CSCI-4110/6110: Computational Social Processes, 2016 Fall
Homework 1

You can either send it through email or hand in an hard copy before the class (electronic version is strongly preferred). If you have questions about the statements please post on Piazza.

Problem 1. Utility Theory (2pts) The **Allais paradox** arises when people are tested in the two experiments illustrated in Table 1. In each experiment, a person can choose one of the two options (A or B).

Suppose a person chooses Lottery A for Experiment 1 and Lottery B for Experiment 2 (in fact many people do so). Show that this is a failure of utility theory.

Experiment 1		Experiment 2	
Lottery A	Lottery B	Lottery A	Lottery B
\$1M@100%	\$1M@89% + \$5M@10% + 0@1%	\$1M@11%+0@89%	\$0M@90% + \$5M@10%

Table 1: Allais paradox.

Problem 2. Voting rules (9pt): If there are multiple winners you should list all of them. Please briefly describe your calculations (or proofs). You will get 0 point without calculations. Consider the following profile P:

$$P = 10@[a \succ b \succ c \succ d]+7@[d \succ a \succ b \succ c]+6@[c \succ d \succ a \succ b]+3@[b \succ c \succ d \succ a]$$

1. Calculate the winner(s) for plurality, Borda, veto, 3-approval.

2. Calculate the winner(s) for plurality with runoff.
3. Calculate the winner(s) for Bucklin.
4. Draw the weighted majority graphs. You only need to show positive edges and their weights.
5. Calculate the winner(s) for Copeland.
6. Calculate the winner(s) for Maximin.
7. Calculate the winner(s) for Ranked Pairs.
8. Calculate the winner(s) for Schulze.
9. Calculate the winner(s) for Kemeny.

Problem 3. (3pt) Let the voting rule be STV.

1. Consider the following profile:

   $$27@[a \succ b \succ c] \qquad 42@[c \succ a \succ b] \qquad 24@[b \succ c \succ a]$$

   What happens when four votes switch from $a \succ b \succ c$ to $c \succ a \succ b$, and what axiomatic property does this violate?

2. For the same profile in (a), what paradoxical outcome occurs when four voters with $a \succ b \succ c$ don't vote?

3. Prove that STV does *not* satisfy consistency.

Problem 4. (3pt) Prove that all positional-scoring rules satisfy consistency. You can assume that there are no ties in the profiles.

Problem 5. (2pt) Prove that for any profile P, let $WMG(P)$ denote the weighted majority graph. Prove that one of the following two cases must hold: (1) weights on all edges of in $WMG(P)$ are even numbers; or (2) weights on all edges of in $WMG(P)$ are odd numbers.

Problem 6. Bonus question (hard): (5pt) Let $\vec{s}_B = (m-1, \ldots, 0)$ denote the scoring vector for Borda.

1. Prove that for any $p > 0, q \in \mathbb{R}$, the positional scoring rule r with the scoring vector $p \cdot \vec{s}_B + q = (p(m-1)+q, p(m-2)+q, \ldots, q)$ is equivalent to Borda. That is, for any profile P, $r(P) = \text{Borda}(P)$.

2. Prove the reverse of (a). That is, prove that a position scoring rule r with scoring vector $\vec{s} = (s_1, \ldots, s_m)$ is equivalent to Borda only if there exist $p > 0, q \in \mathbb{R}$ such that $\vec{s} = p \cdot \vec{s}_B + q$.
   Hint: show that $s_1 - s_2 = s_2 - s_3 = \cdots = s_{m-1} - s_m$.