Manipulation

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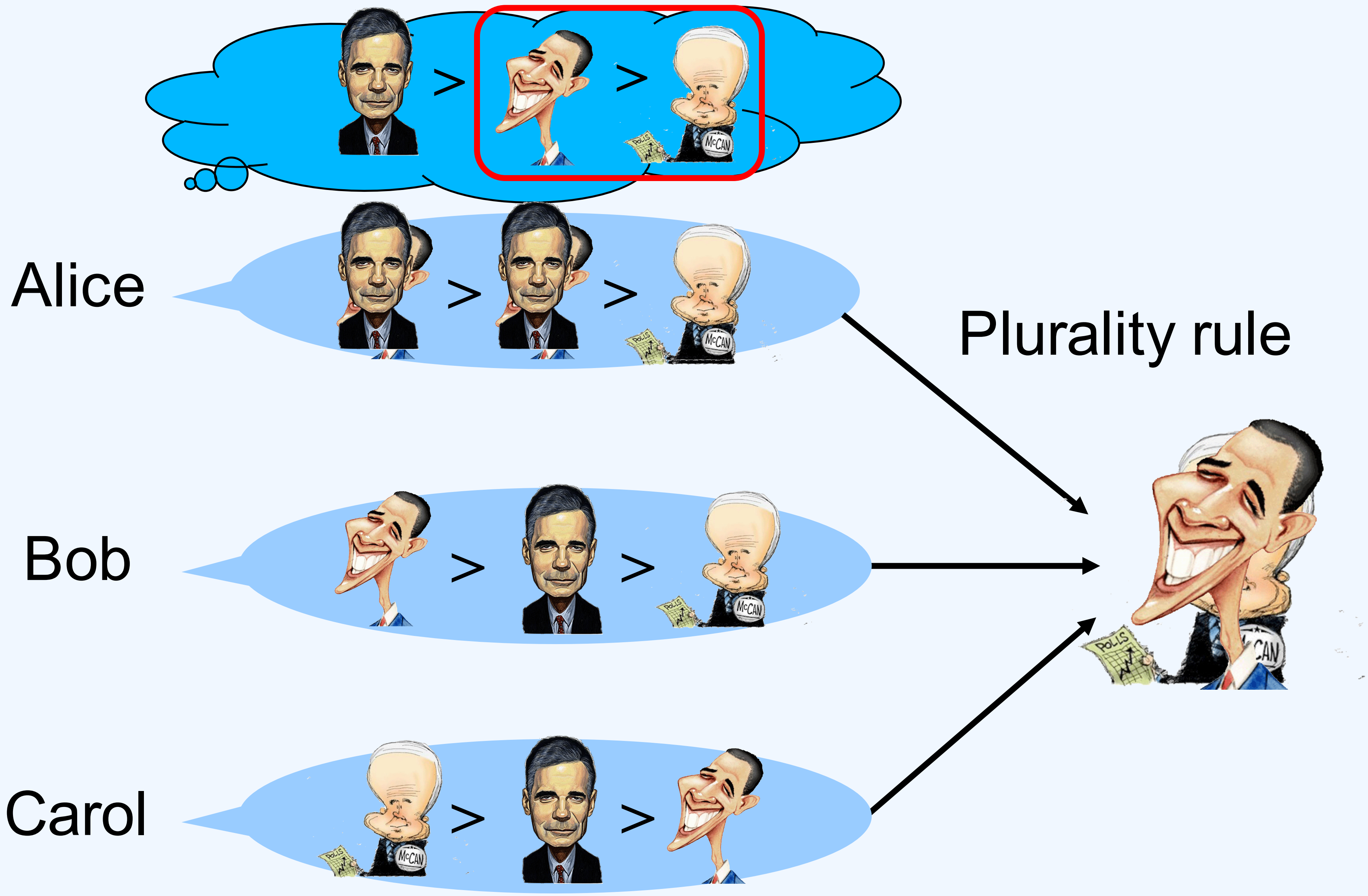
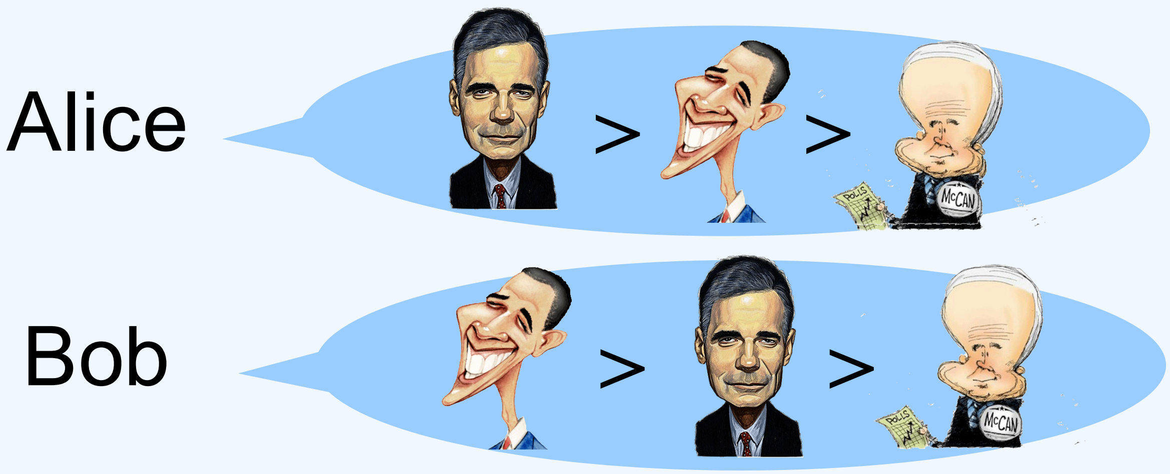
Manipulation under plurality rule (lexicographic tie-breaking)



Strategic behavior (of the agents)

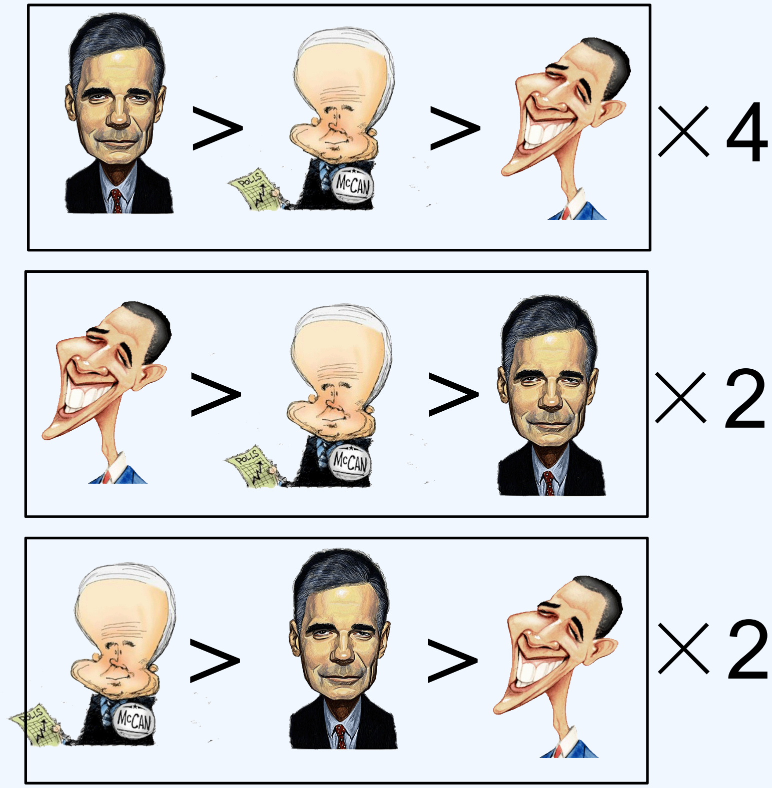
- Manipulation: an agent (manipulator) casts a vote that does not represent her true preferences, to make herself better off
- A voting rule is strategy-proof if there is never a (beneficial) manipulation under this rule

Using Borda?



- Inverse the tie-breaking order?

Using STV?



- N>M>O → O>M>N

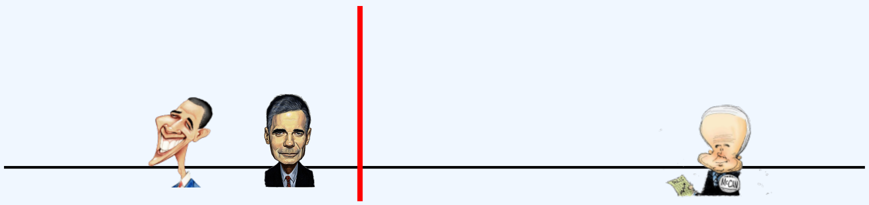
Any strategy-proof voting rule?

No reasonable voting rule is strategyproof

- Gibbard-Satterthwaite Theorem [Gibbard Econometrica-73, Satterthwaite JET-75]: When there are at least three alternatives, no voting rules except dictatorships satisfy
  - non-imposition: every alternative wins for some profile
  - unrestricted domain: voters can use any linear order as their votes
  - strategy-proofness
- Axiomatic characterization for dictatorships!
- Randomized version [Gibbard Econometrica-77]

A few ways out

- Relax non-dictatorship: use a dictatorship
- Restrict the number of alternatives to 2
- Relax unrestricted domain: mainly pursued by economists
  - Single-peaked preferences:

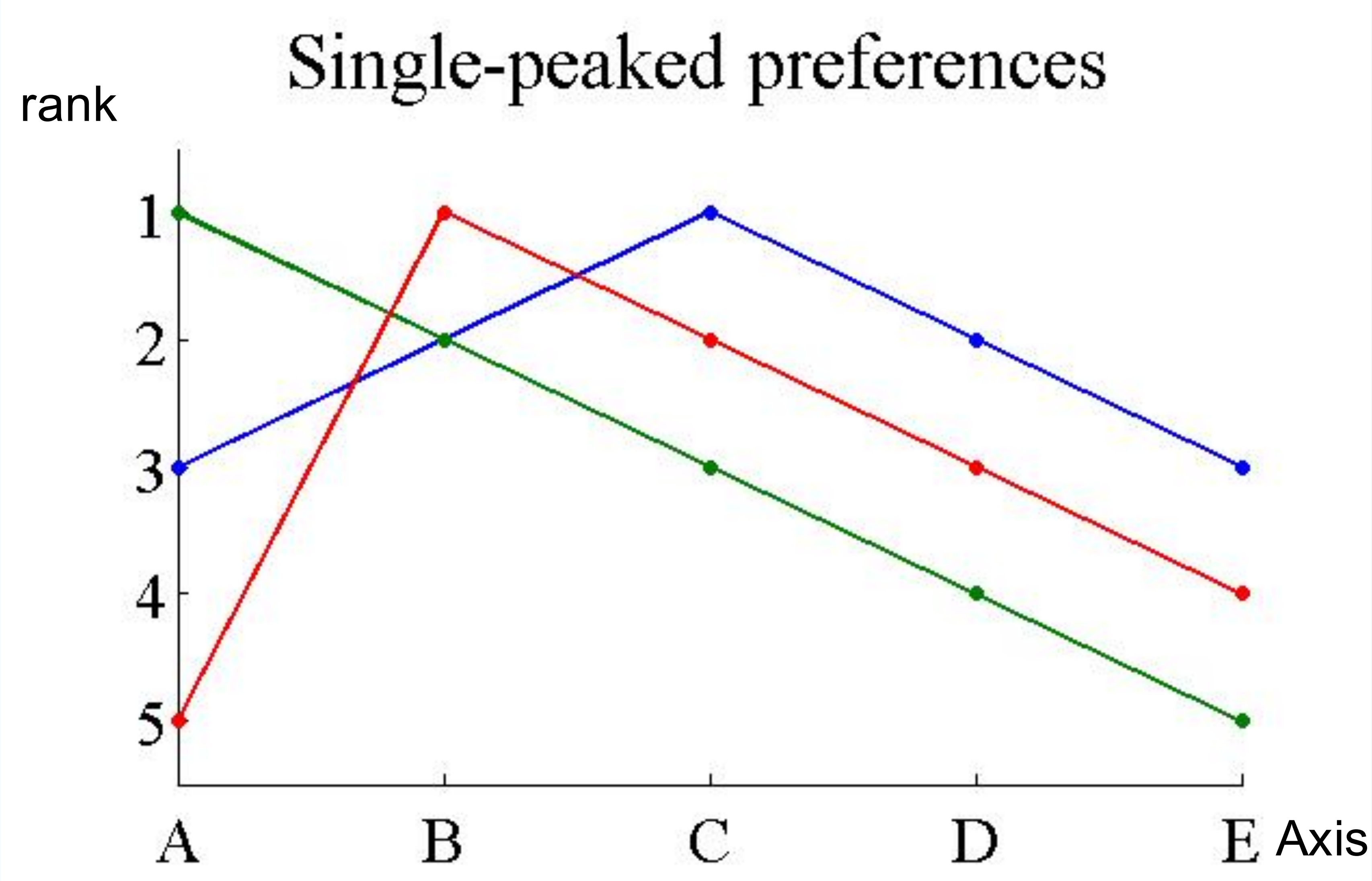


  - Range voting: A voter submit any natural number between 0 and 10 for each alternative
  - Approval voting: A voter submit 0 or 1 for each alternative

Single-peaked preferences

- There exists a social axis S
  - linear order over the alternatives
- Each voter's preferences V are compatible with the social axis S
  - there exists a "peak" a such that
    - [b<c<a in S] implies [c>b in V]
    - [a>c>b in S] implies [c>b in V]
    - alternatives closer to the peak are more preferred
  - different voters may have different peaks

Examples



Strategy-proof rules for single-peaked preferences

- The median rule
  - given a profile of “peaks”
  - choose the median in the social axis
- Theorem. The Median rule is strategy-proof.
- The median rule with phantom voters
  - parameterized by a fixed set of “peaks” of phantom voters
  - chooses the median of the peaks of the regular voters and the phantom voters
- Theorem. Any strategy-proof rule for single-peaked preferences are median rules with phantom voters
- Talk announcement: Dominik Peters 9/21 3-4pm Sage 3713

Computational thinking

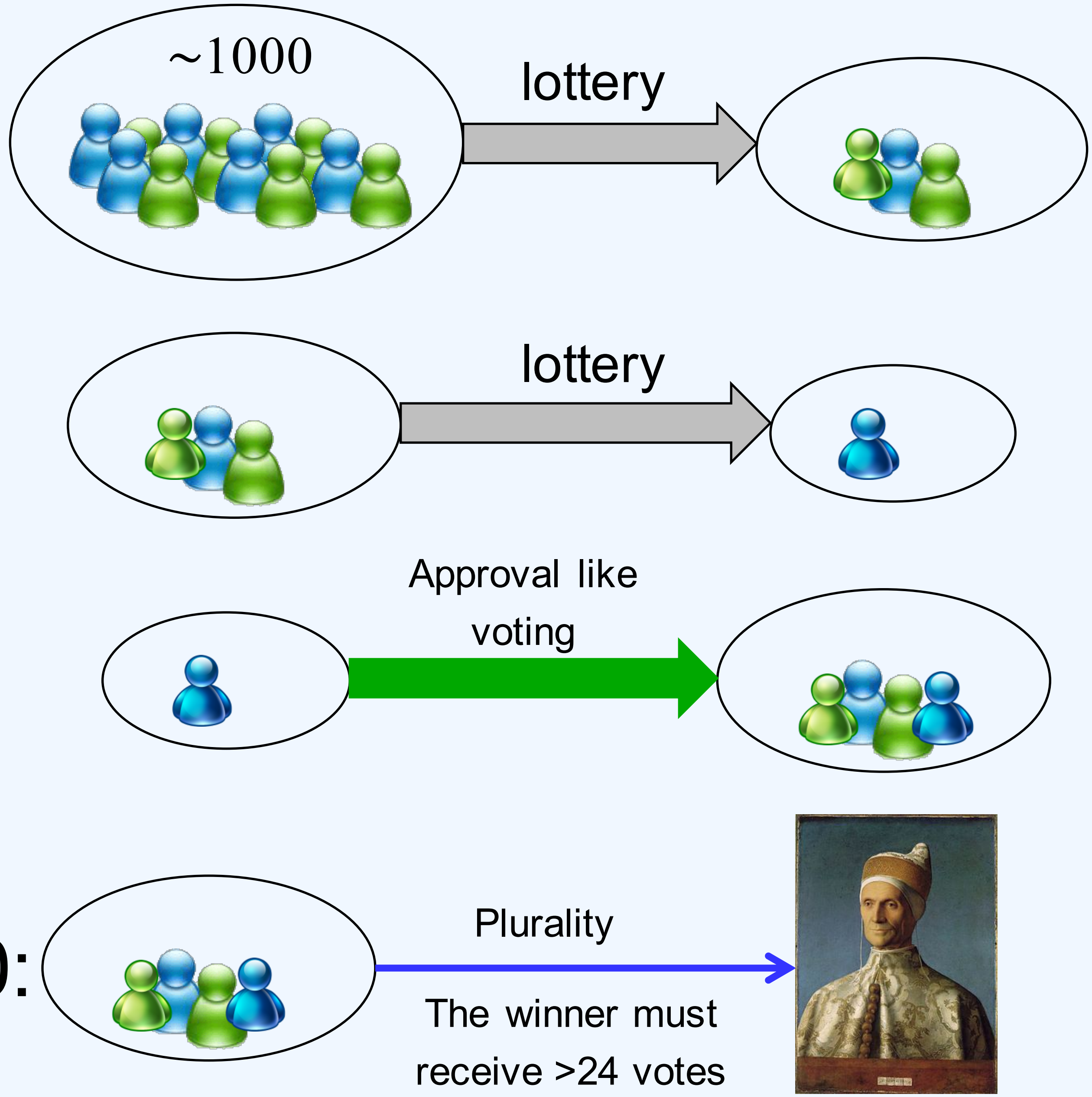
- Use a voting rule that is too complicated so that nobody can easily predict the winner
  - Dodgson
  - Kemeny
  - The randomized voting rule used in Venice Republic for more than 500 years [Walsh&Xia AAMAS-12]
- We want a voting rule where
  - Winner determination is easy
  - Manipulation is hard
- The hard-to-manipulate axiom: manipulation under the given voting rule is NP-hard

Example 3: Venetian election
(1268--1797)

- Round 1:
- Round 2:
- Round 3:

⋮

- Round 10:



Manipulation: A computational complexity perspective

- If it is computationally too hard for a manipulator to compute a manipulation, she is best off voting truthfully
  - – Similar as in cryptography

- For which common voting rules manipulation is computationally hard?

Unweighted coalitional manipulation (UCM) problem

- Given
  - The voting rule r
  - The non-manipulators' profile P^{NM}
  - The number of manipulators n'
  - The alternative c preferred by the manipulators
- We are asked whether or not there exists a profile P^M (of the manipulators) such that c is the winner of $P^{NM} \cup P^M$ under r

The stunningly big table for UCM

#manipulators	One manipulator		At least two	
Copeland	P	[BTT SCW-89b]	**NPC**	[FHS AAMAS-08,10]
STV	**NPC**	[BO SCW-91]	**NPC**	[BO SCW-91]
Veto	P	[ZPR AIJ-09]	P	[ZPR AIJ-09]
Plurality with runoff	P	[ZPR AIJ-09]	P	[ZPR AIJ-09]
Cup	P	[CSL JACM-07]	P	[CSL JACM-07]
Borda	P	[BTT SCW-89b]	**NPC**	[DKN+ AAAI-11] [BNW IJCAI-11]
Maximin	P	[BTT SCW-89b]	**NPC**	[XZP+ IJCAI-09]
Ranked pairs	**NPC**	[XZP+ IJCAI-09]	**NPC**	[XZP+ IJCAI-09]
Bucklin	P	[XZP+ IJCAI-09]	P	[XZP+ IJCAI-09]
Nanson's rule	**NPC**	[NWX AAA-11]	**NPC**	[NWX AAA-11]
Baldwin's rule	**NPC**	[NWX AAA-11]	**NPC**	[NWX AAA-11]

What can we conclude?

- For some common voting rules, computational complexity provides some protection against manipulation
- Is computational complexity a strong barrier?
  - NP-hardness is a worst-case concept