Introduction to mechanism design
Game theory: predicting the outcome with strategic agents

- Games and solution concepts
  - general framework: NE
  - normal-form games: mixed/pure-strategy NE
  - extensive-form games: subgame-perfect NE
Election game of strategic voters

Alice
Strategic vote

Bob
Strategic vote

Carol
Strategic vote
Game theory is predictive

• How to design the “rule of the game”? 
  – so that when agents are strategic, we can achieve a designated outcome w.r.t. their true preferences?
  – “reverse” game theory

• Example: design a social choice mechanism \( f \) so that
  – for every true preference profile \( D^* \)
  – \( \text{OutcomeOfGame}(f, D^*) = \text{Plurality}(D^*) \)
Today’s schedule: mechanism design

- Mechanism design: Nobel prize in economics 2007
  
  Leonid Hurwicz 1917-2008
  Eric Maskin
  Roger Myerson

- VCG Mechanism: Vickrey won Nobel prize in economics 1996
  
  William Vickrey 1914-1996
A game and a solution concept implement a function $f^*$, if
- for every true preference profile $D^*$
- $f^*(D^*) = \text{OutcomeOfGame}(f, D^*)$

$f^*$ is defined for the true preferences
A non-trivial truthful DRM

- Auction for one indivisible item
- $n$ bidders
- Outcomes: { (allocation, payment) }
- Preferences: represented by a quasi-linear utility function
  - every bidder $j$ has a private value $v_j$ for the item. Her utility is
    - $v_j - \text{payment}_j$, if she gets the item
    - 0, if she does not get the item
  - suffices to only report a bid (rather than a total preorder)
- Vickrey auction (second price auction)
  - allocate the item to the agent with the highest bid
  - charge her the second highest bid
Example

- Kyle: $10
- Stan: $70
- Eric: $100

$70

$70
A general workflow of mechanism design

1. Choose a target function $f^*$ to implement

2. Model the situation as a game

3. Choose a solution concept SC

4. Design $f$ such that the game and SC implements $f^*$

- Pareto optimal outcome
- utilitarian optimal
- egalitarian optimal
- allocation + payments
- etc

- dominant-strategy NE
- mixed-strategy NE
- SPNE
- etc

- normal form
- extensive form
- etc
• Agents (players): $N=\{1,\ldots,n\}$
• Outcomes: $O$
• Preferences (private): total preorders over $O$
• Message space (c.f. strategy space): $S_j$ for agent $j$
• Mechanism: $f : \Pi_j S_j \to O$
Frameworks of social choice, game theory, mechanism design

• Agents = players: $N=\{1,\ldots,n\}$

• Outcomes: $O$

• True preference space: $P_j$ for agent $j$
  – consists of total preorders over $O$
  – sometimes represented by utility functions

• Message space = reported preference space = strategy space: $S_j$ for agent $j$

• Mechanism: $f: \prod_j S_j \rightarrow O$
Step 1: choose a target function
(social choice mechanism w.r.t. truth preferences)

- Nontrivial, later after revelation principle
Step 2: specify the game

- Agents: often obvious
- Outcomes: need to design
  - require domain expertise, beyond mechanism design
- Preferences: often obvious given the outcome space
  - usually by utility functions
- Message space: need to design
Step 3: choose a solution concept

- If the solution concept is too weak (general)
  - equilibrium selection
  - e.g. mixed-strategy NE

- If the solution concept is too strong (specific)
  - unlikely to exist an implementation
  - e.g. SPNE

- We will focus on dominant-strategy NE for the rest of today
Step 4: Design a mechanism
Direct-revelation mechanisms (DRMs)

• A special mechanism where for agent $j$, $S_j = P_j$
  – true preference space = reported preference space

• A DRM $f$ is truthful (incentive compatible) w.r.t. a solution concept SC (e.g. NE), if
  – In SC, $R_j = R_j^*$
  – i.e. everyone reports her true preferences
  – A truthful DRM implements itself!

• Examples of truthful DRMs
  – always outputs outcome “$a$”
  – dictatorship
A non-trivial truthful DRM

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$10
$70
$100
Indirect mechanisms (IM)

• No restriction on $S_j$
  – includes all DRM$s$
  – If $S_j \neq P_j$ for some agent $j$, then truthfulness is not defined
  – not clear what a “truthful” agent will do under IM

• Example
  – Second-price auction where agents are required to report an integer bid
Another example

- English auction
  
  "arguably the most common form of auction in use today" ---wikipedia

- Every bidder can announce a higher price

- The last-standing bidder is the winner

- Implements Vickrey (second price) auction
Truthful DRM vs. IM: usability

• Truthful DRM: $f^*$ is implemented for truthful and strategic agents
  – Truthfulness:
    • if an agent is truthful, she reports her true preferences
    • if an agent is strategic (as indicated by the solution concept), she still reports her true preferences
  – Communication: can be a lot
  – Privacy: no

• Indirect Mechanisms
  – Truthfulness: no
  – Communication: can be little
  – Privacy: may preserve privacy
Truthful DRM vs. IM: easiness of design

• Implementation w.r.t. DSNE

• Truthful DRM:
  – $f$ itself!
  – only needs to check the incentive conditions,
    i.e. for every $j$, $R'_j$,
    • for every $R_{-j}$: $f(R_j^*, R_{-j}) \geq_j f(R'_j, R_{-j})$
    • the inequality is strict for some $R_{-j}$

• Indirect Mechanisms
  – Hard to even define the message space
Truthful DRM vs. IM: implementability

- Can IMs implement more social choice mechanisms than truthful DRMs?
  - depends on the solution concept

- Implementability
  - the set of social choice mechanisms that can be implemented (by the game + mechanism + solution concept)
Revelation principle

- **Revelation principle.** Any social choice mechanism $f^*$ implemented by a mechanism w.r.t. DSNE can be implemented by a truthful DRM (itself) w.r.t. DSNE
  - truthful DRMs is as powerful as IMs in implementability w.r.t. DSNE
  - If the solution concept is DSNE, then designing a truthful DRM implication is equivalent to checking that agents are truthful under $f^*$
- has a Bayesian-Nash Equilibrium version
Proof

• \( DS_j(R_j^*) \): the dominant strategy of agent \( j \)

• Prove that \( f^* \) is a truthful DRM that implements itself
  – **truthfulness**: suppose on the contrary that \( f^* \) is not truthful
  – W.l.o.g. suppose \( f^*(R_1, R_{-1}^*) >_1 f^*(R_1^*, R_{-1}^*) \)
  – \( DS_1(R_1^*) \) is not a dominant strategy
    • compared to \( DS_1(R_1) \), given \( DS_2(R_2^*), \ldots, DS_n(R_n^*) \)

\[ \begin{align*}
  R_1^* & \rightarrow DS_1(R_1^*) \\
  R_2^* & \rightarrow DS_2(R_2^*) \\
  \vdots & \quad \vdots \\
  R_n^* & \rightarrow DS_n(R_n^*) \\
\end{align*} \]
Interpreting the revelation principle

- It is a powerful, useful, and negative result
- **Powerful**: applies to any mechanism design problem
- **Useful**: only need to check if truth-reporting is the dominant strategy in $f^*$
- **Negative**: If any agent has incentive to lie under $f^*$, then $f^*$ cannot be implemented by any mechanism w.r.t. DSNE
Step 1: Choosing the function to implement (w.r.t. DSNE)
Mechanism design with money

- Modeling situations with monetary transfers
- Set of alternatives: $A$
  - e.g. allocations of goods
- Outcomes: \{ (alternative, payments) \}
- Preferences: represented by a quasi-linear utility function
  - every agent $j$ has a private value $v_j^*(a)$ for every $a \in A$. Her utility is
    \[
    u_j^*(a, p) = v_j^*(a) - p_j
    \]
  - It suffices to report a value function $v_j$
Can we adjust the payments to maximize social welfare?

- Social welfare of $a$
  - $\text{SCW}(a) = \sum_j v_j^*(a)$

- Can any $(\text{argmax}_a \text{SCW}(a)$, payments) be implemented w.r.t. DSNE?
The Vickrey-Clarke-Groves mechanism (VCG)

- The Vickrey-Clarke-Groves mechanism (VCG) is defined by
  - Alternative in outcome: \( a^* = \text{argmax}_a \text{SCW}(a) \)
  - Payments in outcome: for agent \( j \)
    \[
    p_j = \max_a \sum_{i \neq j} v_i(a) - \sum_{i \neq j} v_i(a^*)
    \]
    - negative externality of agent \( j \) of its presence on other agents
- Truthful, efficient
- A special case of Groves mechanism
Example: one item auction

- Alternatives = (give to K, give to S, give to E)
- \( a^* \)
- \( p_1 = 100 - 100 = 0 \)
- \( p_2 = 100 - 100 = 0 \)
- \( p_3 = 70 - 0 = 70 \)
Wrap up

- Mechanism design:
  - the social choice mechanism $f^*$
  - the game and the mechanism to implement $f^*$
- The revelation principle: implementation w.r.t. DSNE = checking incentive conditions
- VCG mechanism: a generic truthful and efficient mechanism for mechanism design with money
Looking forward

• The end of “pure economics” classes
  – Social choice: 1972 (Arrow), 1998 (Sen)
  – Game theory: 1994 (Nash, Selten and Harsanyi), 2005 (Schelling and Aumann)
  – Mechanism design: 2007 (Hurwicz, Maskin and Myerson)
  – Auctions: 1996 (Vickrey)

• The next class: introduction to computation
  – Linear programming
  – Basic computational complexity theory

• Then
  – Computation + Social choice

• HW1 is due on Friday before class
NE of the plurality election game

• Players: \{ YOU, Bob, Carol\}, \( n=3 \)
• Outcomes: \( O = \{ \text{YOU}, \text{Bob}, \text{Carol} \} \)
• Strategies: \( S_j = \text{Rankings}(O) \)
• Preferences: \text{Rankings}(O)
• Mechanism: the plurality rule
Proof (1)

- Given
  - $f^*$ implemented by $f'$ w.r.t. DSNE
- Construct a DRM $f$ that "simulates" the strategic behavior of the agents under $f'$, $DS_j(u_j)$

\[ f(u_1, \ldots, u_n) = f'(DS_1(u_1), \ldots, DS_n(u_n)) \]