Computational social choice
Combinatorial voting

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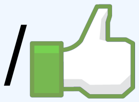
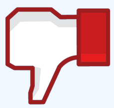
Last class: the easy-to-compute axiom

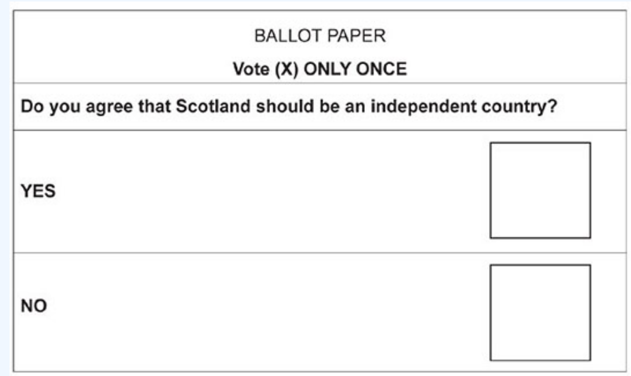
- We hope that the outcome of a social choice mechanism can be computed in p-time
  - P: positional scoring rules, maximin, Copeland, ranked pairs, etc
  - NP-hard: Kemeny, Slater, Dodgson
- But sometimes P is not enough
  - input size: $nm \log m$
  - preference representation: ask a human to give a full ranking over 2000 alternatives
  - preference aggregation

Today: Combinatorial voting

- In California, voters voted on 11 binary issues (/)
  - 2^{11}=2048 combinations in total
  - 5/11 are about budget and taxes

- Prop.30 Increase sales and some income tax for education
- Prop.38 Increase income tax on almost everyone for education

Referendum voting


BALLOT PAPER

Vote (X) ONLY ONCE

Do you agree that Scotland should be an independent country?

YES

NO

- Other interesting facts
- A 12-pages ballot
  - http://www.miamidade.gov/elections/s_ballots/11-6-12_sb.pdf
- Five of the Most Confusing Ballots in the Country
  - http://www.propublica.org/article/five-of-the-most-confusing-ballots-in-the-country

Looking into one proposition

- New York Redistricting Commission Amendment, Proposal 1 (2014)
  - Revising State's Redistricting Procedure The proposed amendment to sections 4 and 5 and addition of new section 5-b to Article 3 of the State Constitution revises the redistricting procedure for state legislative and congressional districts. The proposed amendment establishes an independent redistricting commission every 10 years beginning in 2020, with two members appointed by each of the four legislative leaders and two members selected by the eight legislative appointees; prohibits legislators and other elected officials from serving as commissioners; establishes principles to be used in creating districts; requires the commission to hold public hearings on proposed redistricting plans; subjects the commission's redistricting plan to legislative enactment; provides that the legislature may only amend the redistricting plan according to the established principles if the commission's plan is rejected twice by the legislature; provides for expedited court review of a challenged redistricting plan; and provides for funding and bipartisan staff to work for the commission. Shall the proposed amendment be approved?
- CSCI 4979/6976 reformation Amendment, Proposal 1 (2014)
  - All students should get A+ immediately; all students have right not coming to the class any time for any reason; students can throw rotten eggs and tomatoes at the instructor; we should fight evil and protect world; we should watch at least one movie per week in class; the instructor should offer pizza every time; everyone should give the instructor one million US dollars. Shall the proposed amendment be approved?

Combinatorial domains (Multi-issue domains)

- The set of alternatives can be uniquely characterized by multiple issues
- Let $I=\{x_1,\ldots,x_p\}$ be the set of p issues
- Let D_i be the set of values that the i-th issue can take, then $A=D_1\times\ldots\times D_p$
- Example:
  - Issues={ Main course, Wine }
  - Alternatives={   } ×{   }

Potential problems

- Preference representation
- Communication
- Preference aggregation
- Which one do you think is the most serious problem?

Where is the bottleneck?

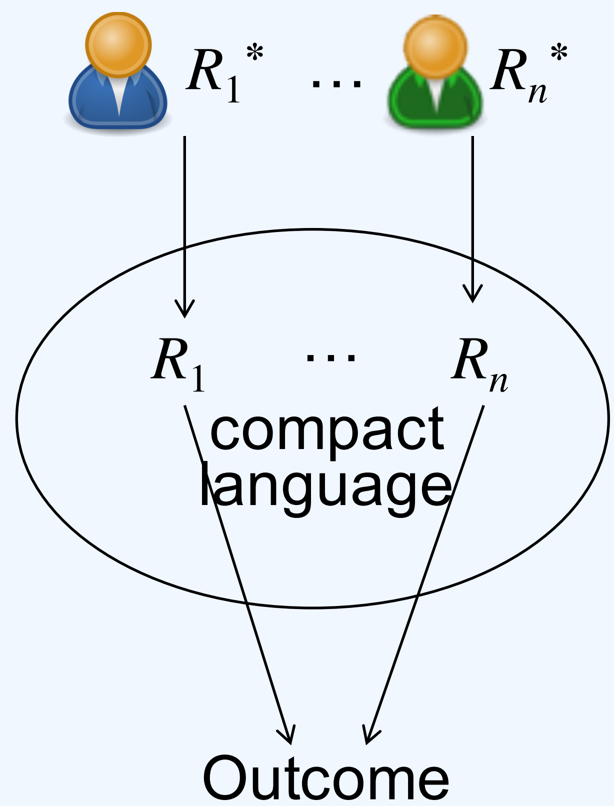
- Ballot propositions
  - preference representation: big problem
    - rank 2000 alternatives
  - communication: not a big problem
    - internet is fast and almost free for use
  - Computation: not a big problem
    - computers can easily handle 2000 alternatives

Where is the bottleneck?

- Robots on Mars
  - preference representation: sometimes not a big problem
    - robots can come up a ranking over millions of alternatives
  - communication: big problem
  - computation: sometimes not a big problem

Where is the bottleneck?

- Use a compact representation
  - preference representation: a big problem
    - tradeoff between efficiency and expressiveness
  - communication: not a problem
  - computation: a big problem
    - many voting rules becomes NP-hard to compute

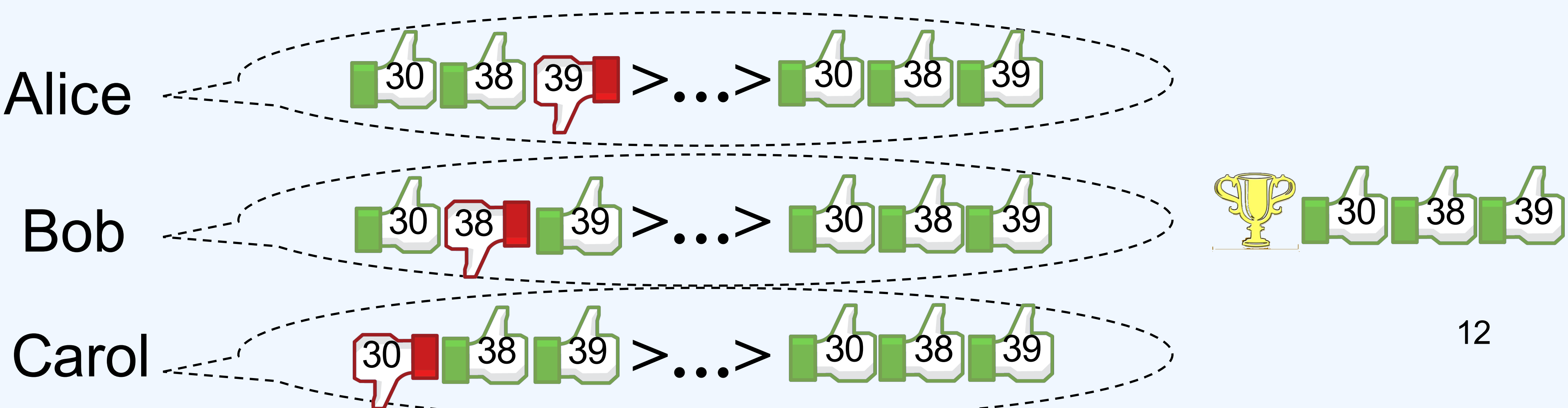


Econ vs. CS in Combinatorial voting

Combinatorial voting	**Economics**	**CS**
Representation	one value per issue	CP-nets
Aggregation	issue-by-issue voting	sequential voting
Evaluation	paradoxes	“numerical” paradoxes
	satisfiability of axioms	
Strategic behavior	equilibrium analysis	evaluation of equilibrium outcome

Issue-by-issue voting

- Issue-by-issue voting (binary variables)
  - representation: each voter mark one value for each issue
    - similar to the plurality rule
  - for each issue, use the majority rule to decide the winner



Computational aspects of issue-by-issue voting

- Language
  - one value per issue
  - $\Sigma_i \log |D_i|$
- Low communication
- Fast computation

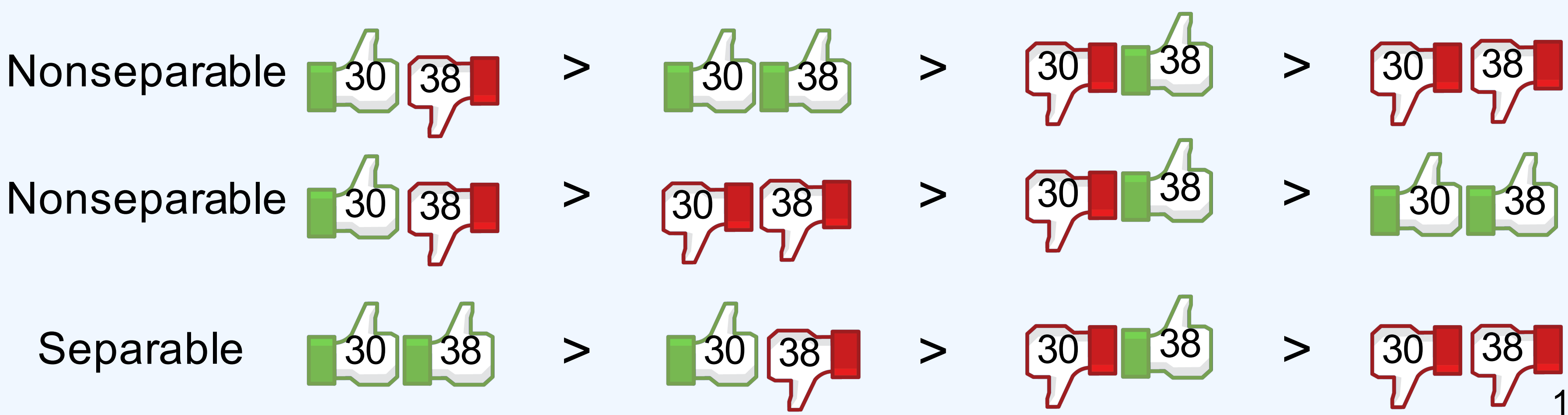
Social choice aspects of issue-by-issue voting

- Representation
  - agents are likely to feel uncomfortable with reporting unconditional preferences
- Hard to analyze
  - not clear what an agent will report
- Outcome is sometimes extremely bad
  - multiple-election paradoxes
    - winner ranked in the bottom
    - winner is not Pareto optimal
- No issue-by-issue voting rule satisfies neutrality or Pareto efficient [Benoit & Kornhauser GEB-10]
  - If the domain is not composed of two binary issues
- Strategic aspects: [Ahn & Oliveros Econometrica-12]

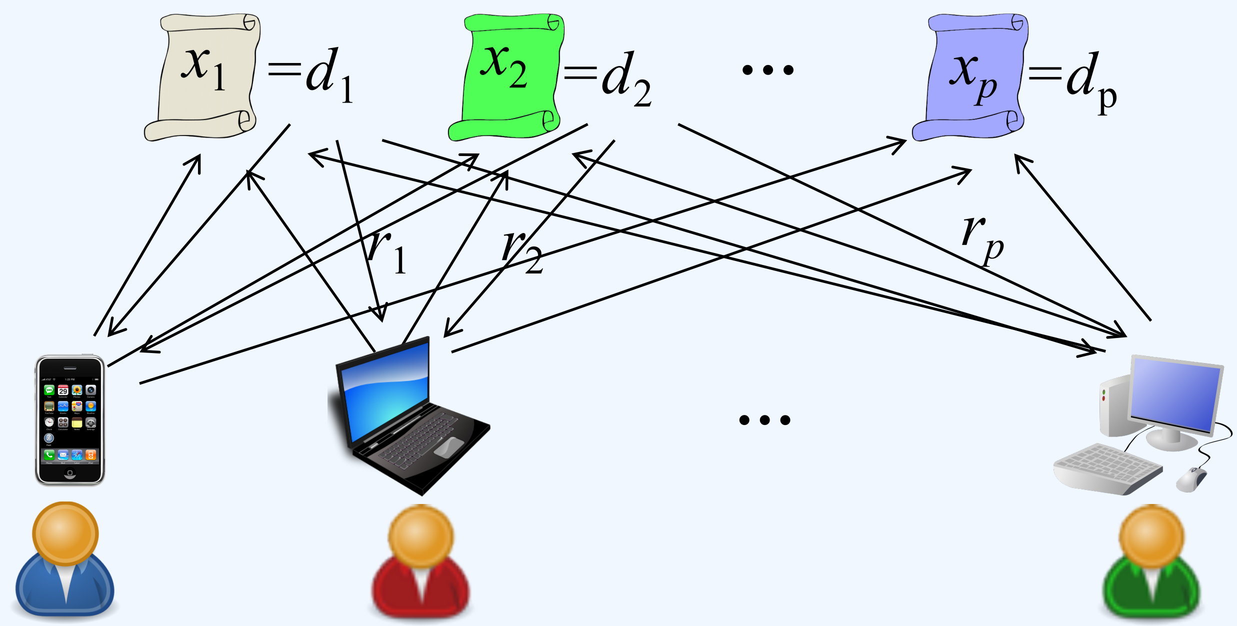
Separable preferences

- Agents are comfortable reporting their preferences when these preferences are separable
  - for any issue i, any agent's preferences over issue i does not depend on the value of other issues
  - for any agent j, any a_i, $b_i \in D_i$ and any c_{-i}, $d_{-i} \in D_{-i}$,

$$(a_i, c_{-i}) >_j (b_i, c_{-i}) \text{ if and only if } (a_i, d_{-i}) >_j (b_i, d_{-i})$$



Sequential voting [Lang IJCAI-07]



- Given
  - an order over issues, w.l.o.g. $x_1 \rightarrow \ldots \rightarrow x_p$
  - p local rules $r_1,\ldots,r_p$
    - r_j is a social choice mechanism for x_j

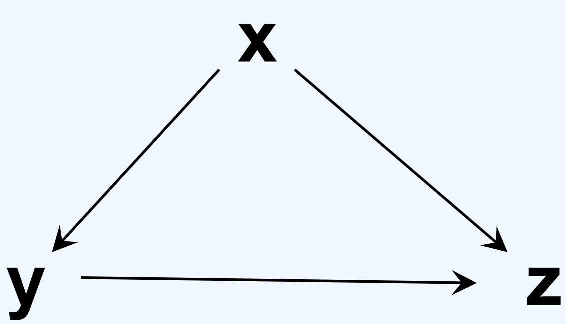
Seems better, but

- Practically: hard to have all agents vote for p times
- Theoretically: How to formally analyze this process?
  - are agents more comfortable?
  - any multiple-election paradoxes?
  - axiomatic properties?

Preference representation: CP-nets

[Boutilier et al. JAIR-04]

Variables: x,y,z. $D_x = \{x, \bar{x}\}, D_y = \{y, \bar{y}\}, D_z = \{z, \bar{z}\}$.



Graph

$CPT(\mathbf{x})$: $x \succ \bar{x}$

$CPT(\mathbf{y})$:
$x : y \succ \bar{y}$
$\bar{x} : \bar{y} \succ y$

$CPT(\mathbf{z})$:
$xy : z \succ \bar{z}$
$x\bar{y} : z \succ \bar{z}$
$\bar{x}y : z \succ \bar{z}$
$\bar{x}\bar{y} : \bar{z} \succ z$

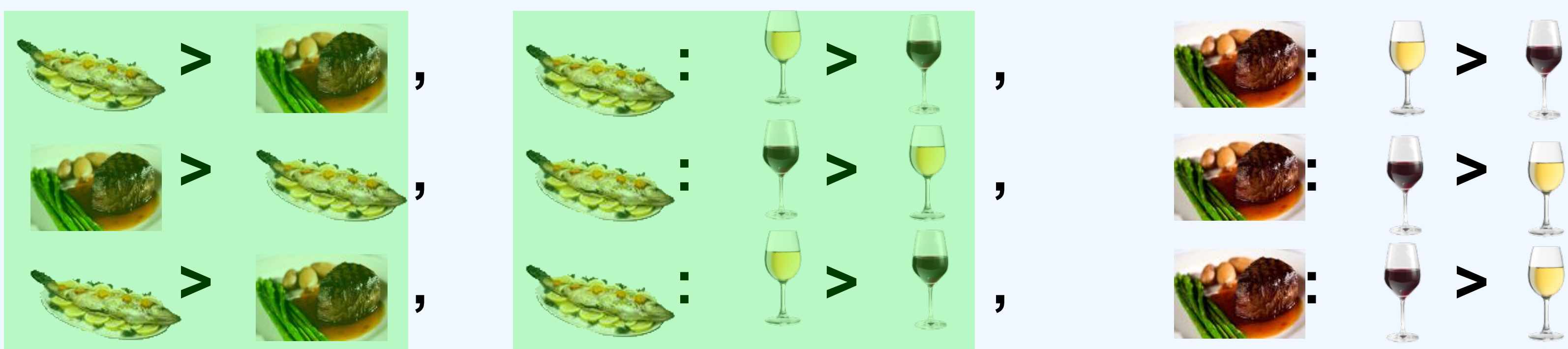
CPTs

This CP-net encodes the following partial order:

$$xyz \rightarrow \{x\bar{y}z,\ xy\bar{z}\} \rightarrow x\bar{y}\bar{z} \rightarrow \bar{x}\bar{y}\bar{z} \rightarrow \bar{x}\bar{y}z \rightarrow \bar{x}yz \rightarrow \bar{x}y\bar{z}$$

Sequential voting under CP-nets

- Issues: main course, wine
- Order: main course > wine
  - agents' CP-nets are compatible with this order
- Local rules are majority rules
- V_1:
- V_2:
- V_3:



- Step 1: 
- Step 2: given  ,  is the winner for wine
- Winner: ( , )

Computational aspects of sequential voting

- More flexible
  - separable preferences are a special case (CP-nets with no edges)
- Language
  - CP-nets
  - CPT for x_i: $2^{\text{\#parents of } x_i} |D_i| \log |D_i|$
  - Total: $\Sigma_i 2^{\text{\#parents of } x_i} |D_i| \log |D_i|$
- Low-high communication
- Fast computation

Social choice aspects of sequential voting

- Representation
  - agents feel more comfortable than using issue-by-issue voting
- Easier to analyze
- Outcome is sometimes very bad, but better than issue-by-issue voting
  - multiple-election paradoxes when agents' preferences are represented by CP-nets compatible with the same order
    - winner ranked almost in the bottom
    - winner is not Pareto optimal
- No sequential voting rule satisfies neutrality or Pareto efficient [Xia&Lang IJCAI-09]
  - If the domain is not composed of two binary issues
  - Strategic behavior: next

Other social choice axioms?

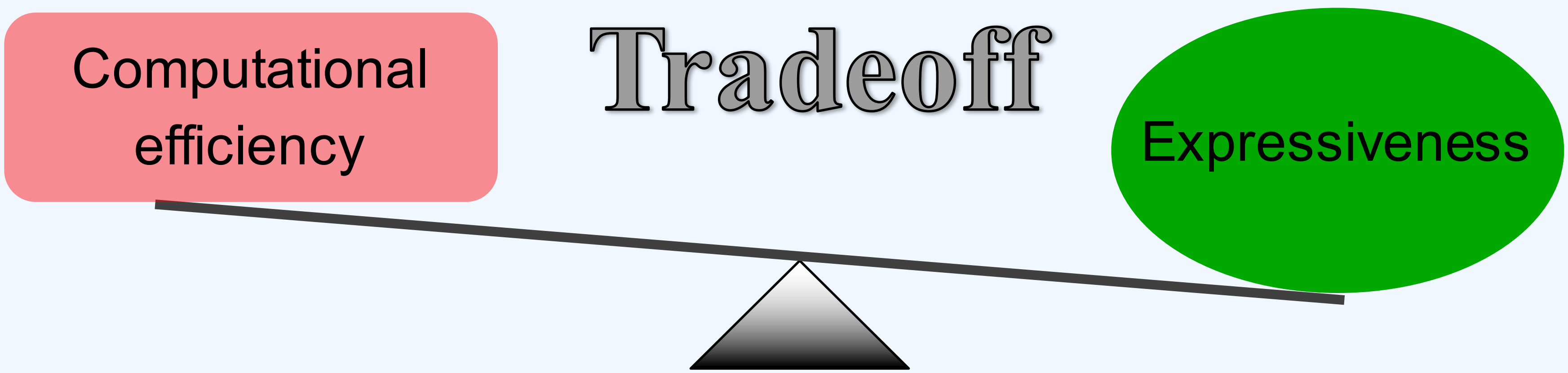
- Depends on whether "local" rules satisfy the property [LX MSS-09, CLX IJCAI-11]
  - E.g., the sequential rule satisfies anonymity ⇔ all local rules satisfy anonymity

Axiom	Global to local	Local to global
Anonymity	Y	Y
Monotonicity	Only last local rule	Only last local rule
Consistency	Y	Y
Participation	Y	N
Strong monotonicity	Y	Y

- Other axioms: open

Bottom line

- Design the language for your application
  - other languages: GAI networks, soft constraints, TCP nets
    - cf combinatorial auctions
  - coding theory may help



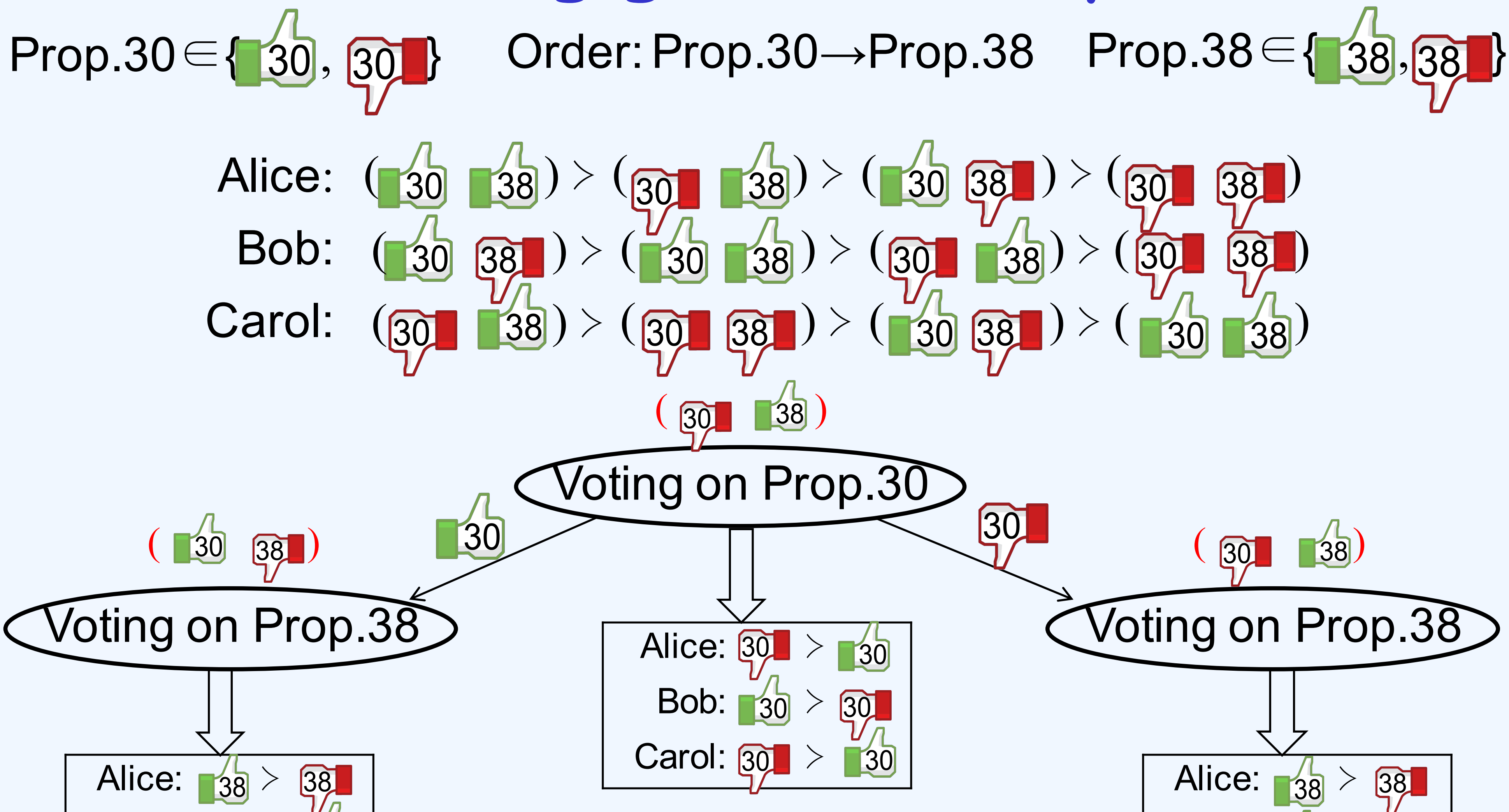
Strategic agents

- Do we need to worry about agents' strategic behavior?
  - Manipulation, bribery, agenda control…
- Evaluate the effect of strategic behavior
  - Game theory
  - Price of anarchy [KP STACS-99]

$$\frac{\text{Optimal truthful social welfare}}{\text{Social welfare in the worst equilibrium}}$$

  - Social welfare is not defined for ordinal cases [AD SIGecom Exchange-10]

Analyzing strategic sequential voting using game theory



Backward induction

Majority rule is strategy-proof

Game of strategic sequential voting (SSP) [XCL EC-11]

- k binary issues
- Agents vote simultaneously on issues, one issue after another
- For each issue, the majority rule is used to determine the value
- Complete information
- Observation. SSP (backward induction) winner is unique

Strategic behavior is extremely harmful in the worst case

- Theorem [xCL EC-11]. For any $p \geq 2$ and any $n \geq 3$, there exists a situation such that
  - for every order over issues,
  - the SSP winner is ranked below the (2^p-2p)th position in every agent's true preferences
- Average case: open

Wrap up

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Next class: the hard-to-manipulate axiom

- So far

- Next class