Computational social choice
The hard-to-manipulate axiom

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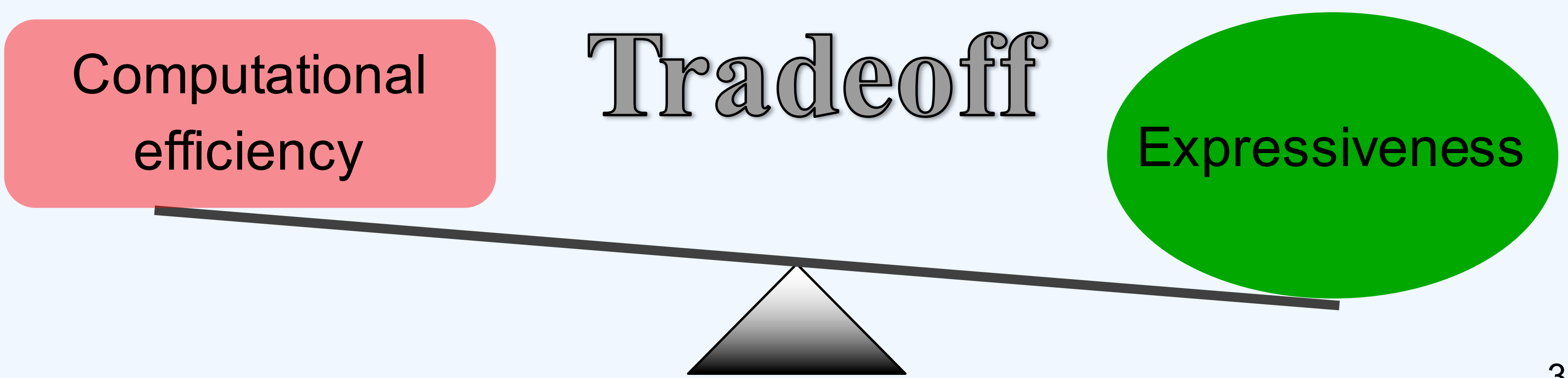
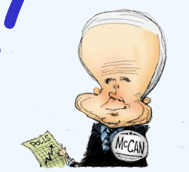
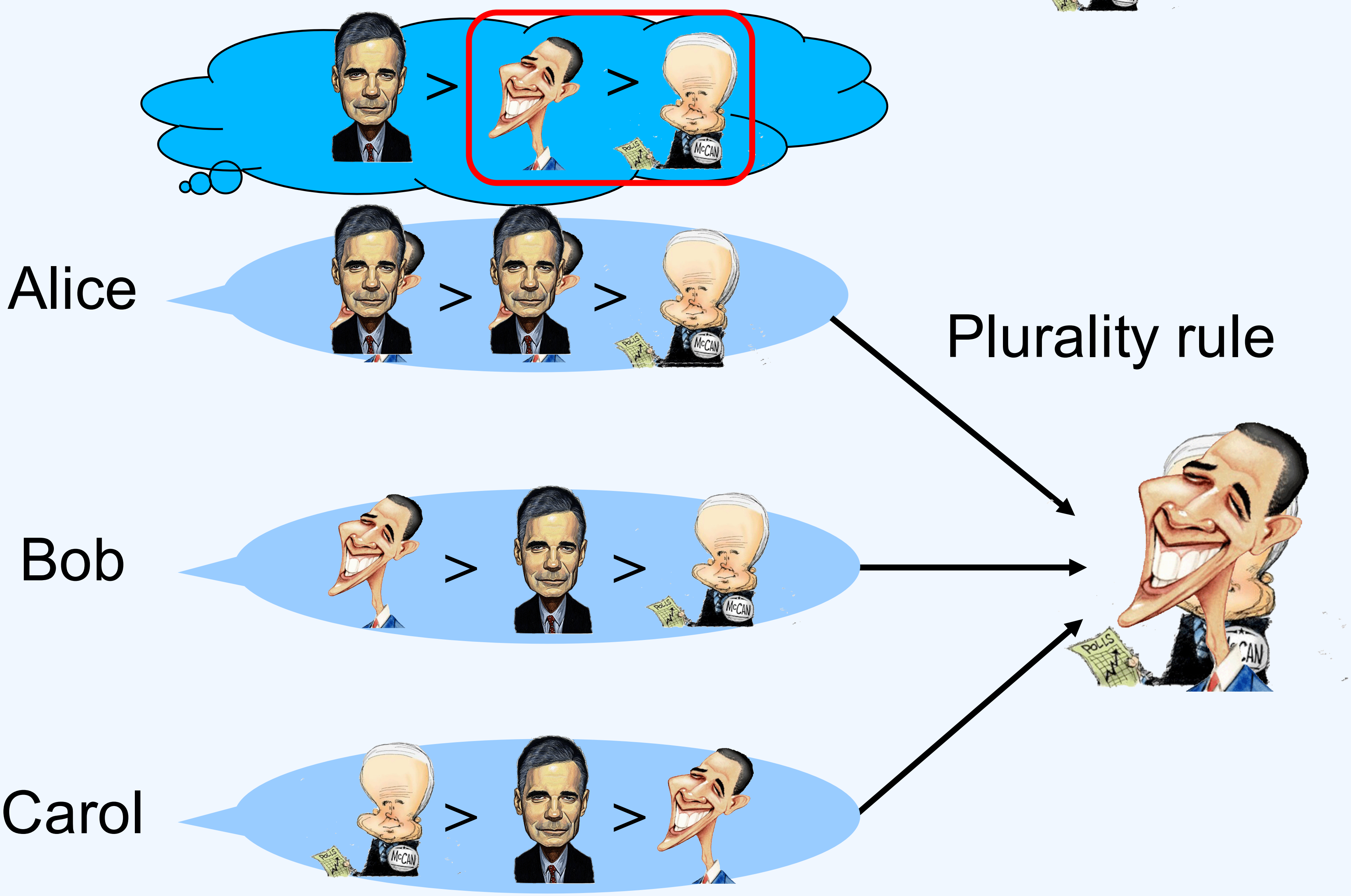
Old news in the world

- Scottish independence referendum
  - 45% yes vs 55% no
- The YouGov survey predicts Scots have rejected independence by a margin of 54% to 46%
  - based on the responses of 1,828 people after they voted, as well as 800 people who had already cast their ballots
  - Peter Kellner of YouGov said: "At risk of looking utterly ridiculous in a few hours time, I would say it's a 99% chance of a No victory."
- Where does this 99% come from?
  - we will learn in the hypothesis testing class

Last class: combinatorial voting



Manipulation under plurality rule (ties are broken in favor of )



Strategic behavior (of the agents)

- Manipulation: an agent (manipulator) casts a vote that does not represent her true preferences, to make herself better off
- A voting rule is strategy-proof if there is never a (beneficial) manipulation under this rule
  - truthful direct revelation mechanism
- Is strategy-proofness compatible with other axioms?

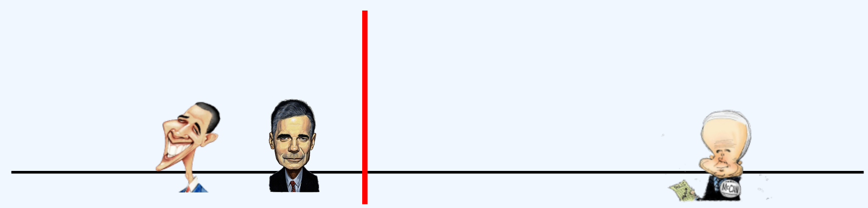
Any strategy-proof voting rule?

No reasonable voting rule is strategyproof

- Gibbard-Satterthwaite Theorem [Gibbard Econometrica-73, Satterthwaite JET-75]: When there are at least three alternatives, no voting rules except dictatorships satisfy
  - non-imposition: every alternative wins for some profile
  - unrestricted domain: voters can use any linear order as their votes
  - strategy-proofness
- Axiomatic characterization for dictatorships!
- Revelation principle: among all voting rules that satisfy non-imposition and unrestricted domain, only dictatorships can be implemented w.r.t. dominant strategy
- Randomized version [Gibbard Econometrica-77]

A few ways out

- Relax non-dictatorship: use a dictatorship
- Restrict the number of alternatives to 2
- Relax unrestricted domain: mainly pursued by economists

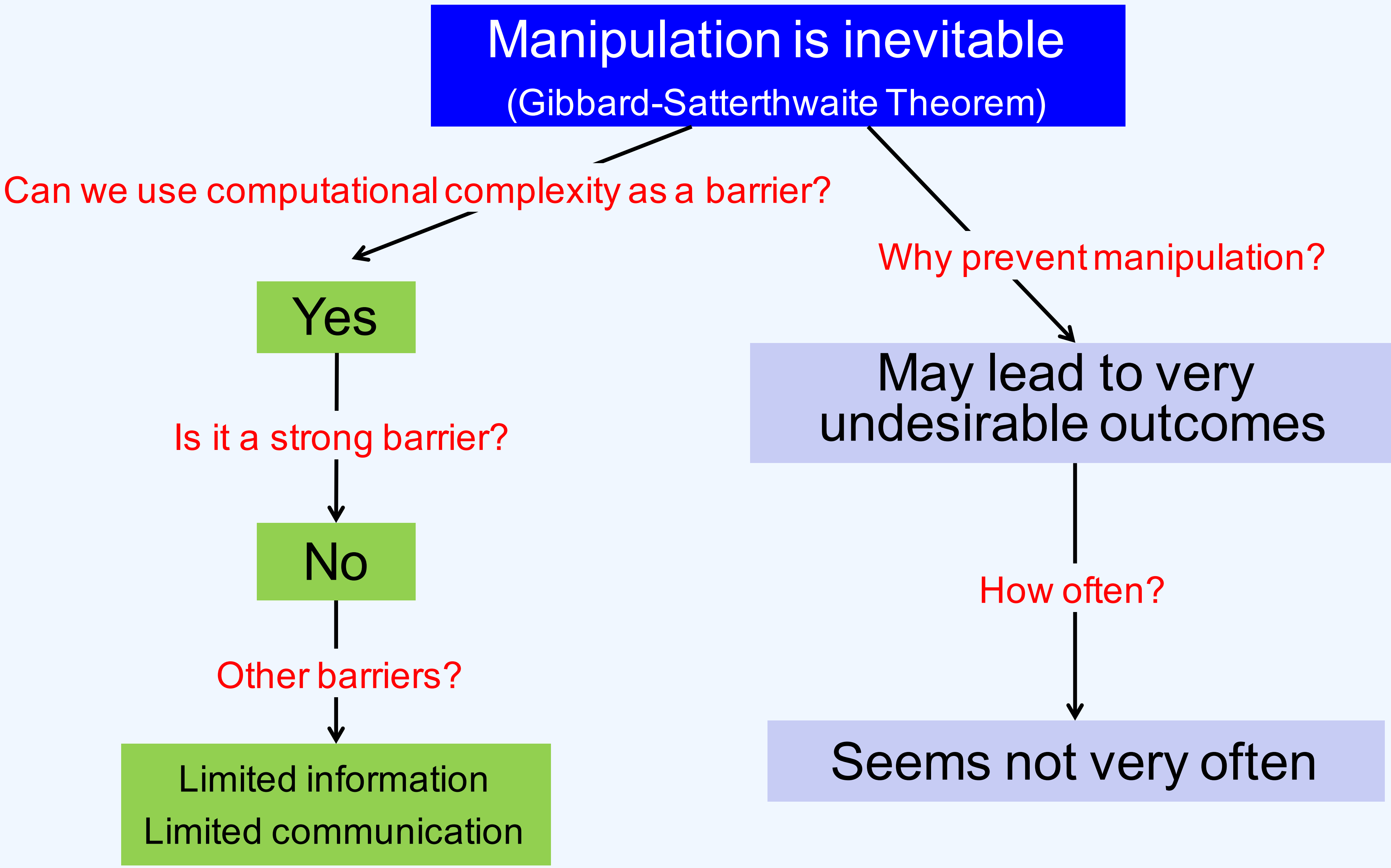


  – Single-peaked preferences:
  – Range voting: A voter submit any natural number between 0 and 10 for each alternative
  – Approval voting: A voter submit 0 or 1 for each alternative

Computational thinking

- Use a voting rule that is too complicated so that nobody can easily predict the winner
  - Dodgson
  - Kemeny
  - The randomized voting rule used in Venice Republic for more than 500 years [Walsh&Xia AAMAS-12]
- We want a voting rule where
  - Winner determination is easy
  - Manipulation is hard
- The hard-to-manipulate axiom: manipulation under the given voting rule is NP-hard

Overview


Manipulation is inevitable
(Gibbard-Satterthwaite Theorem)
Can we use computational complexity as a barrier?
Yes
Is it a strong barrier?
No
Other barriers?
Limited information
Limited communication
Why prevent manipulation?
May lead to very undesirable outcomes
How often?
Seems not very often

Manipulation: A computational complexity perspective

- If it is computationally too hard for a manipulator to compute a manipulation, she is best off voting truthfully
  - Similar as in cryptography

- For which common voting rules manipulation is computationally hard?

Unweighted coalitional manipulation (UCM) problem

- Given
  - The voting rule r
  - The non-manipulators' profile P^{NM}
  - The number of manipulators n'
  - The alternative c preferred by the manipulators
- We are asked whether or not there exists a profile P^{M} (of the manipulators) such that c is the winner of $P^{NM} \cup P^{M}$ under r

The stunningly big table for UCM

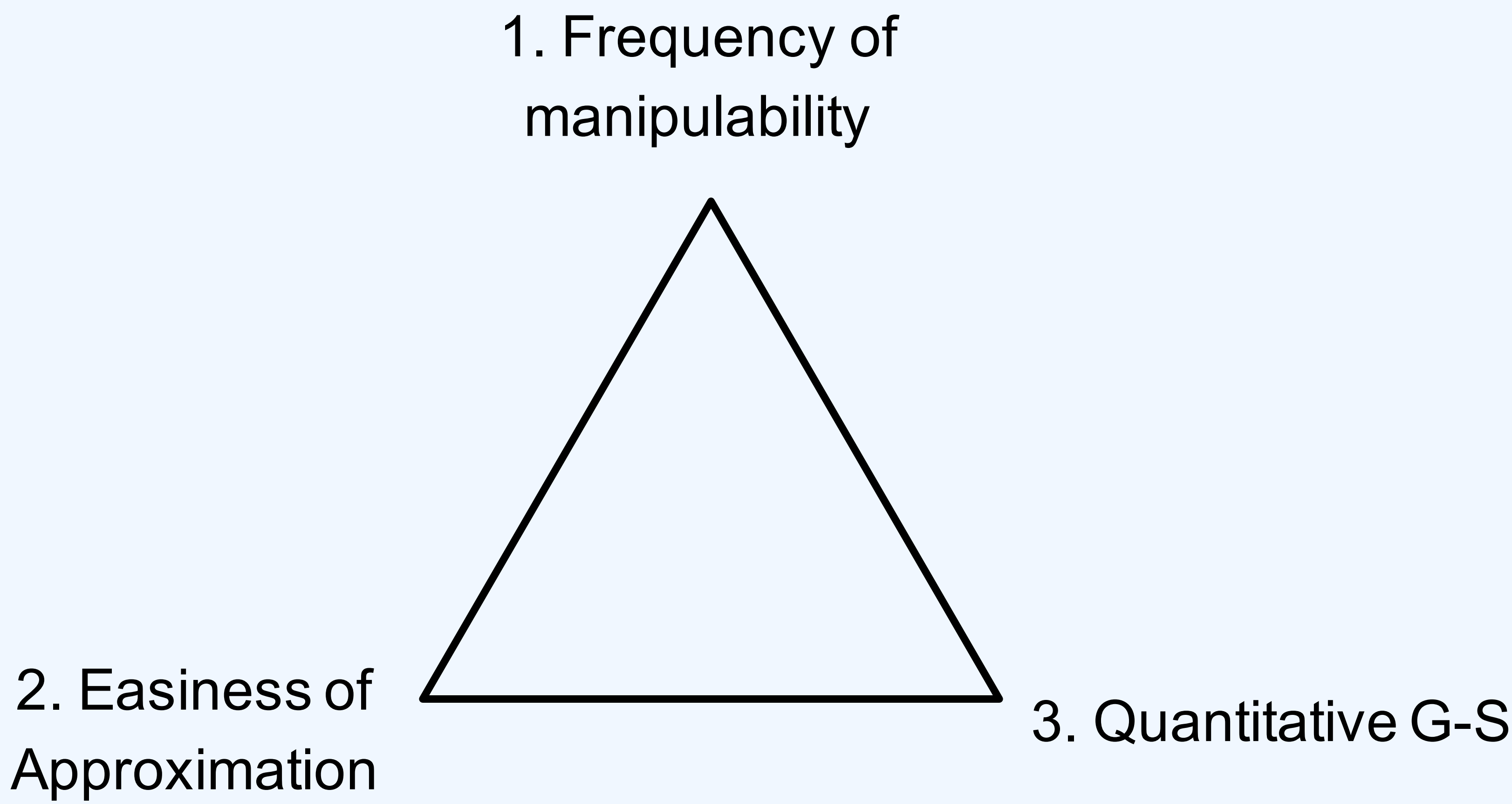
#manipulators	One manipulator		At least two	
Copeland	P	[BTT SCW-89b]	**NPC**	[FHS AAMAS-08,10]
STV	**NPC**	[BO SCW-91]	**NPC**	[BO SCW-91]
Veto	P	[ZPR AIJ-09]	P	[ZPR AIJ-09]
Plurality with runoff	P	[ZPR AIJ-09]	P	[ZPR AIJ-09]
Cup	P	[CSL JACM-07]	P	[CSL JACM-07]
Borda	P	[BTT SCW-89b]	**NPC**	[DKN+ AAAI-11] [BNW IJCAI-11]
Maximin	P	[BTT SCW-89b]	**NPC**	[XZP+ IJCAI-09]
Ranked pairs	**NPC**	[XZP+ IJCAI-09]	**NPC**	[XZP+ IJCAI-09]
Bucklin	P	[XZP+ IJCAI-09]	P	[XZP+ IJCAI-09]
Nanson's rule	**NPC**	[NWX AAA-11]	**NPC**	[NWX AAA-11]
Baldwin's rule	**NPC**	[NWX AAA-11]	**NPC**	[NWX AAA-11]

Nanson & Baldwin in the news

What can we conclude?

- For some common voting rules, computational complexity provides some protection against manipulation
- Is computational complexity a strong barrier?
  - NP-hardness is a worst-case concept

Probably NOT a strong barrier



A first angle: frequency of manipulability

- Non-manipulators' votes are drawn i.i.d.
  - E.g. i.i.d. uniformly over all linear orders (the impartial culture assumption)
- How often can the manipulators make c win?
  - Specific voting rules [Peleg T&D-79, Baharad&Neeman RED-02, Slinko T&D-02, Slinko MSS-04, Procaccia and Rosenschein AAMAS-07]

A general result [Xia&Conitzer EC-08a]

- **Theorem.** For any generalized scoring rule
  - Including many common voting rules

$$\#\text{ manipulators} \quad \frac{\text{All-powerful}}{\text{No power}} \quad \Theta(\sqrt{n})$$

- Computational complexity is not a strong barrier against manipulation
  - UCM as a decision problem is easy to compute in most cases
  - The case of $\Theta(\sqrt{n})$ has been studied experimentally in [Walsh IJCAI-09]

A second angle: approximation

- Unweighted coalitional optimization (UCO): compute the smallest number of manipulators that can make c win
  - A greedy algorithm has additive error no more than 1 for Borda [Zuckerman, Procaccia, &Rosenschein AIJ-09]

An approximation algorithm for positional scoring rules [Xia, Conitzer, & Procaccia EC-10]

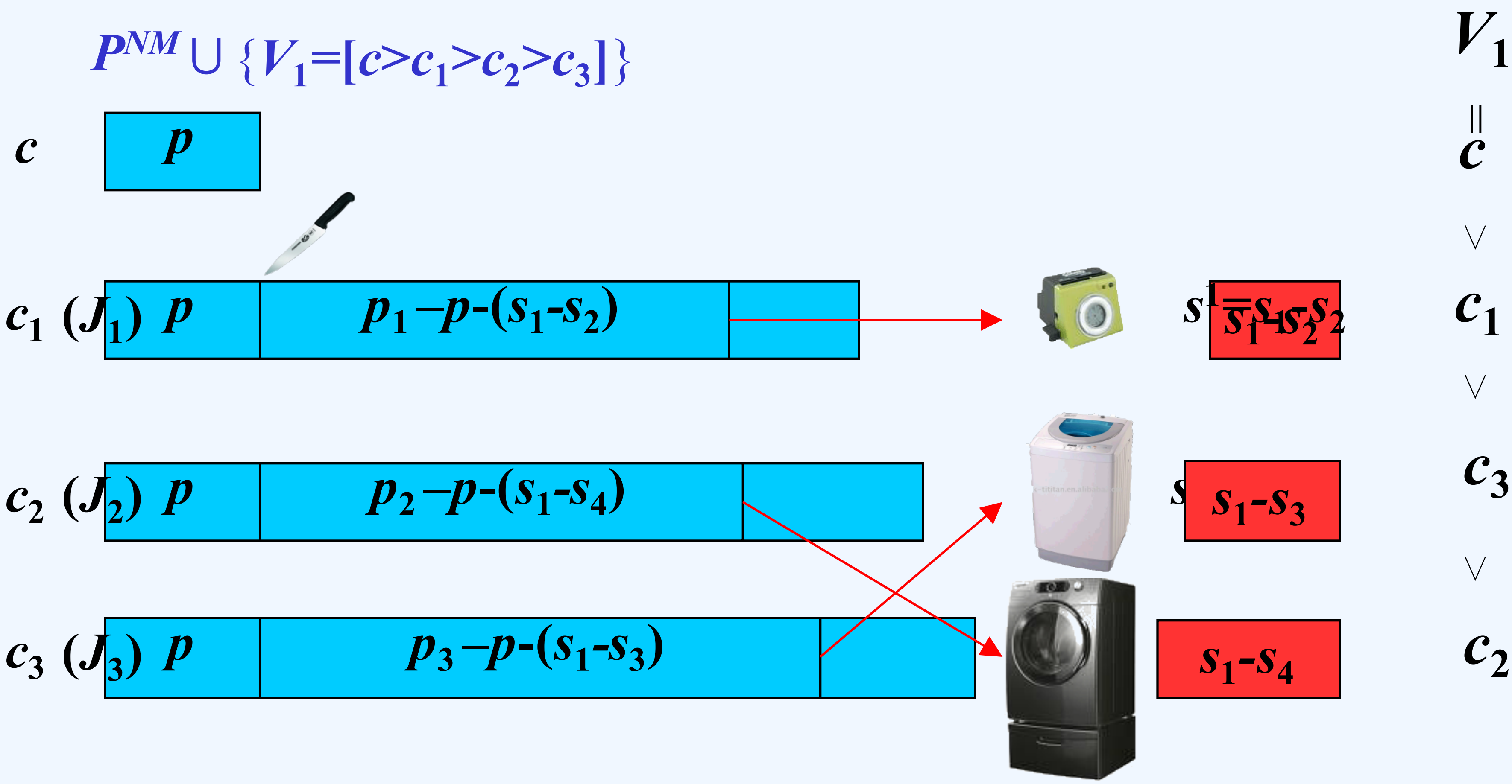
- A polynomial-time approximation algorithm that works for all positional scoring rules
  - Additive error is no more than m-2
  - Based on a new connection between UCO for positional scoring rules and a class of scheduling problems
- Computational complexity is not a strong barrier against manipulation
  - The cost of successful manipulation can be easily approximated (for positional scoring rules)

The scheduling problems $Q|pmtn|C_{max}$

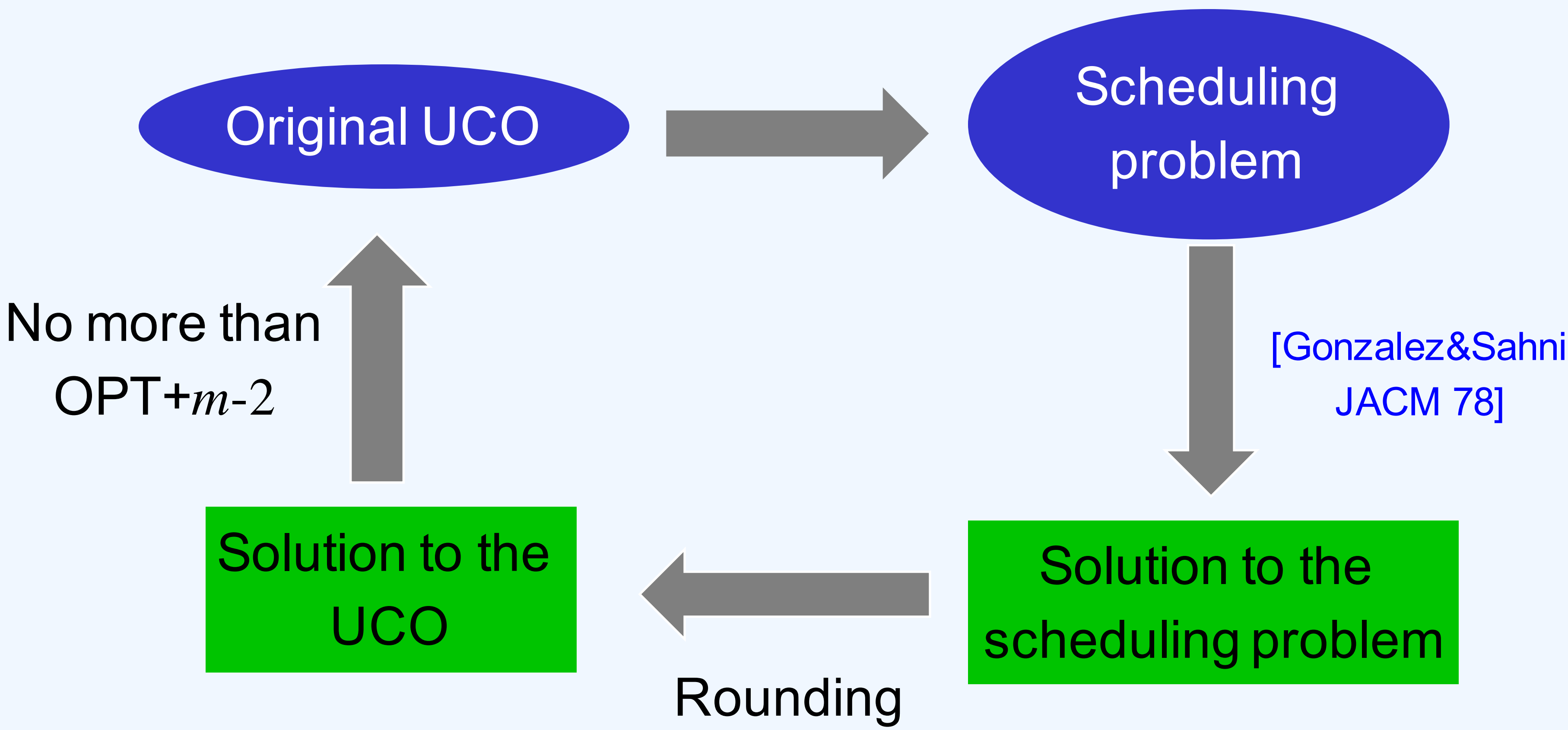
- m^* parallel uniform machines $M_1,\ldots,M_{m^*}$
  – Machine i's speed is s^i (the amount of work done in unit time)
- n^* jobs $J_1,\ldots,J_{n^*}$
- preemption: jobs are allowed to be interrupted (and resume later maybe on another machine)
- We are asked to compute the minimum makespan
  – the minimum time to complete all jobs

Thinking about UCO_{pos}

- Let $p, p_1, \ldots, p_{m\text{-}1}$ be the total points that $c, c_1, \ldots, c_{m\text{-}1}$ obtain in the non-manipulators' profile



The approximation algorithm



Complexity of UCM for Borda

- Manipulation of positional scoring rules = scheduling (preemptions at integer time points)
  - Borda manipulation corresponds to scheduling where the machines speeds are m-1, m-2, …, 0
    - NP-hard [Yu, Hoogeveen, & Lenstra J.Scheduling 2004]
  - UCM for Borda is NP-C for two manipulators
    - [Davies et al. AAAI-11 best paper]
    - [Betzler, Niedermeier, & Woeginger IJCAI-11 best paper]

A third angle: quantitative G-S

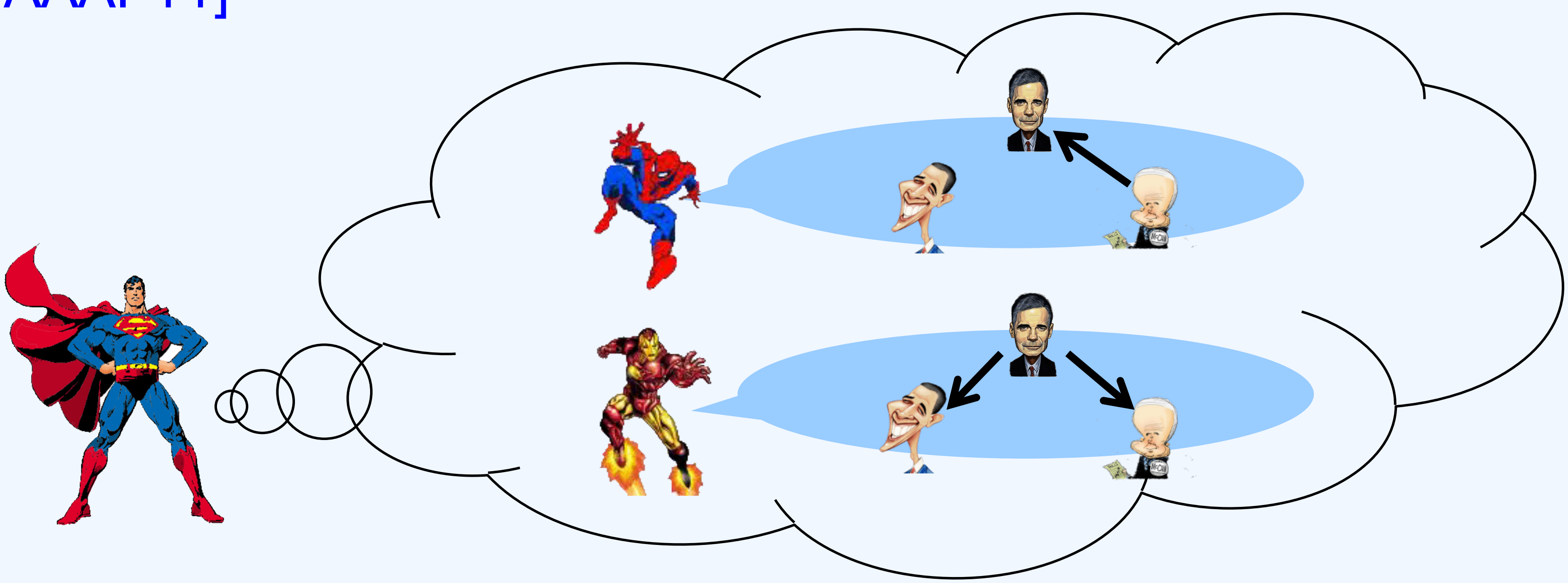
- G-S theorem: for any reasonable voting rule there exists a manipulation
- Quantitative G-S: for any voting rule that is “far away” from dictatorships, the number of manipulable situations is non-negligible
  - First work: 3 alternatives, neutral rule [Friedgut, Kalai, &Nisan FOCS-08]
  - Extensions: [Dobzinski&Procaccia WINE-08, Xia&Conitzer EC-08b, Isaksson,Kindler,&Mossel FOCS-10]
  - Finally proved: [Mossel&Racz STOC-12]

Next steps

- The first attempt seems to fail
- Can we obtain positive results for a restricted setting?
  - The manipulators has complete information about the non-manipulators' votes
  - The manipulators can perfectly discuss their strategies

Limited information

- Limiting the manipulator's information can make dominating manipulation computationally harder, or even impossible [Conitzer,Walsh,&Xia AAAI-11]



- Bayesian information [Lu et al. UAI-12]

Limited communication among manipulators

- The leader-follower model
  - The leader broadcast a vote W, and the potential followers decide whether to cast W or not
    - The leader and followers have the same preferences
  - Safe manipulation [Slinko&White COMSOC-08]: a vote W that
    - No matter how many followers there are, the leader/potential followers are not worse off
    - Sometimes they are better off
  - Complexity: [Hazon&Elkind SAGT-10, Ianovski et al. IJCAI-11]

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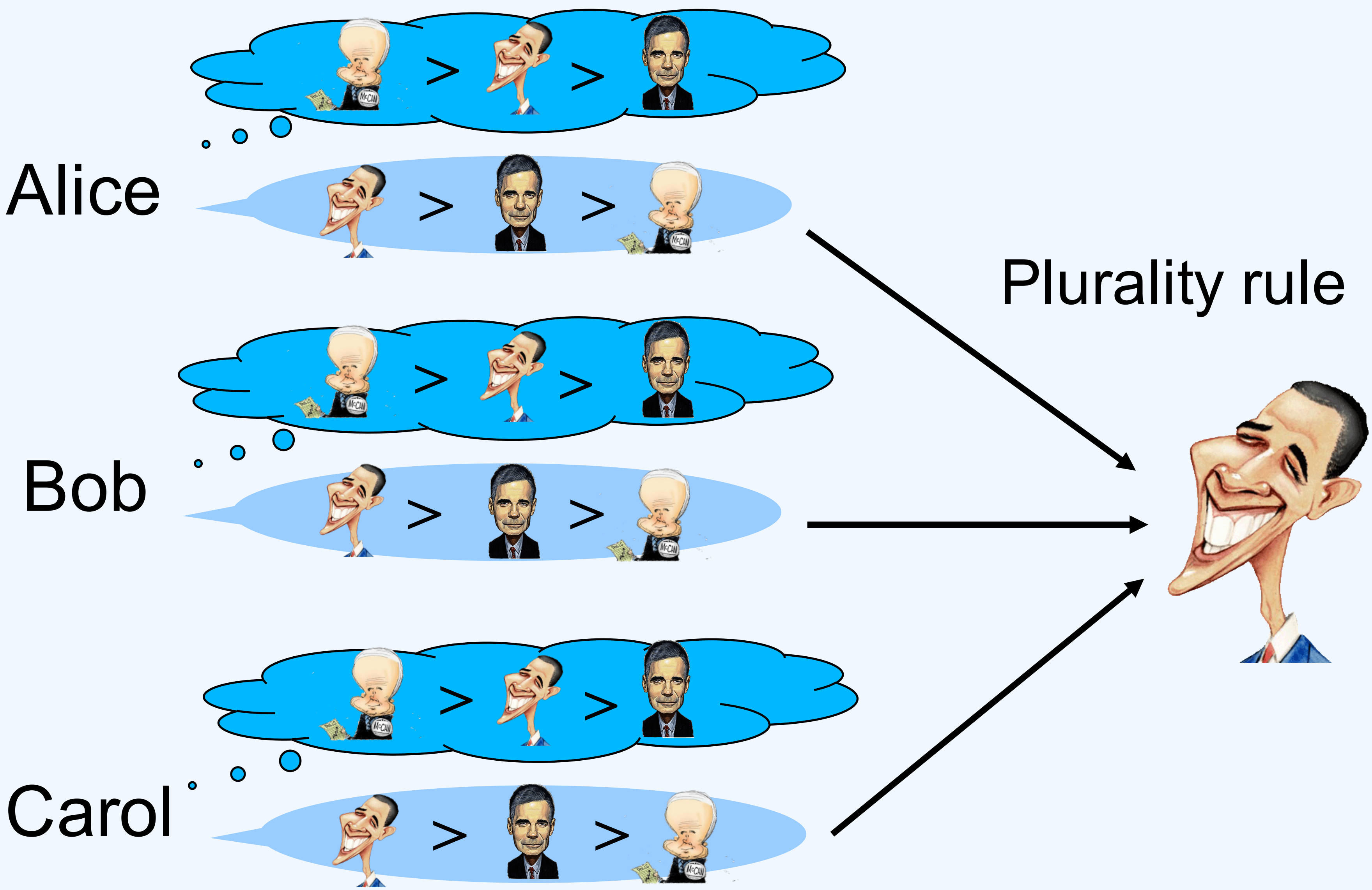
Research questions

- How to predict the outcome?
  - Game theory
- How to evaluate the outcome?
- Price of anarchy [Koutsoupias&Papadimitriou STACS-99]
  - $\dfrac{\text{Optimal welfare when agents are truthful}}{\text{Worst welfare when agents are fully strategic}}$
  - Not very applicable in the social choice setting
    - Equilibrium selection problem
    - Social welfare is not well defined
    - Use best-response game to select an equilibrium and use scores as social welfare [Brânzei et al. AAAI-13]

Simultaneous-move voting games

- Players: Voters $1,\ldots,n$
- Strategies / reports: Linear orders over alternatives
- Preferences: Linear orders over alternatives
- Rule: $r(P')$, where P' is the reported profile

Equilibrium selection problem



Stackelberg voting games
[Xia&Conitzer AAAI-10]

- Voters vote sequentially and strategically
  - voter $1 \to$ voter $2 \to$ voter $3 \to \ldots \to$ voter n
  - any terminal state is associated with the winner under rule r
- Called a Stackelberg voting game
  - Unique winner in SPNE (not unique SPNE)
  - Similar setting in [Desmedt&Elkind EC-10]

Other types of strategic behavior (of the chairperson)

- Procedure control by
  - {adding, deleting} $\times$ {voters, alternatives}
  - partitioning voters/alternatives
  - introducing clones of alternatives
  - changing the agenda of voting
  - [Bartholdi, Tovey, &Trick MCM-92, Tideman SCW-07, Conitzer,Lang,&Xia IJCAI-09]
- Bribery [Faliszewski, Hemaspaandra, &Hemaspaandra JAIR-09]
- See [Faliszewski, Hemaspaandra, &Hemaspaandra CACM-10] for a survey on their computational complexity
- See [Xia EC-15] for a framework for studying many of these for generalized scoring rules

Next class: statistical approaches

GOAL1: democracy

Axiomatic approaches

GOAL2: truth

Statistical approaches