Manipulation

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Manipulation under plurality rule (lexicographic tie-breaking)

Alice

Bob

Carol

Plurality rule
Strategic behavior (of the agents)

- **Manipulation**: an agent (manipulator) casts a vote that does not represent her true preferences, to make herself better off.

- A voting rule is **strategy-proof** if there is never a (beneficial) manipulation under this rule.
Using Borda?

- Inverse the tie-breaking order?
Using STV?

- N > M > O  →  O > M > N
Any strategy-proof voting rule?

No reasonable voting rule is strategyproof

• Gibbard-Satterthwaite Theorem [Gibbard Econometrica-73, Satterthwaite JET-75]: When there are at least three alternatives, no voting rules except dictatorships satisfy
  – non-imposition: every alternative wins for some profile
  – unrestricted domain: voters can use any linear order as their votes
  – strategy-proofness

• Axiomatic characterization for dictatorships!

• Randomized version [Gibbard Econometrica-77]
A few ways out

• Relax non-dictatorship: use a dictatorship
• Restrict the number of alternatives to 2
• Relax unrestricted domain: mainly pursued by economists
  – Single-peaked preferences:
  – Range voting: A voter submit any natural number between 0 and 10 for each alternative
  – Approval voting: A voter submit 0 or 1 for each alternative
There exists a social axis $S$

- linear order over the alternatives

Each voter’s preferences $V$ are compatible with the social axis $S$

- there exists a “peak” $a$ such that
  - $[b \prec c \prec a$ in $S]$ implies $[c \succ b$ in $V]$
  - $[a \succ c \succ b$ in $S]$ implies $[c \succ b$ in $V]$

- alternatives closer to the peak are more preferred

- different voters may have different peaks
Examples

Single-peaked preferences

rank

1
2
3
4
5
A B C D E

Axis
Strategy-proof rules for single-peaked preferences

• The median rule
  – given a profile of “peaks”
  – choose the median in the social axis

• **Theorem.** The Median rule is strategy-proof.

• The median rule with phantom voters
  – parameterized by a fixed set of “peaks” of phantom voters
  – chooses the median of the peaks of the regular voters and the phantom voters

• **Theorem.** Any strategy-proof rule for single-peaked preferences are median rules with phantom voters

• Talk announcement: Dominik Peters 9/21 3-4pm Sage 3713
Computational thinking

• Use a voting rule that is too complicated so that nobody can easily predict the winner
  – Dodgson
  – Kemeny
  – The randomized voting rule used in Venice Republic for more than 500 years [Walsh&Xia AAMAS-12]

• We want a voting rule where
  – Winner determination is easy
  – Manipulation is hard

• The hard-to-manipulate axiom: manipulation under the given voting rule is NP-hard
Example 3: Venetian election (1268--1797)

- **Round 1:**
  - 

- **Round 2:**
  - 

- **Round 3:**
  - Approval like voting

- **Round 10:**
  - Plurality
  - The winner must receive >24 votes
Manipulation: A computational complexity perspective

💡 If it is **computationally too hard** for a manipulator to compute a manipulation, she is best off voting truthfully

– Similar as in cryptography

❓ For which common voting rules manipulation is computationally hard?
Unweighted coalitional manipulation (UCM) problem

• Given
  – The voting rule $r$
  – The non-manipulators’ profile $P_{NM}$
  – The number of manipulators $n'$
  – The alternative $c$ preferred by the manipulators

• We are asked whether or not there exists a profile $P_{M}$ (of the manipulators) such that $c$ is the winner of $P_{NM} \cup P_{M}$ under $r$
# The stunningly big table for UCM

<table>
<thead>
<tr>
<th>#manipulators</th>
<th>One manipulator</th>
<th>At least two</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Copeland</strong></td>
<td>P [BTT SCW-89b]</td>
<td>NPC [FHS AAMAS-08,10]</td>
</tr>
<tr>
<td><strong>STV</strong></td>
<td>NPC [BO SCW-91]</td>
<td>NPC [BO SCW-91]</td>
</tr>
<tr>
<td><strong>Veto</strong></td>
<td>P [ZPR AIJ-09]</td>
<td>P [ZPR AIJ-09]</td>
</tr>
<tr>
<td><strong>Plurality with runoff</strong></td>
<td>P [ZPR AIJ-09]</td>
<td>P [ZPR AIJ-09]</td>
</tr>
<tr>
<td><strong>Cup</strong></td>
<td>P [CSL JACM-07]</td>
<td>P [CSL JACM-07]</td>
</tr>
<tr>
<td><strong>Borda</strong></td>
<td>P [BTT SCW-89b]</td>
<td>NPC [DKN+ AAAI-11]</td>
</tr>
<tr>
<td><strong>Maximin</strong></td>
<td>P [BTT SCW-89b]</td>
<td>NPC [XZP+ IJCAI-09]</td>
</tr>
<tr>
<td><strong>Ranked pairs</strong></td>
<td>NPC [XZP+ IJCAI-09]</td>
<td>NPC [XZP+ IJCAI-09]</td>
</tr>
<tr>
<td><strong>Bucklin</strong></td>
<td>P [XZP+ IJCAI-09]</td>
<td>P [XZP+ IJCAI-09]</td>
</tr>
<tr>
<td><strong>Nanson’s rule</strong></td>
<td>NPC [NWX AAA-11]</td>
<td>NPC [NWX AAA-11]</td>
</tr>
<tr>
<td><strong>Baldwin’s rule</strong></td>
<td>NPC [NWX AAA-11]</td>
<td>NPC [NWX AAA-11]</td>
</tr>
</tbody>
</table>
What can we conclude?

• For some common voting rules, computational complexity provides some protection against manipulation

• Is computational complexity a strong barrier?
  – NP-hardness is a worst-case concept