# Computational Social Processes

Lirong Xia

Rensselaer

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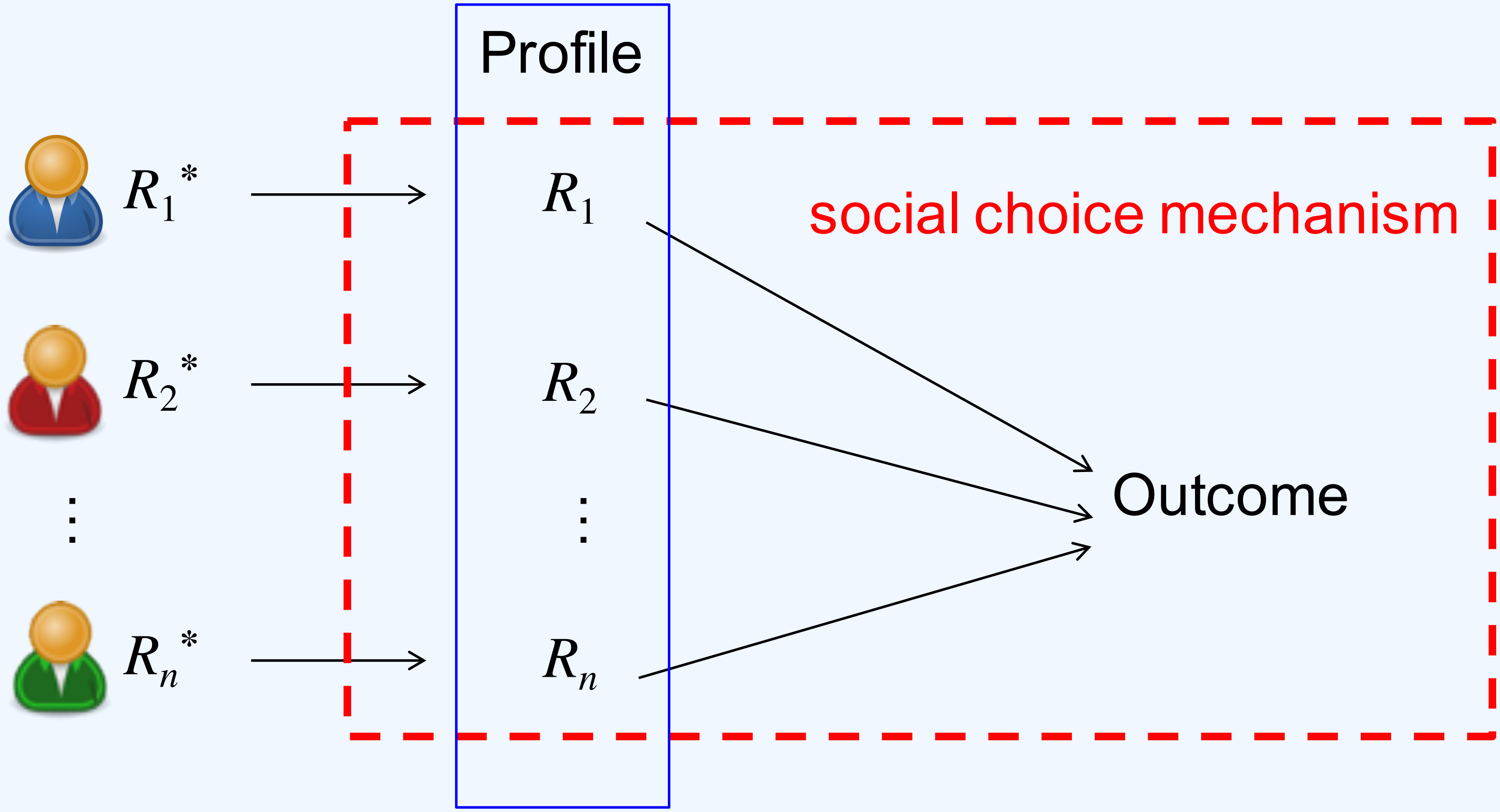
# Logistics

- Register now!
- Final grade calculation revote
  - call for nominations
  - Winner: Midterm 30%; Project 40%
  - Midterm 20%; Project 50%
  - Midterm 35%; Final 35%
- Project ideas online
- Homeworks
  - must do it yourself
  - must acknowledge discussions
- 0 tolerance on cheating and plagiarism
  - Homework and exams: 0 if caught
  - Participation: 0 if caught
  - Project: 0 if caught

# Last class

- Braess' paradox
  - Incentive matters in system design
- Social choice problems
  - voting
  - auction
  - school match
  - resource allocation
- The Borda rule

# Social choice problems



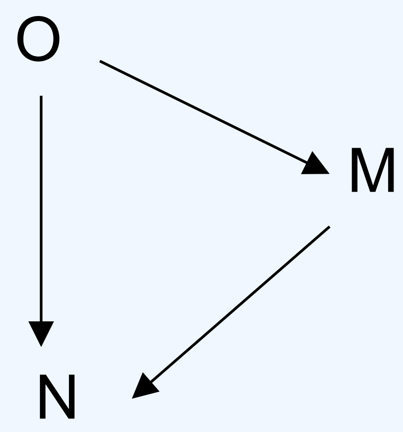

- Agents
- Alternatives
- Outcomes
- Preferences (true and reported)
- Social choice mechanism

# Today: Preferences

- How to model agents' preferences?
- Order theory
  - linear orders: rankings without ties
  - weak orders: rankings with ties
  - partial orders: allowing incomparable alternatives
    - top-k order: ranking over top-k alternatives
- Utility theory
  - preferences over lotteries

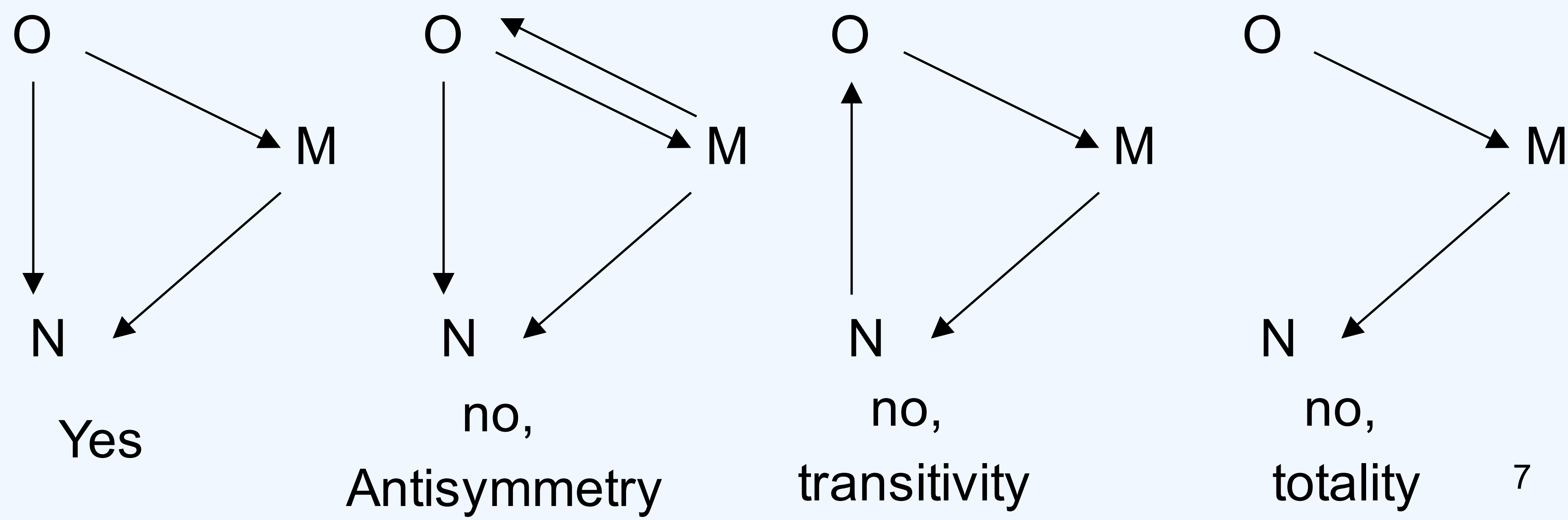
# Mathematical definition

- Given a set of alternatives A
- A binary relation R is a subset of A×A
  - (a,b) ∈ R means "a is preferred to b"
  - Also write $a >_R b$
- Example
  - A = {O, M, N}
  - R = {(O,M), (O,N), (M,N)}
- Graphical representation
  - Vertices are A
  - There is an edge a→b if and only if (a,b) ∈ R



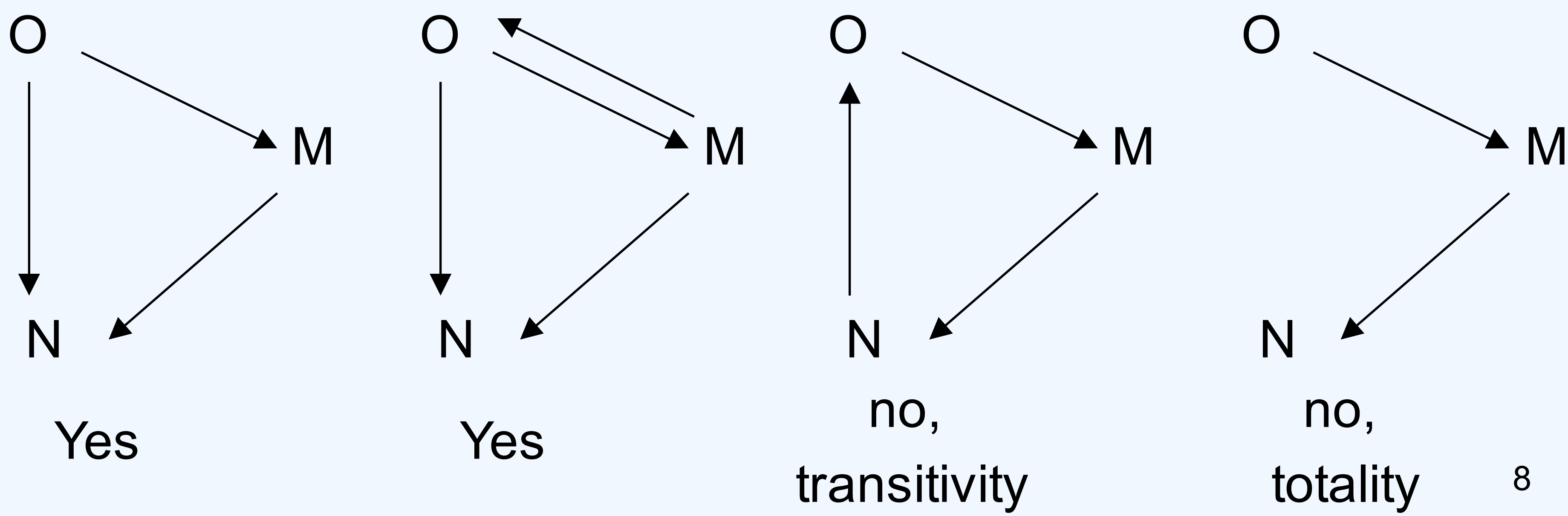
# Linear orders

- Linear orders (rankings without ties): binary relations that satisfies
  - Antisymmetry (no ties): $a >_R b$ and $b >_R a$ implies $a = b$
  - Transitivity: $a >_R b$ and $b >_R c$ implies $a >_R c$
  - Totality: for all a,b, one of $a >_R b$ or $b >_R a$ must hold



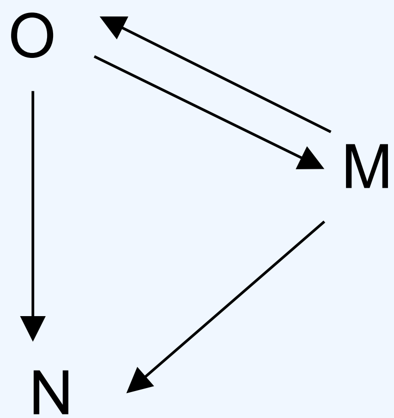
# Weak orders

- Weak orders (rankings with ties): binary relations that satisfies
  - Transitivity: $a>_R b$ and $b>_R c$ implies $a>_R c$
  - Totality: for all a,b, one of $a>_R b$ or $b>_R a$ must hold
  - Not requiring antisymmetry



# Weak orders: Tiers

- Any weak order R can be represented as tiers
  - $A = T_1 \cup T_2 \cup \ldots \cup T_q$
  - alternatives within each tier is tied: for all a,b in $T_i$, $a >_R b$ and $b >_R a$
  - Strict preferences across tiers: for all i<j, a in $T_i$ and b in $T_j$, one of $a >_R b$ and $b \not>_R a$

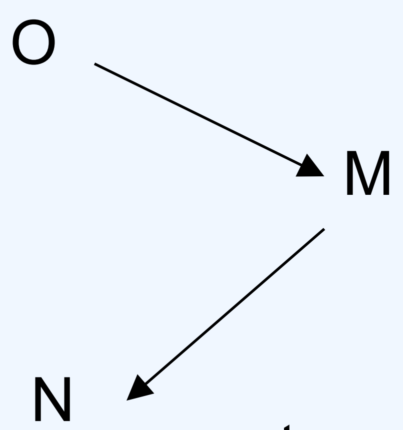



$T_1$: {O, M}

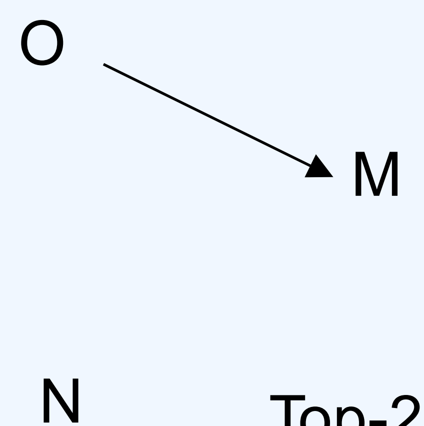
$T_2$: {N}

# Partial orders

- Partial orders: binary relations that satisfies
  - Antisymmetry (no ties): $a>_R b$ and $b>_R a$ implies $a=b$
  - Transitivity: $a>_R b$ and $b>_R c$ implies $a>_R c$
  - Reflexivity: for all a, $a>_R a$
- Top-k orders
  - $k \leq m$
  - linear order over k alternatives
  - nothing else



no, transitivity



Top-2

# Prefpy@Github

- https://github.com/PrefPy/prefpy/
- Class Preferences in Preference.py
  - wmgMap: the binary relation
  - containsTie: check if it is a weak order
  - getRankMap: stores the tiers of alternatives
  - getOrderVector: list of alternatives in tiers
  - Not allowing partial orders for now

# Poll

- What is the most general type of preferences OPRA is using?
  - rank the three choices
- Office hours

# Utility theory

# Preferences over lotteries

- Option 1 vs. Option 2
  - Option 1: $0@50%+$30@50%
  - Option 2: $5 for sure
- Option 3 vs. Option 4
  - Option 3: $0@50%+$30M@50%
  - Option 4: $5M for sure

# Lotteries

- There are $m$ objects. Obj={$o_1,\ldots,o_m$}
- Lot(Obj): all lotteries (distributions) over Obj
- In general, an agent's preferences can be modeled by a weak order (ranking with ties) over Lot(Obj)
  - But there are infinitely many outcomes

# Utility theory

- Utility function: $u$: Obj $\rightarrow \mathbb{R}$

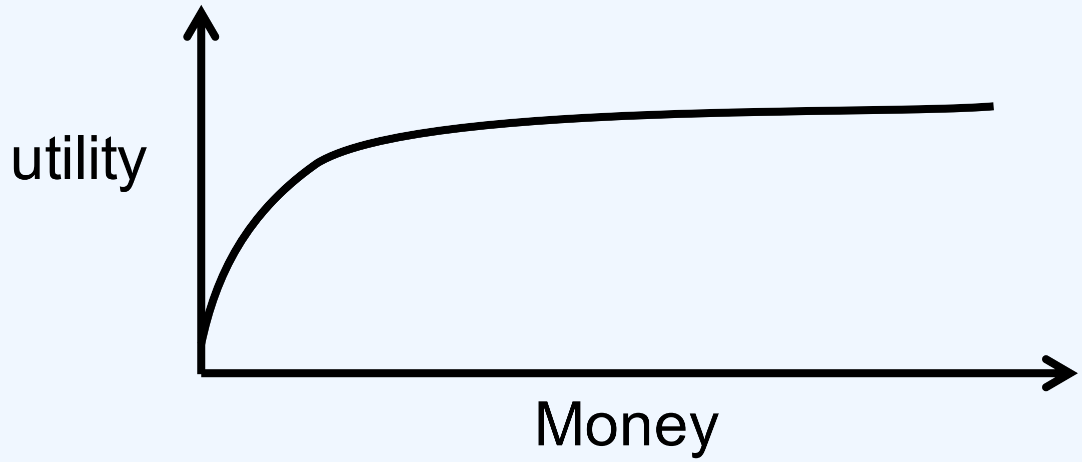
➢ For any $p \in$ Lot(Obj)

  - $u(p) = \sum_{o \in \text{Obj}} p(o)u(o)$

➢ $u$ represents a weak order over Lot(Obj)

  - $p_1 > p_2$ if and only if $u(p_1) > u(p_2)$

# Example

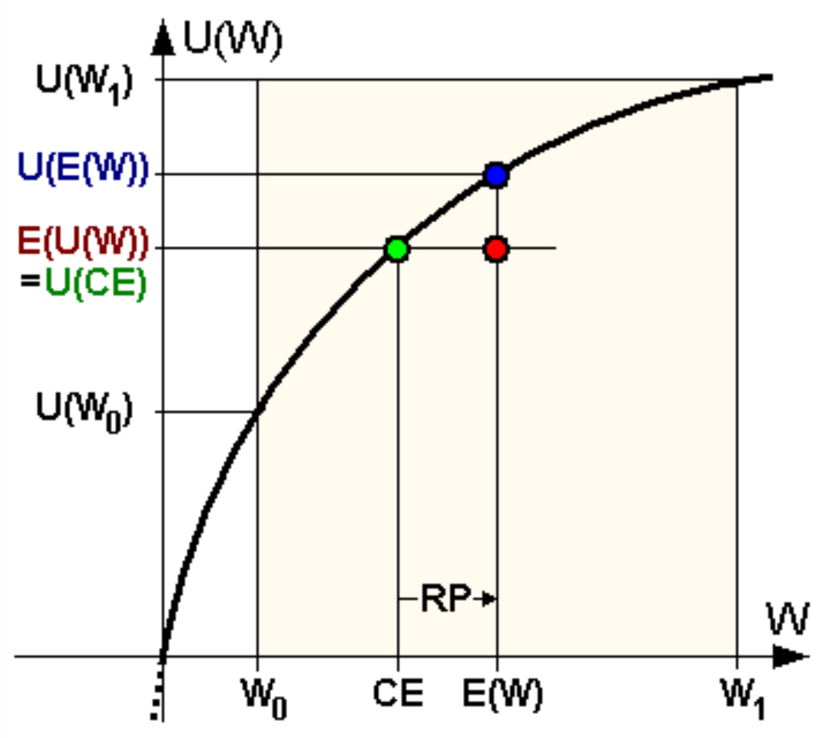



| Utility | 1 | 3 | 10 | 100 | 150 |
|---|---|---|---|---|---|
| Money | 0 | 5 | 30 | 5M | 30M |

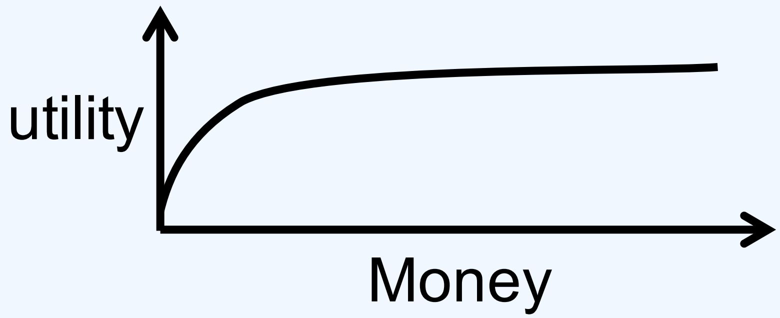
- $u(\text{Option 1}) = u(0) \times 50\% + u(30) \times 50\% = 5.5$
- $u(\text{Option 2}) = u(5) \times 100\% = 3$
- $u(\text{Option 3}) = u(0) \times 50\% + u(30M) \times 50\% = 75.5$
- $u(\text{Option 4}) = u(5M) \times 100\% = 100$

# Risk aversion

- Concave utility curve
- Lottery:
  - $W_0=10$, $W_1=90$
  - $W=W_0@50\%+W_1@50\%$
  - $E(W) = 50$
- Certainty equivalent
  - money equally desirable to the lottery
  - Suppose CE = 40
- Risk premium
  - minimum compensation to take the risk
  - max amount to avoid the risk
  - RP = 10



# Example: house insurance

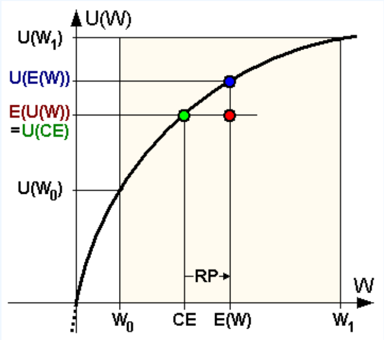



| Utility | 0 | 990 | 1000 |
|---|---|---|---|
| Money | 0 | 900K | 1M |

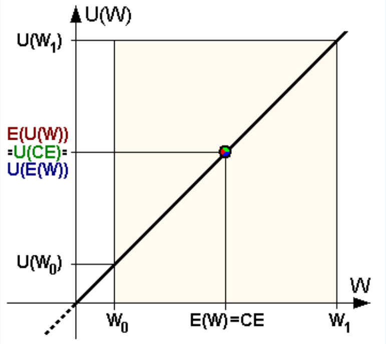
- Your house is worth 1M
  - 1% chance of fire
- Option 1: not doing anything
  - 1%@0+99%1M
  - Expected monetary loss 1K
- Option 2: buy an insurance of 100K
  - 100%@900K
- CE of option 1: 900K
- Risk premium: 100K
- Why is the insurance company willing to provide option 1?

# Risk attitudes

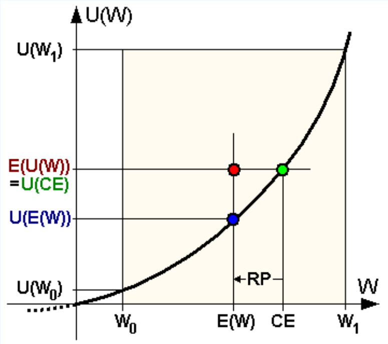
- Risk averse (concave)




- Risk neutral (line)




- Risk seeking (convex)






credit: wikipedia

# Recap

- How to model agents' preferences?
- Order theory
  - linear orders: rankings without ties
  - weak orders: rankings with ties
  - partial orders: allowing incomparable alternatives
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# Next class

- Social choice
  - many voting rules

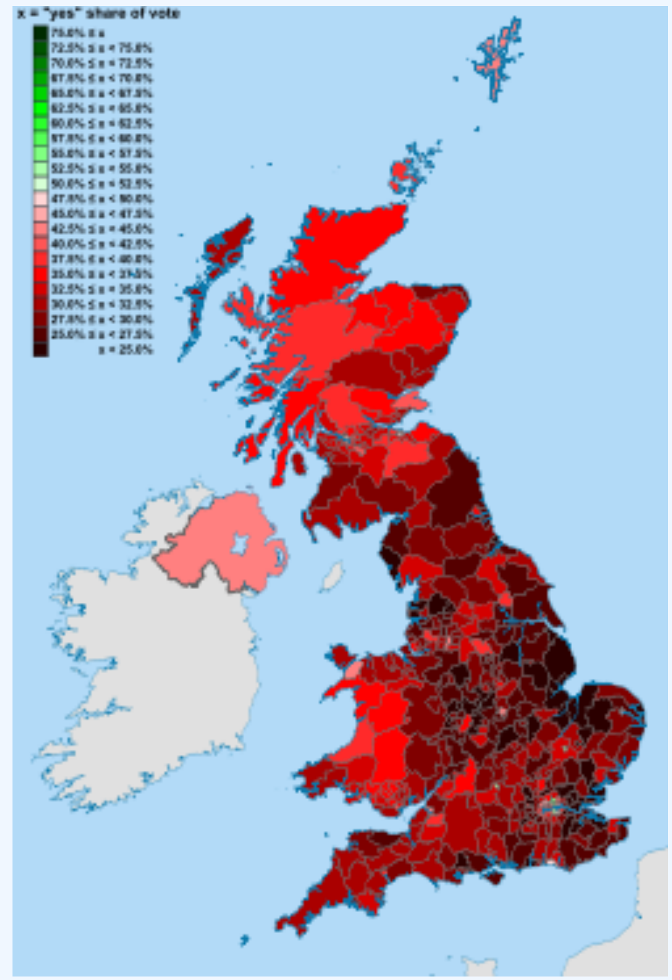
# Why different from MOOC (e.g. coursera)

- Credits
- More interaction
  - Do feel free to interrupt with questions
- Hands-on research experience
- No similar course online
- I will be back to school eventually…

# Change the world: 2011 UK Referendum

- The second nationwide referendum in UK history
  - The first was in 1975
- Member of Parliament election:

  Plurality rule ➔ Alternative vote rule
- 68% No vs. 32% Yes
- Why people want to change?
- Why it was not successful?
- Can we do better?

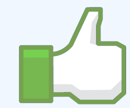
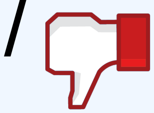


# Example2: Multiple referenda

- In California, voters voted on 11 binary issues ( / )
  - $2^{11}$=2048 combinations in total
  - 5/11 are about budget and taxes

- Prop.30 Increase sales and some income tax for education
- Prop.38 Increase income tax on almost everyone for education

# Why this is social choice?

- Agents: voters
- Alternatives: $2^{11}$=2048 combinations of  / 
- Outcomes: combinations
- Preferences (vote): Top-ranked combination
- Mechanisms: issue-by-issue voting
- More in the "combinatorial voting" class
- Goal: democracy