Introduction to mechanism design

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Rensselaer

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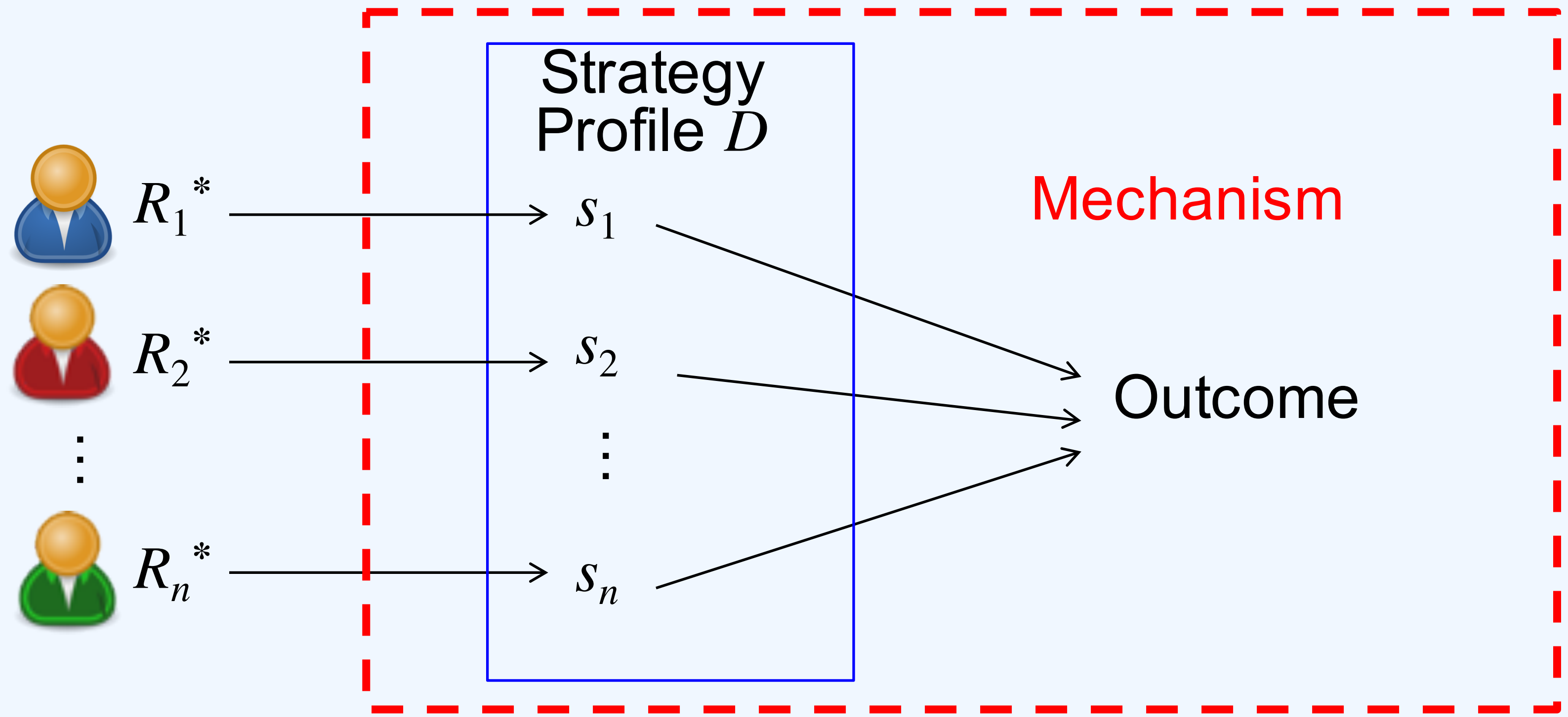
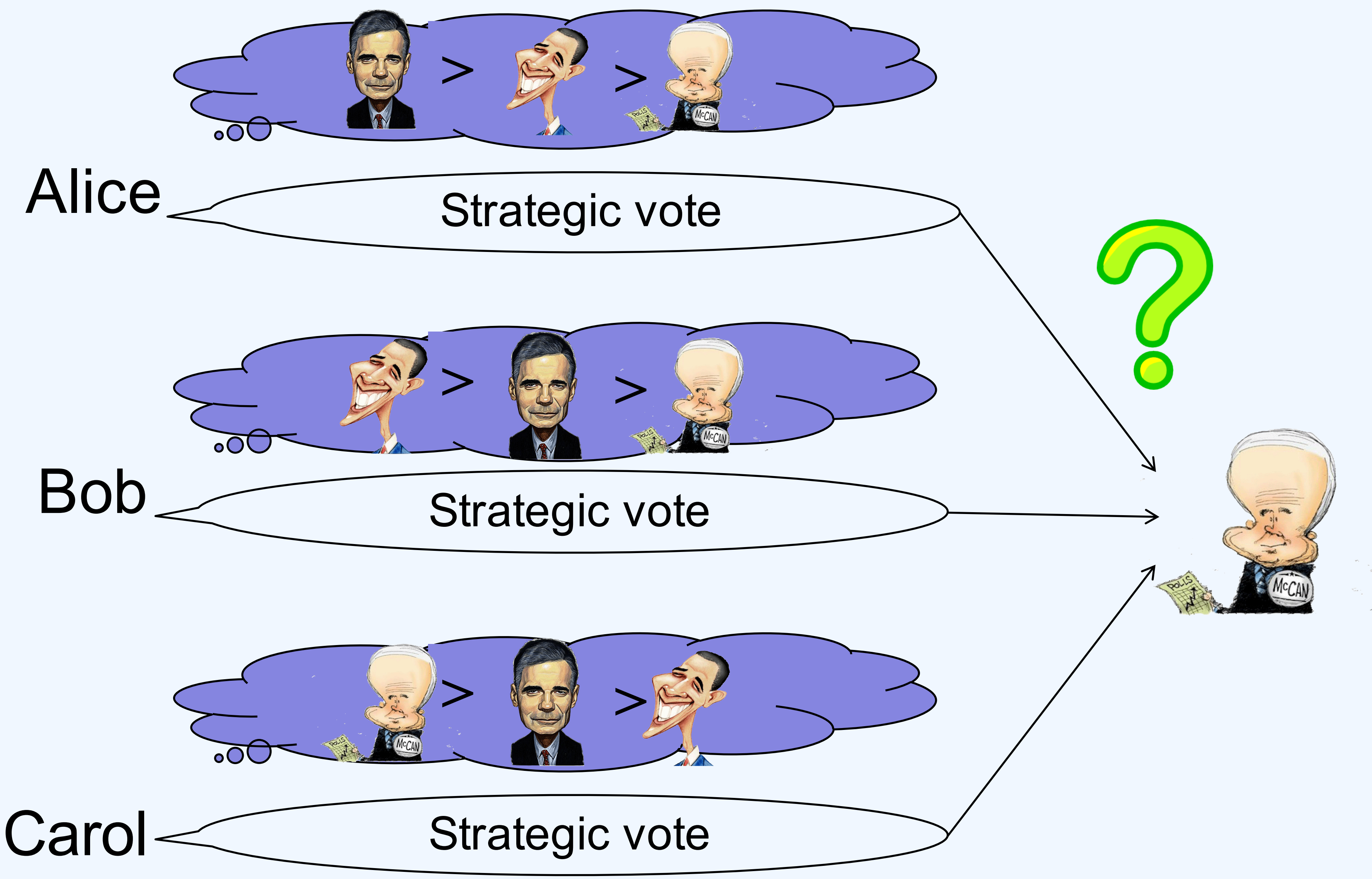
Last class: game theory



- Game theory: predicting the outcome with strategic agents
- Games and solution concepts
  - general framework: NE
  - normal-form games: mixed/pure-strategy NE
  - extensive-form games: subgame-perfect NE

Election game of strategic voters



Game theory is predictive

- How to design the “rule of the game”?
  - so that when agents are strategic, we can achieve a designated outcome w.r.t. their true preferences?
  - “reverse” game theory
- Example: design a social choice mechanism f so that
  - for every true preference profile D^*
  - OutcomeOfGame(f, D^*)=Plurality(D^*)

Today's schedule: mechanism design

- Mechanism design: Nobel prize in economics 2007

Leonid Hurwicz
1917-2008

Eric Maskin

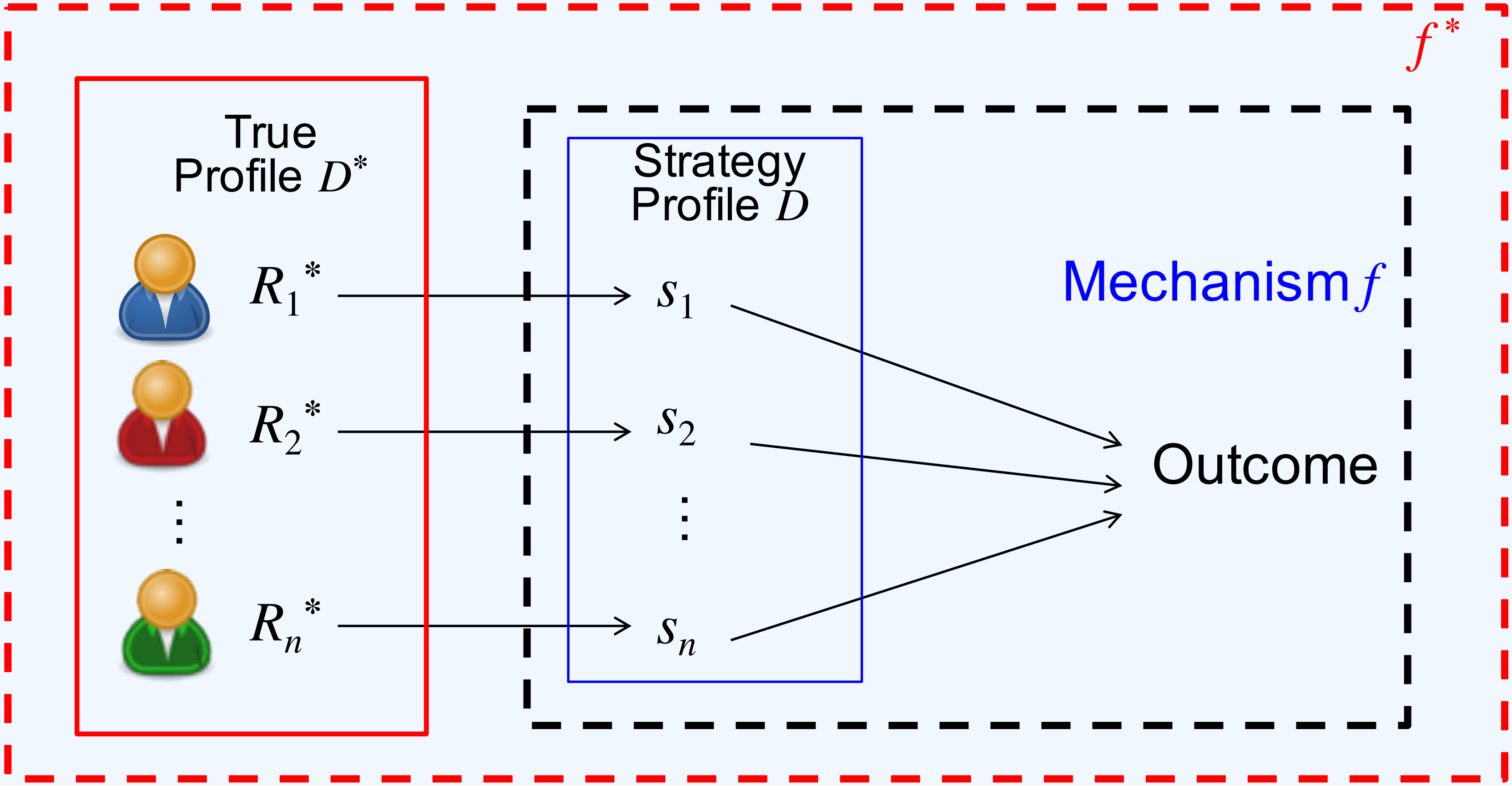
Roger Myerson

- VCG Mechanism: Vickrey won Nobel prize in economics 1996

William Vickrey
1914-1996

- What? Your homework
- Why? Your homework
- How? Your homework

Implementation



- A game and a solution concept implement a function f^*, if
  - for every true preference profile D^*
  - $f^*(D^*)$ =OutcomeOfGame(f, D^*)
- f^* is defined for the true preferences

A general workflow of mechanism design

- Pareto optimal outcome
- utilitarian optimal
- egalitarian optimal
- allocation+ payments
- etc

1. Choose a target function f^* to implement

↓

2. Model the situation as a game

- normal form
- extensive form
- etc

↓

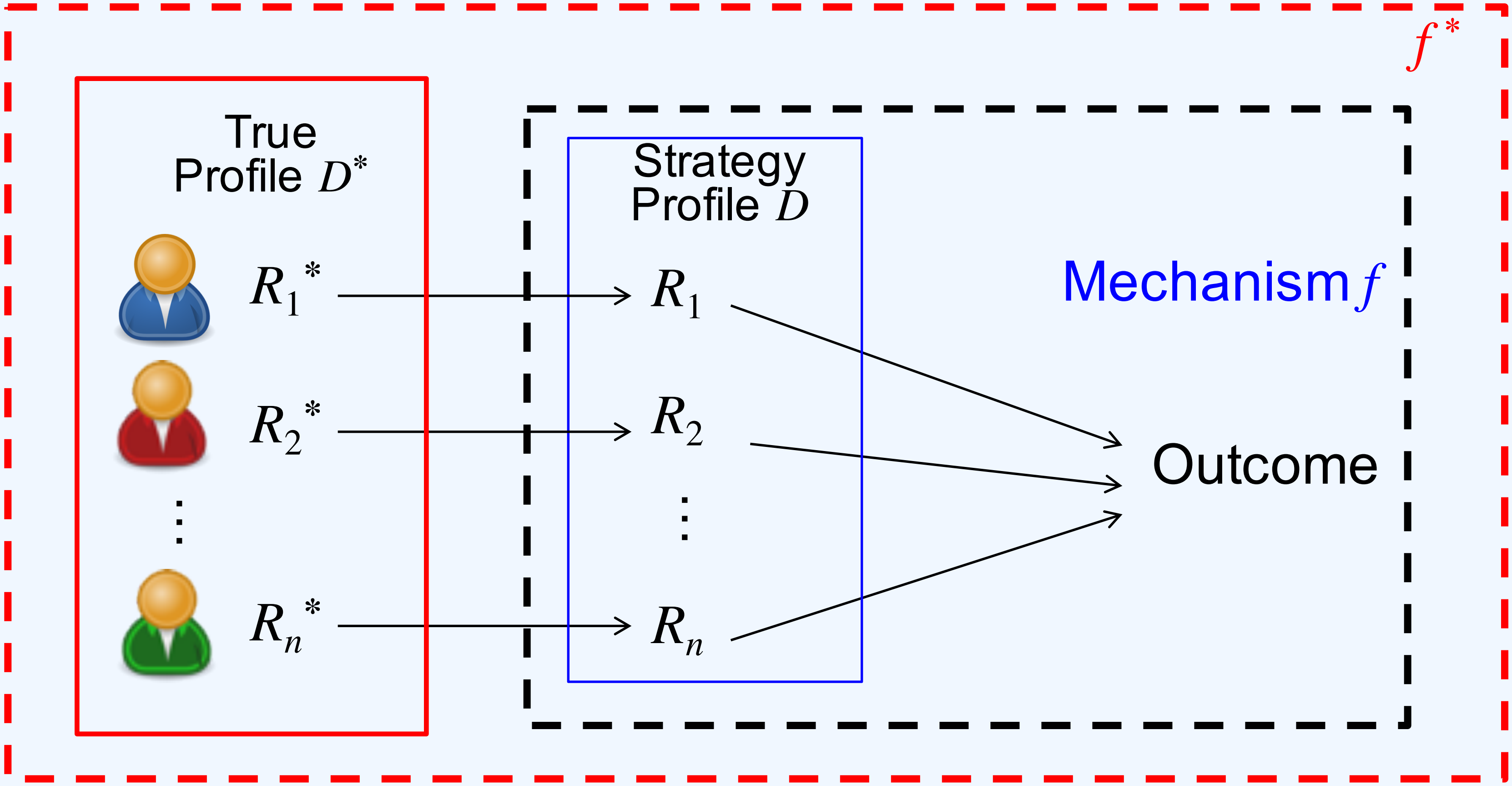
3. Choose a solution concept SC

- dominant-strategy NE
- mixed-strategy NE
- SPNE
- etc

↓

4. Design f such that the game and SC implements f^*

Framework of mechanism design



- Agents (players): $N=\{1,\ldots,n\}$
- Outcomes: O
- Preferences (private): total preorders over O
- Message space (c.f. strategy space): S_j for agent j
- Mechanism: $f : \Pi_j S_j \rightarrow O$

Frameworks of social choice, game theory, mechanism design

- Agents = players: $N=\{1,\ldots,n\}$
- Outcomes: O
- True preference space: P_j for agent j
  - consists of total preorders over O
  - sometimes represented by utility functions
- Message space = reported preference space = strategy space: S_j for agent j
- Mechanism: $f : \Pi_j S_j \rightarrow O$

Step 1: choose a target function
(social choice mechanism w.r.t. truth preferences)

- Nontrivial, later after revelation principle

Step 2: specify the game

- Agents: often obvious
- Outcomes: need to design
  - require domain expertise, beyond mechanism design
- Preferences: often obvious given the outcome space
  - usually by utility functions
- Message space: need to design

Step 3: choose a solution concept

- If the solution concept is too weak (general)
  - equilibrium selection
  - e.g. mixed-strategy NE
- If the solution concept is too strong (specific)
  - unlikely to exist an implementation
  - e.g. SPNE
- We will focus on dominant-strategy NE for the rest of today

Dominant-strategy NE

- Recall that an NE exists when every player has a dominant strategy
  - s_j is a dominant strategy for player j, if for every $s_j' \in S_j$,
    1. for every s_{-j} , $f(s_j, s_{-j}) \geq_j f(s_j', s_{-j})$
    2. the preference is strict for some s_{-j}
- A dominant-strategy NE (DSNE) is an NE where
  - every player takes a dominant strategy
  - may not exists, but if exists, then must be unique

Prisoner's dilemma

Column player

Row player

	Cooperate	Defect
Cooperate	(-1 , -1)	(-3 , 0)
Defect	(0 , -3)	(-2 , -2)

Defect is the dominant strategy for both players

Step 4: Design a mechanism

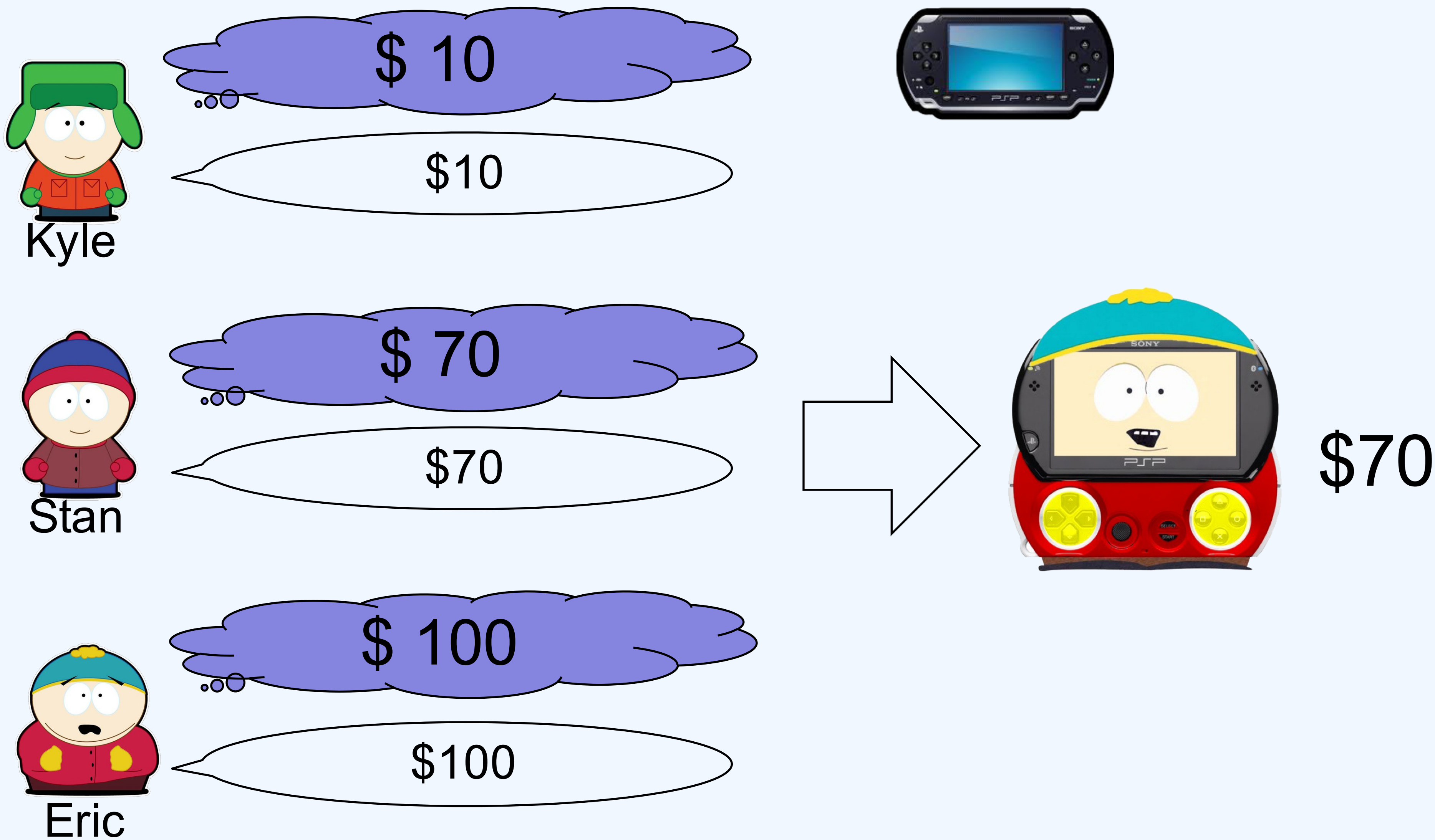
Direct-revelation mechanisms (DRMs)

- A special mechanism where for agent j, $S_j = P_j$
  - true preference space = reported preference space
- A DRM f is truthful (incentive compatible) w.r.t. a solution concept SC (e.g. NE), if
  - In SC, $R_j = R_j^*$
  - i.e. everyone reports her true preferences
  - A truthful DRM implements itself!
- Examples of truthful DRMs
  - always outputs outcome "a"
  - dictatorship

A non-trivial truthful DRM

- Auction for one indivisible item
- n bidders
- Outcomes: { (allocation, payment) }
- Preferences: represented by a quasi-linear utility function
  - every bidder j has a private value v_j for the item. Her utility is
    - v_j - payment$_j$, if she gets the item
    - 0, if she does not get the item
  - suffices to only report a bid (rather than a total preorder)
- Vickrey auction (second price auction)
  - allocate the item to the agent with the highest bid
  - charge her the second highest bid

Example



Indirect mechanisms (IM)

- No restriction on S_j
  - includes all DRMs
  - If $S_j \neq P_j$ for some agent j, then truthfulness is not defined
  - not clear what a "truthful" agent will do under IM
- Example
  - Second-price auction where agents are required to report an integer bid

Another example

- English auction

  “*arguably the most common form of auction in use today*” ---wikipedia

- Every bidder can announce a higher price
- The last-standing bidder is the winner
- Implements Vickrey (second price) auction

Truthful DRM vs. IM: usability

- Truthful DRM: f^* is implemented for truthful and strategic agents
  - Truthfulness:
    - if an agent is truthful, she reports her true preferences
    - if an agent is strategic (as indicated by the solution concept), she still reports her true preferences
  - Communication: can be a lot
  - Privacy: no
- Indirect Mechanisms
  - Truthfulness: no
  - Communication: can be little
  - Privacy: may preserve privacy

Truthful DRM vs. IM: easiness of design

- Implementation w.r.t. DSNE
- Truthful DRM:
  - f itself!
  - only needs to check the incentive conditions, i.e. for every j, R_j',
    - for every R_{-j} : $f(R_j^*, R_{-j}) \geq_j f(R_j', R_{-j})$
    - the inequality is strict for some R_{-j}
- Indirect Mechanisms
  - Hard to even define the message space

Truthful DRM vs. IM: implementability

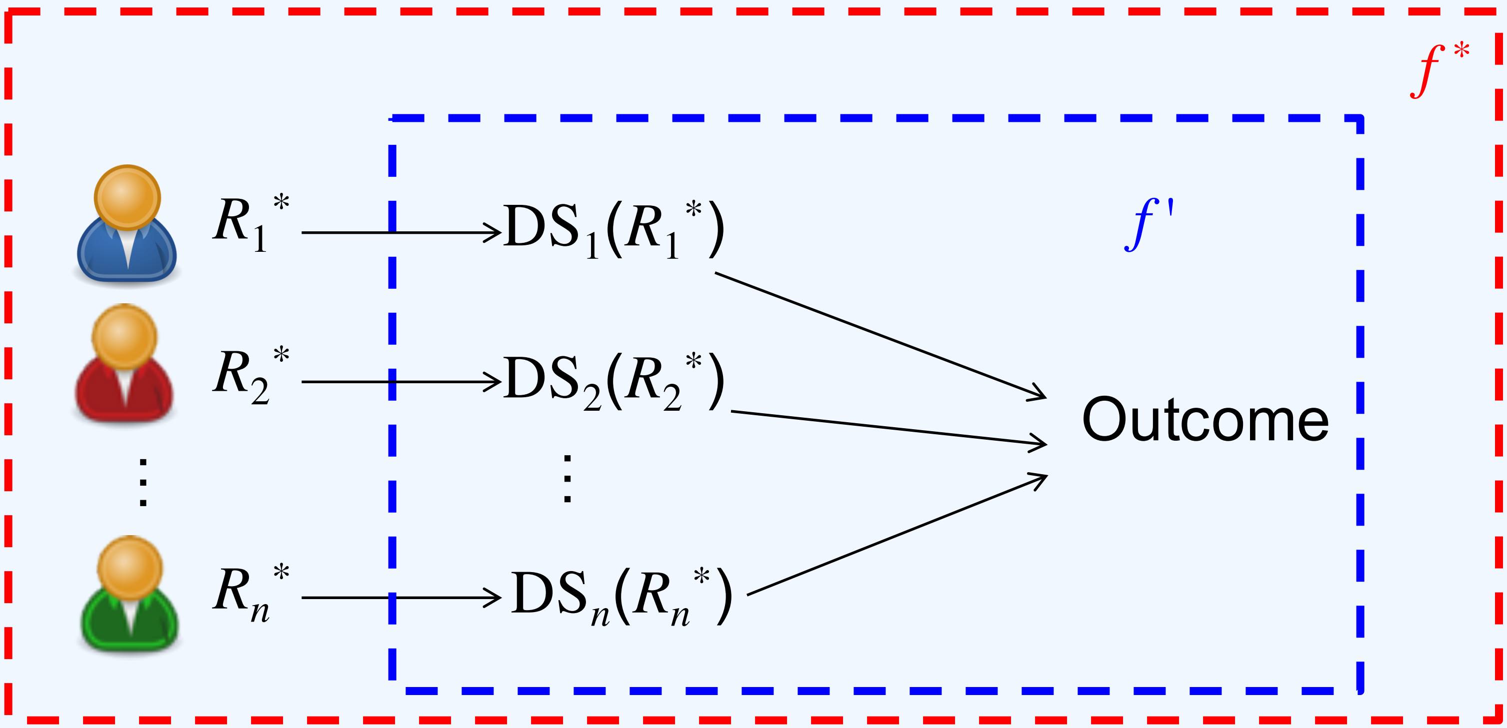
- Can IMs implement more social choice mechanisms than truthful DRMs?
  - depends on the solution concept
- Implementability
  - the set of social choice mechanisms that can be implemented (by the game + mechanism + solution concept)

Revelation principle

- Revelation principle. Any social choice mechanism f^* implemented by a mechanism w.r.t. DSNE can be implemented by a truthful DRM (itself) w.r.t. DSNE
  - truthful DRMs is as powerful as IMs in implementability w.r.t. DSNE
  - If the solution concept is DSNE, then designing a truthful DRM implication is equivalent to checking that agents are truthful under f^*
- has a Bayesian-Nash Equilibrium version

Proof

- $\mathrm{DS}_j(R_j^*)$: the dominant strategy of agent j
- Prove that f^* is a truthful DRM that implements itself
  - truthfulness: suppose on the contrary that f^* is not truthful
  - W.l.o.g. suppose $f^*(R_1, R_{-1}^*) >_1 f^*(R_1^*, R_{-1}^*)$
  - $\mathrm{DS}_1(R_1^*)$ is not a dominant strategy
    - compared to $\mathrm{DS}_1(R_1)$, given $\mathrm{DS}_2(R_2^*)$, …, $\mathrm{DS}_n(R_n^*)$



Interpreting the revelation principle

- It is a powerful, useful, and negative result
- Powerful: applies to any mechanism design problem
- Useful: only need to check if truth-reporting is the dominant strategy in f^*
- Negative: If any agent has incentive to lie under f^*, then f^* cannot be implemented by any mechanism w.r.t. DSNE

Step 1: Choosing the function to implement (w.r.t. DSNE)

Mechanism design with money

- Modeling situations with monetary transfers
- Set of alternatives: A
  - e.g. allocations of goods
- Outcomes: { (alternative, payments) }
- Preferences: represented by a quasi-linear utility function
  - every agent j has a private value $v_j^*(a)$ for every $a \in A$. Her utility is

$$u_j^*(a, p) = v_j^*(a) - p_j$$

  - It suffices to report a value function v_j

Can we adjust the payments to maximize social welfare?

- Social welfare of a
  - $\mathrm{SCW}(a)=\Sigma_j v_j^*(a)$
- Can any (argmax_a $\mathrm{SCW}(a)$, payments) be implemented w.r.t. DSNE?

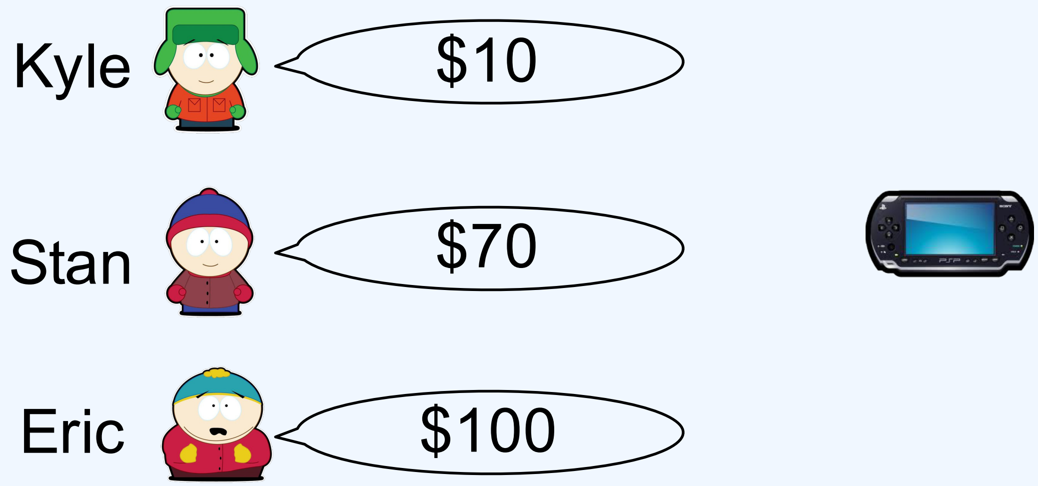
The Vickrey-Clarke-Groves mechanism (VCG)

- The Vickrey-Clarke-Groves mechanism (VCG) is defined by
  - Alterative in outcome: a^*=argmax$_a$ SCW(a)
  - Payments in outcome: for agent j

$$p_j = \max_a \Sigma_{i \neq j} v_i(a) - \Sigma_{i \neq j} v_i(a^*)$$

    - negative externality of agent j of its presence on other agents
- Truthful, efficient
- A special case of Groves mechanism

Example: auction of one item



- Alternatives = (give to K, give to S, give to E)
- a^* = 
- p_1 = 100 – 100 = 0
- p_2 = 100 – 100 = 0
- p_3 = 70 – 0 = 70

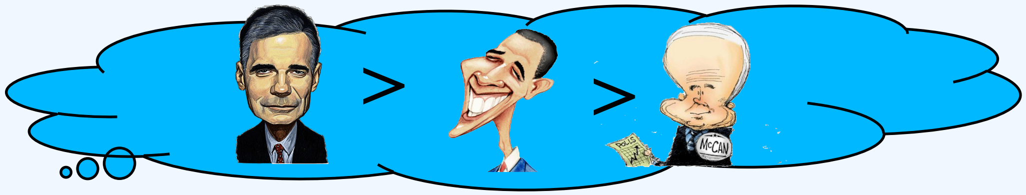
Wrap up

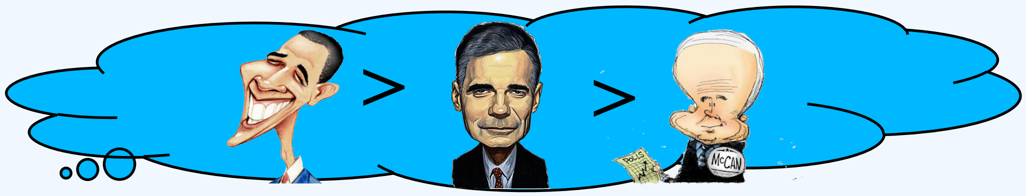
- Mechanism design:
  - the social choice mechanism f^*
  - the game and the mechanism to implement f^*
- The revelation principle: implementation w.r.t. DSNE = checking incentive conditions
- VCG mechanism: a generic truthful and efficient mechanism for mechanism design with money

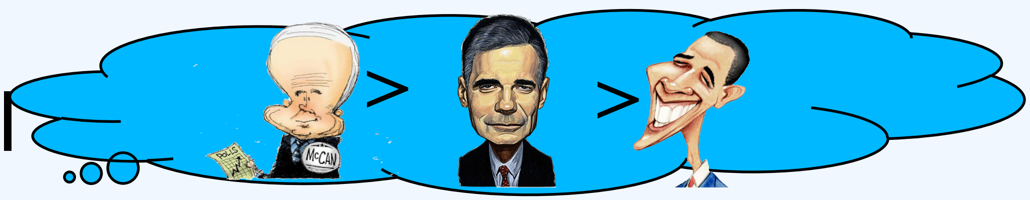
Looking forward

- The end of “pure economics” classes
  - Social choice: 1972 (Arrow), 1998 (Sen)
  - Game theory: 1994 (Nash, Selten and Harsanyi), 2005 (Schelling and Aumann)
  - Mechanism design: 2007 (Hurwicz, Maskin and Myerson)
  - Auctions: 1996 (Vickrey)
- The next class: introduction to computation
  - Linear programming
  - Basic computational complexity theory
- Then
  - Computation + Social choice
- HW1 is due on Friday before class

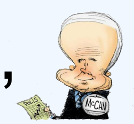
NE of the plurality election game

YOU 

Bob 

Carol 

Plurality rule

- Players: { YOU, Bob, Carol}, n=3
- Outcomes: O = {  ,  ,  }
- Strategies: S_j = Rankings(O)
- Preferences: Rankings(O)
- Mechanism: the plurality rule

Proof (1)

- Given
  - f^* implemented by f' w.r.t. DSNE
- Construct a DRM f that "simulates" the strategic behavior of the agents under f', $\mathrm{DS}_j(u_j)$

$$f(u_1, \ldots, u_n) = f'(\mathrm{DS}_1(u_1), \ldots, \mathrm{DS}_n(u_n))$$