for **all** positional scoring rules
  - Additive error is no more than  $m-2$
  - Based on a new connection between UCO for positional scoring rules and a class of scheduling problems
- Computational complexity is **not** a strong barrier against manipulation
  - The cost of successful manipulation can be easily approximated (for positional scoring rules)

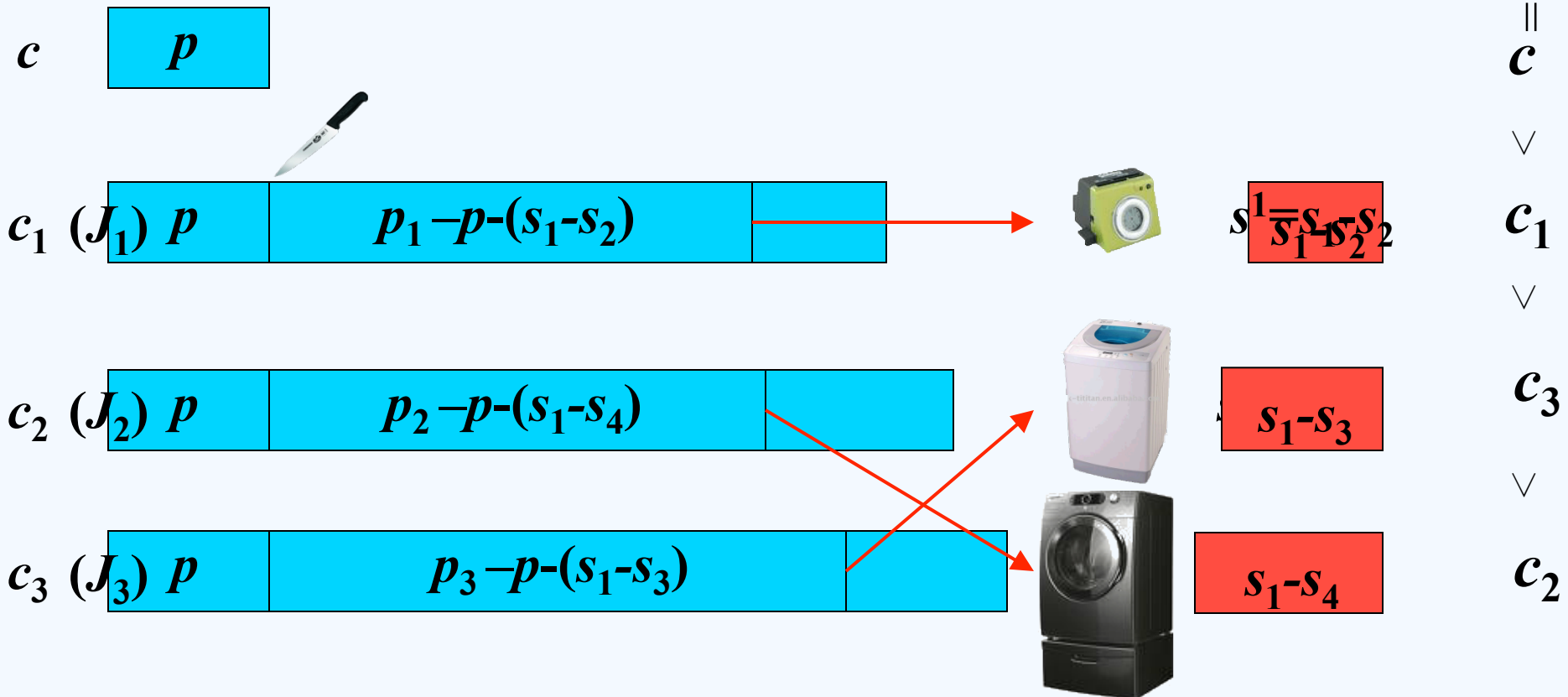
# The scheduling problems $Q|pmtn|C_{max}$

- $m^*$  parallel uniform machines  $M_1, \dots, M_{m^*}$ 
  - Machine  $i$ 's speed is  $s^i$  (the amount of work done in unit time)
- $n^*$  jobs  $J_1, \dots, J_{n^*}$
- preemption: jobs are allowed to be interrupted (and resume later maybe on another machine)
- We are asked to compute the minimum **makespan**
  - the minimum time to complete all jobs

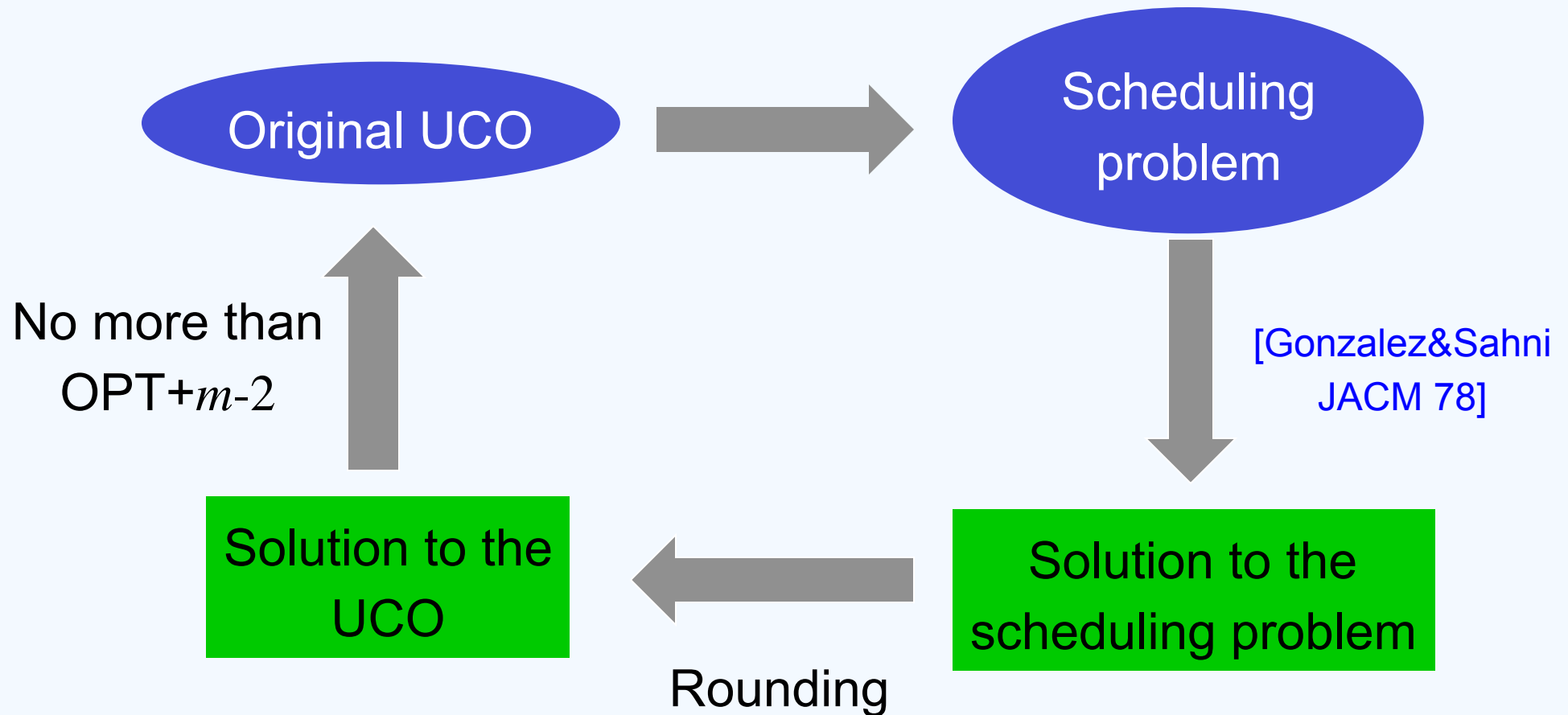
# Thinking about $UCO_{pos}$

- Let  $p, p_1, \dots, p_{m-1}$  be the total points that  $c, c_1, \dots, c_{m-1}$  obtain in the non-manipulators' profile

$$P^{NM} \cup \{V_1 = [c > c_1 > c_2 > c_3]\}$$



# The approximation algorithm



# Complexity of UCM for Borda

- Manipulation of positional scoring rules = scheduling (preemptions at integer time points)
  - Borda manipulation corresponds to scheduling where the machines speeds are  $m-1, m-2, \dots, 0$ 
    - NP-hard [Yu, Hoogeveen, & Lenstra J.Scheduling 2004]
  - UCM for Borda is NP-C for two manipulators
    - [Davies et al. AAAI-11 best paper]
    - [Betzler, Niedermeier, & Woeginger IJCAI-11 best paper]

# A third angle: quantitative G-S

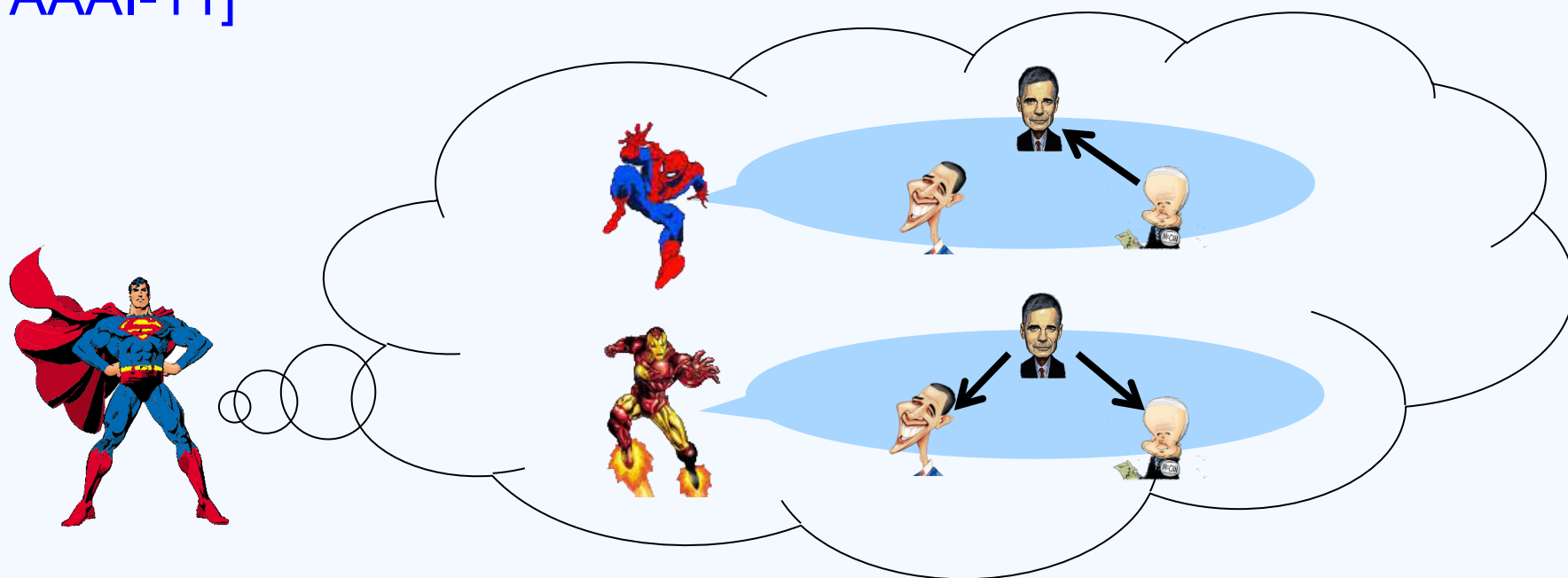
- **G-S theorem**: for any reasonable voting rule there exists a manipulation
- **Quantitative G-S**: for any voting rule that is “far away” from dictatorships, the number of manipulable situations is non-negligible
  - First work: 3 alternatives, neutral rule [Friedgut, Kalai, & Nisan FOCS-08]
  - Extensions: [Dobzinski & Procaccia WINE-08, Xia & Conitzer EC-08b, Isaksson, Kindler, & Mossel FOCS-10]
  - Finally proved: [Mossel & Racz STOC-12]

# Next steps

- The first attempt seems to fail
- Can we obtain positive results for a restricted setting?
  - The manipulators has complete information about the non-manipulators' votes
  - The manipulators can perfectly discuss their strategies

# Limited information

- Limiting the manipulator's information can make **dominating manipulation** computationally harder, or even **impossible** [Conitzer, Walsh, & Xia AAI-11]



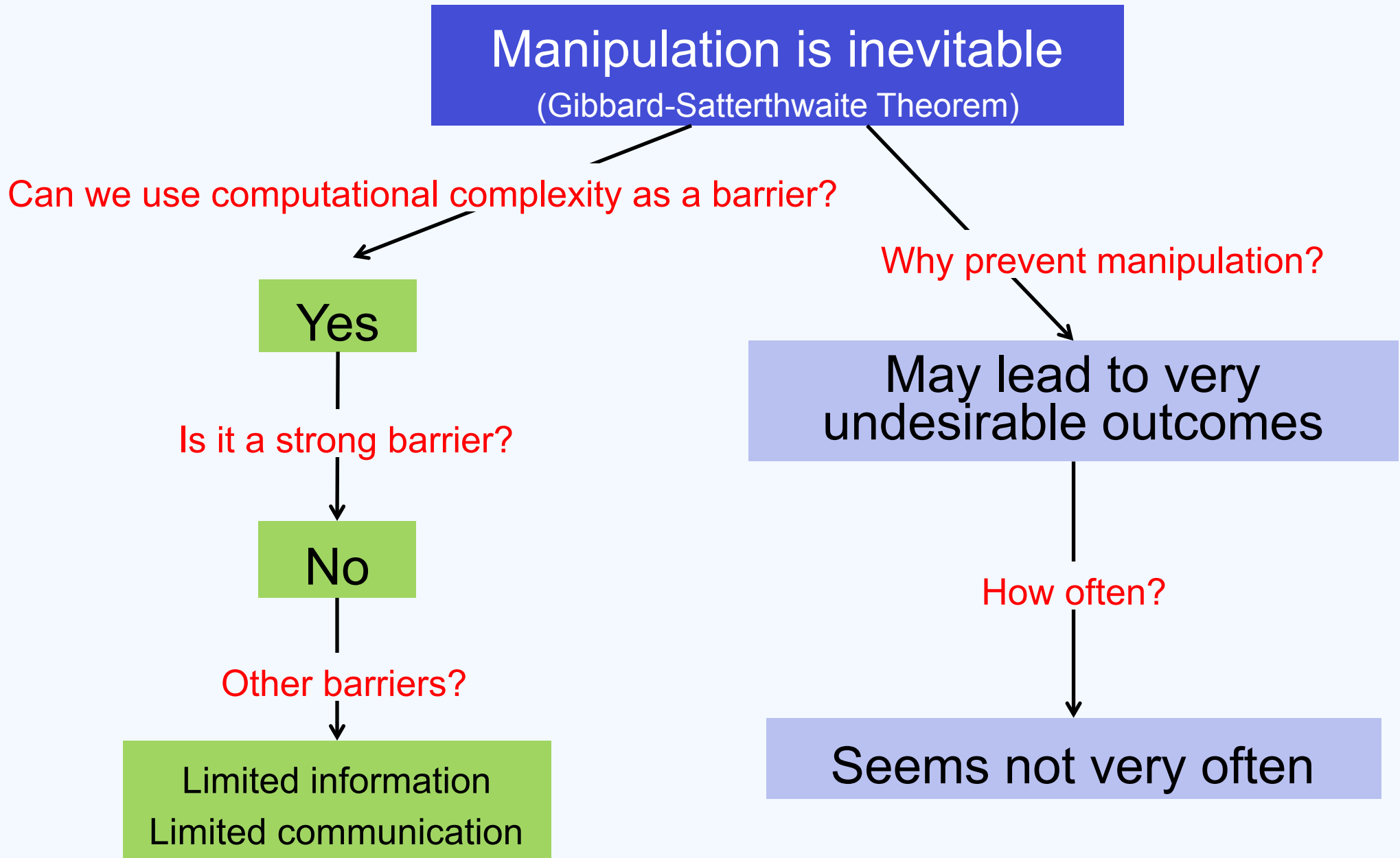
- Bayesian information [Lu et al. UAI-12]



# Limited communication among manipulators

- The leader-follower model
  - The leader broadcast a vote  $W$ , and the potential followers decide whether to cast  $W$  or not
    - The leader and followers have the same preferences
  - **Safe manipulation** [Slinko&White COMSOC-08]: a vote  $W$  that
    - No matter how many followers there are, the leader/potential followers are not worse off
    - Sometimes they are better off
  - Complexity: [Hazon&Elkind SAGT-10, Ianovski et al. IJCAI-11]

# Overview



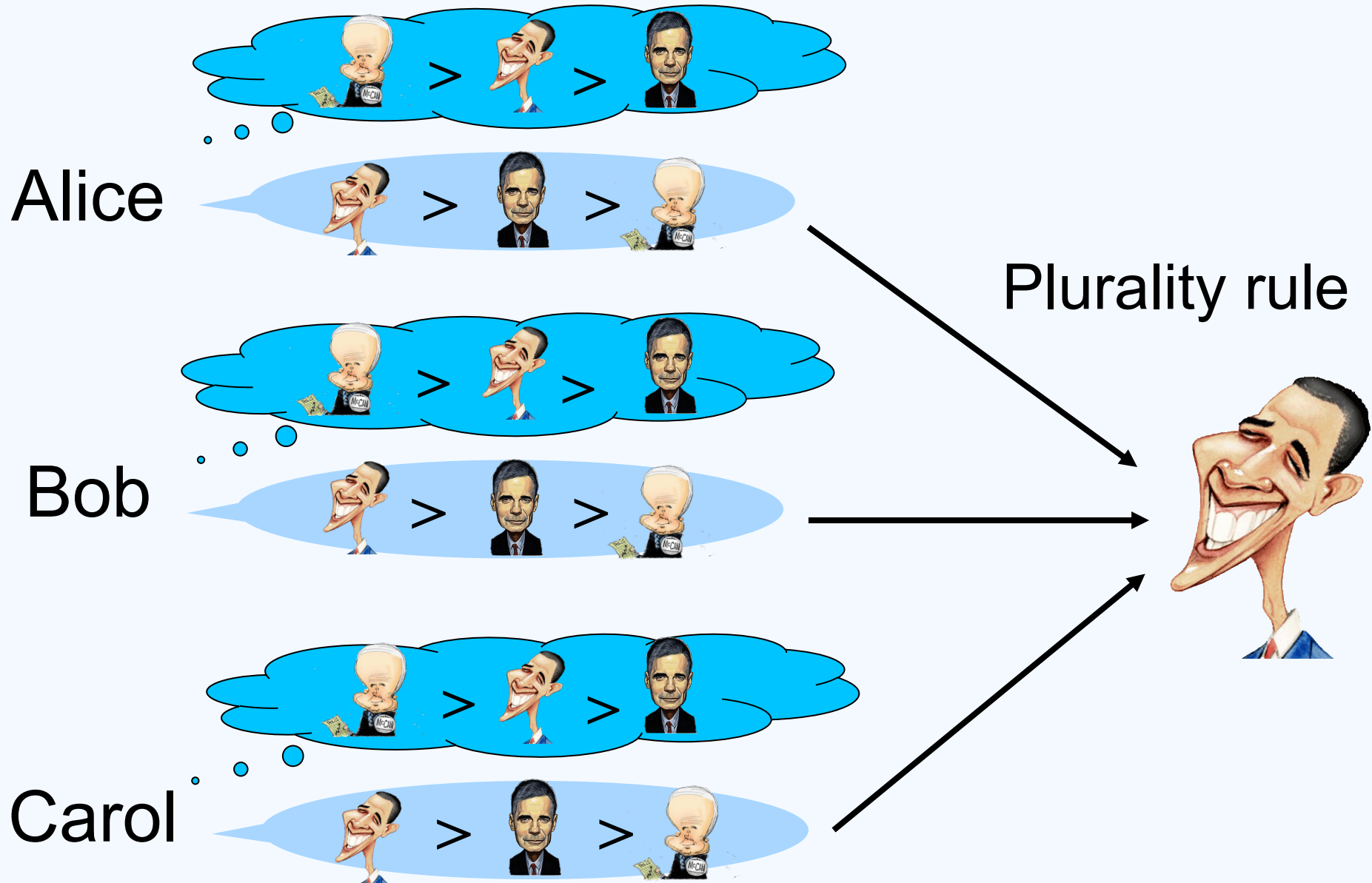
# Research questions

- How to predict the outcome?
  - Game theory
- How to evaluate the outcome?
- Price of anarchy [[Koutsoupias&Papadimitriou STACS-99](#)]
  - $$\frac{\text{Optimal welfare when agents are truthful}}{\text{Worst welfare when agents are fully strategic}}$$
  - Not very applicable in the social choice setting
    - Equilibrium selection problem
    - Social welfare is not well defined
    - Use best-response game to select an equilibrium and use scores as social welfare [[Brânzei et al. AAI-13](#)]

# Simultaneous-move voting games

- **Players:** Voters  $1, \dots, n$
- **Strategies / reports:** Linear orders over alternatives
- **Preferences:** Linear orders over alternatives
- **Rule:**  $r(P')$ , where  $P'$  is the reported profile

# Equilibrium selection problem



# Stackelberg voting games

[Xia&Conitzer AAAI-10]

- Voters vote **sequentially** and **strategically**
  - voter 1  $\rightarrow$  voter 2  $\rightarrow$  voter 3  $\rightarrow$  ...  $\rightarrow$  voter  $n$
  - any terminal state is associated with the winner under rule  $r$
- Called a **Stackelberg voting game**
  - Unique winner in SPNE (not unique SPNE)
  - Similar setting in [Desmedt&Elkind EC-10]

# Other types of strategic behavior (of the chairperson)

- Procedure control by
  - {adding, deleting} × {voters, alternatives}
  - partitioning voters/alternatives
  - introducing clones of alternatives
  - changing the agenda of voting
  - [Bartholdi, Tovey, & Trick MCM-92, Tideman SCW-07, Conitzer, Lang, & Xia IJCAI-09]
- Bribery [Faliszewski, Hemaspaandra, & Hemaspaandra JAIR-09]
- See [Faliszewski, Hemaspaandra, & Hemaspaandra CACM-10] for a survey on their computational complexity
- See [Xia Arxiv-12] for a framework for studying many of these for generalized scoring rules

# Next class: statistical approaches

**GOAL1:** democracy



**Axiomatic approaches**

**GOAL2:** truth



**Statistical approaches**