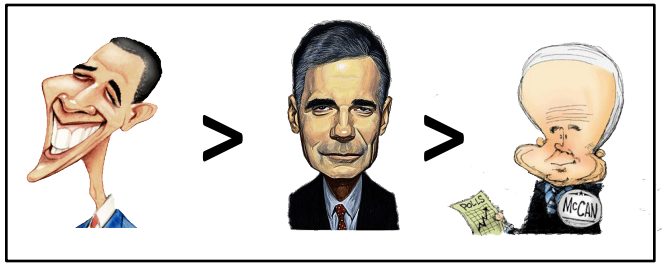
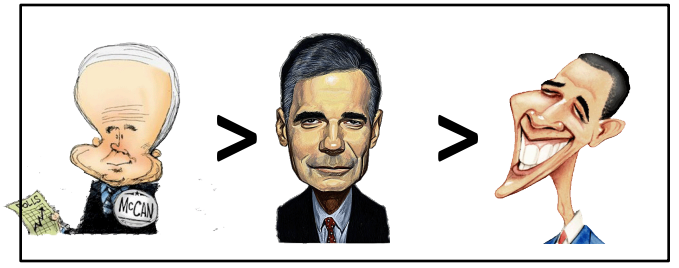
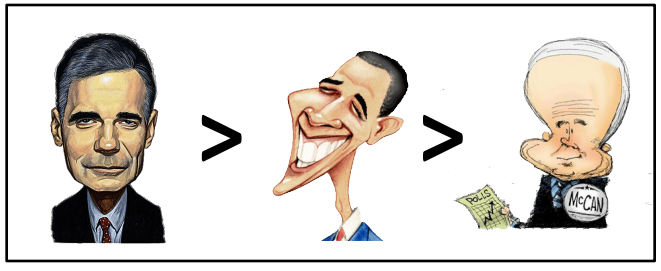
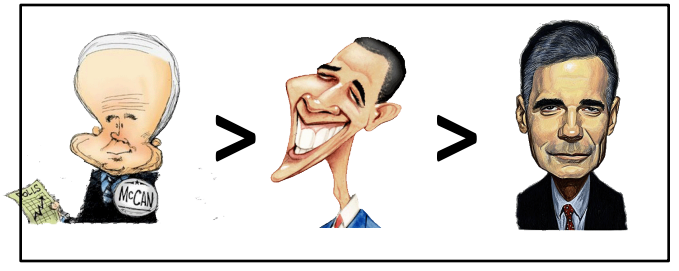
CSCI-4150: Introduction to Artificial Intelligence

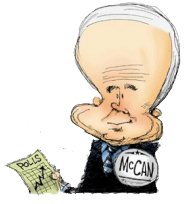
26 March 2018

Borda Rule

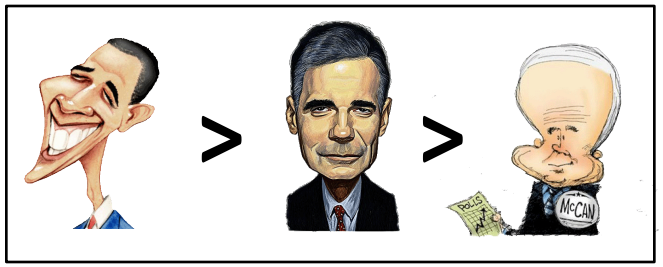
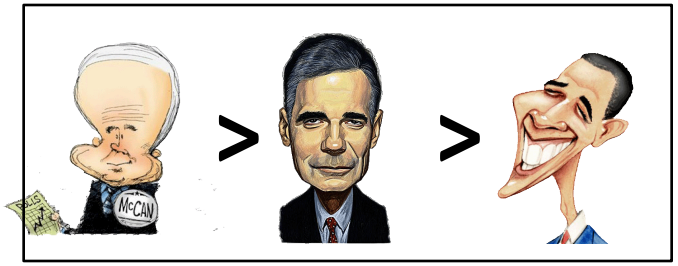
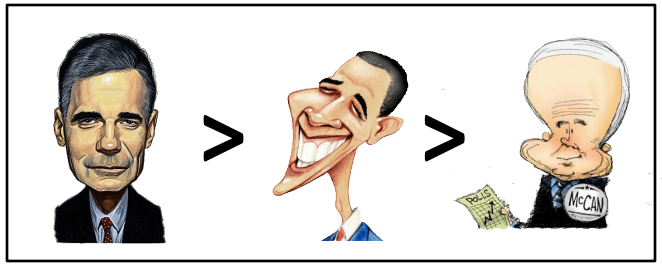
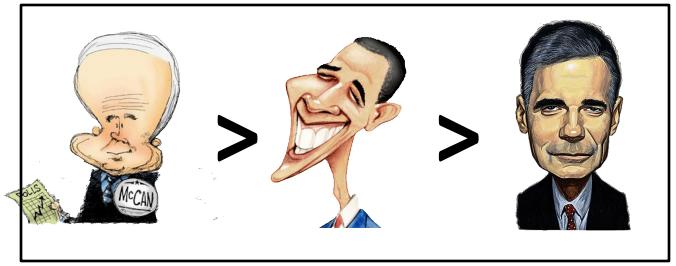
P={  ×4,  ×3,

 ×2,  ×2 }

Borda(P) = 

Borda scores  : 2×4+4=12  : 2*2+7=11  : 2*5=10

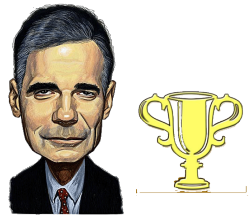
Positional Scoring Rules

$P=\{$  ×4,  ×3,  ×2,  ×2 $\}$

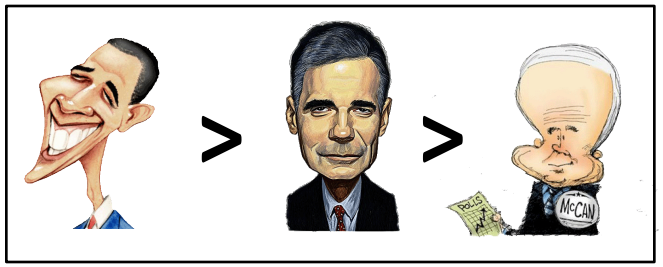
Borda

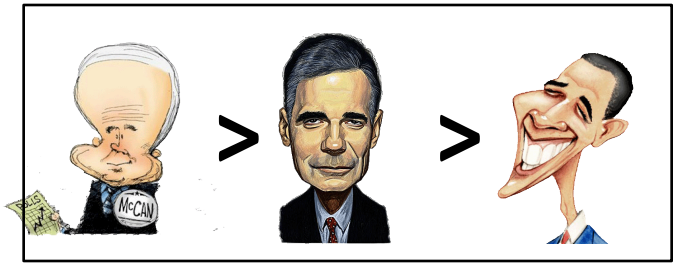
Plurality
(1- approval)

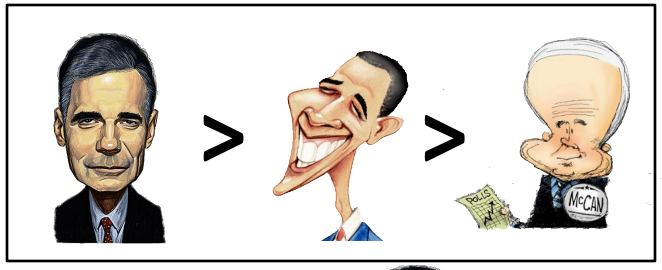
Veto
(2-approval)

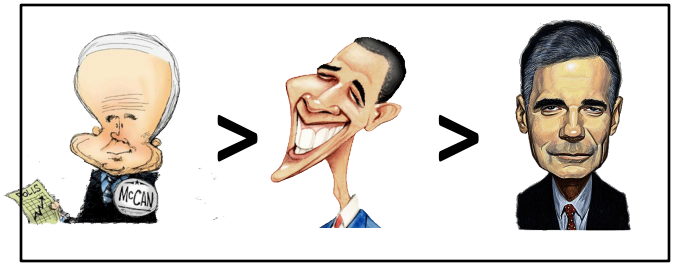


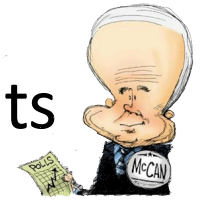
Plurality with Runoff

P={  ×4,

 ×3

 ×2,

 ×2 }

- Round 1:  drops out
- Round 2:  defeats 

Different outcome from Plurality!

Single Transferable Vote

In each round,

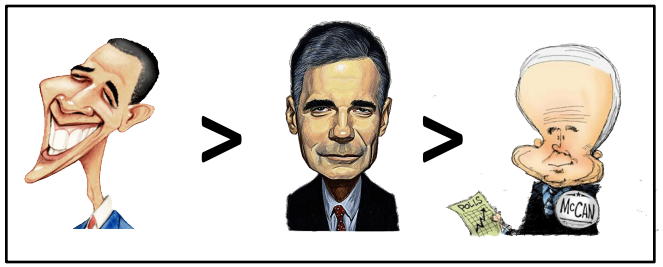
The alternative with the lowest plurality score drops out,

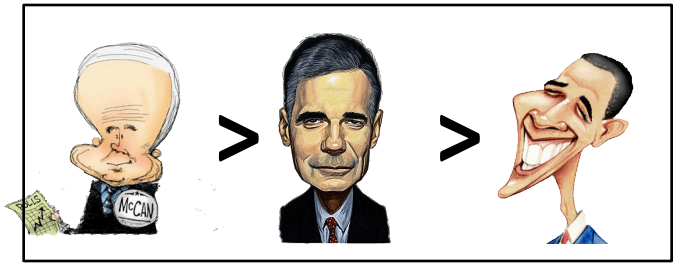
and is removed from all votes

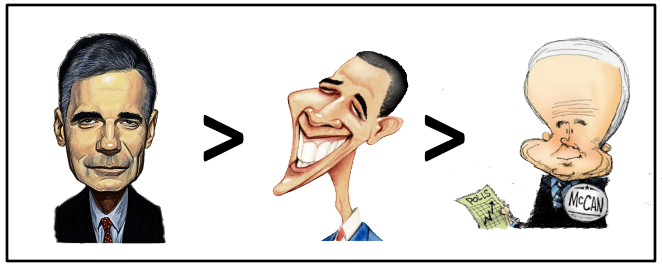
$a > b > c > d$	$d > a > b > c$	$c > d > a > b$	$b > c > d > a$
10	7	6	3

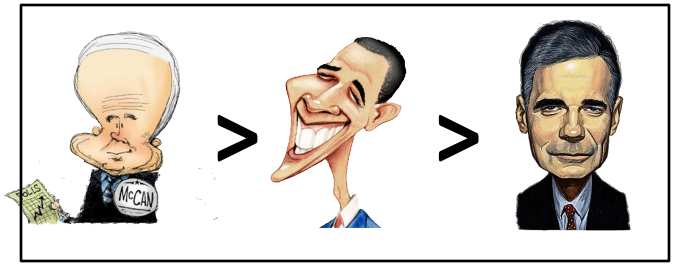
 a

Weighted Majority Graph (WMG)

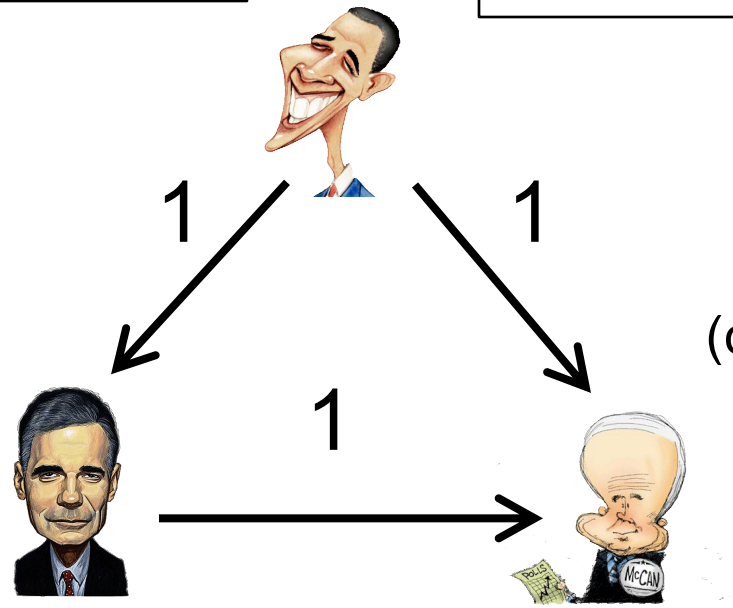
P={  ×4,

 ×3,

 ×2,

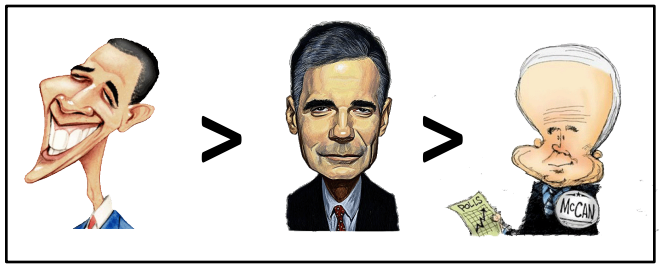
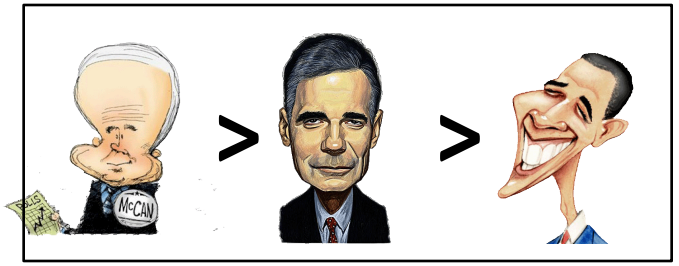
 ×2 }

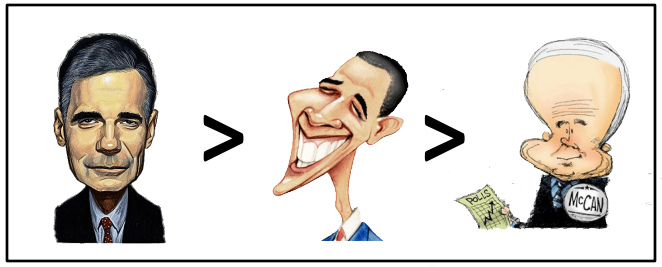
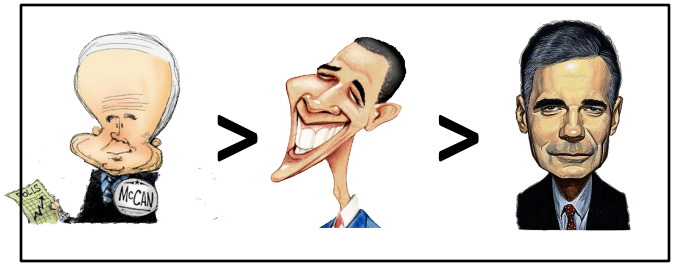
WMG(P) =

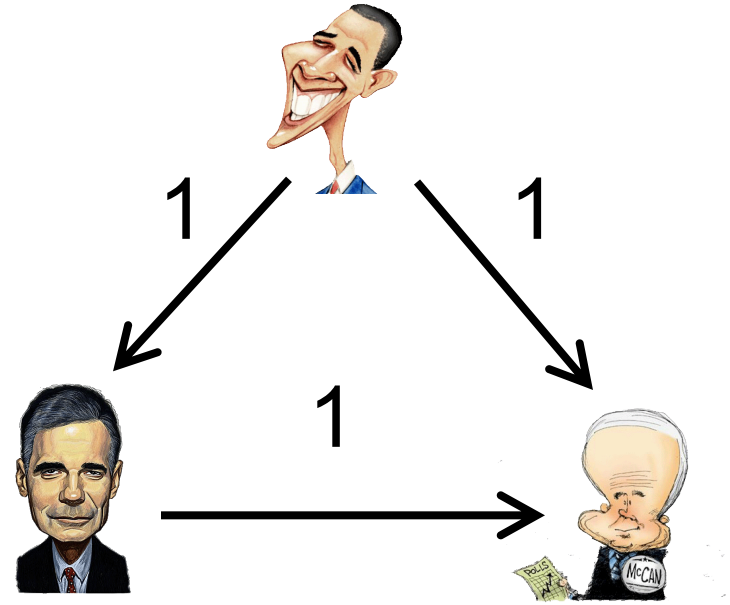


(only showing positive edges)

Copeland Rule

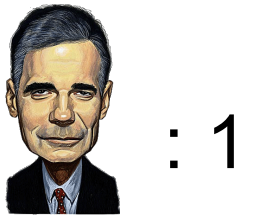
P={  ×4,  ×3,

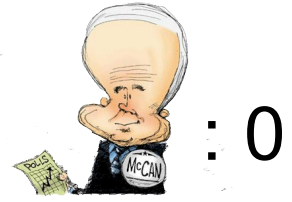
 ×2,  ×2 }

WMG(P) = 

Copeland score:

 : 2

 : 1

 : 0

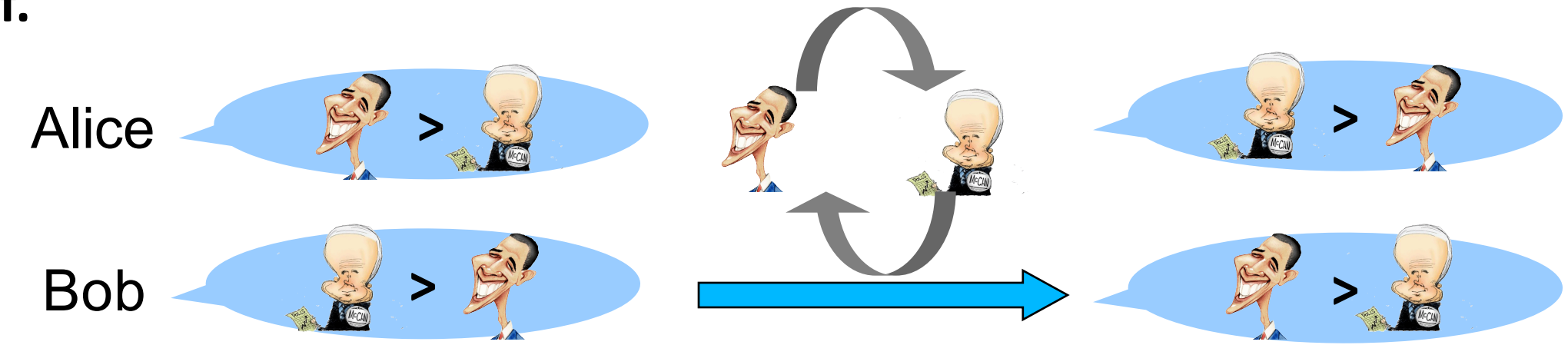
Which axioms? Which voting rule?

	Condorcet criterion	Consistency	Anonymity/neutrality, non-dictatorship, monotonicity
Plurality	N	Y	Y
STV (alternative vote)	Y	N	Y

Some Axioms Are Incompatible with Others

Theorem. For voting rules that select a single winner, anonymity is not compatible with neutrality.

Proof.

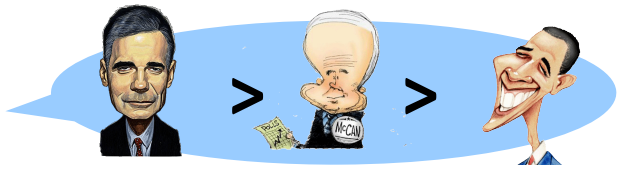


Positional Scoring Rules And Condorcet Consistency

Theorem. No positional scoring rule satisfies Condorcet consistency

Suppose $s_1 > s_2 > s_3$ (scoring vector)

3 Voters 

2 Voters 

1 Voter 

1 Voter 

 is the Condorcet winner

 : $3s_1 + 2s_2 + 2s_3$

^

 : $3s_1 + 3s_2 + 1s_3$

CONTRADICTION

That’s a Lot!

Now try a slightly larger tip of the iceberg at wiki, wiki

Wrap Up

- Voting rules
  - positional scoring rules
  - multi-round elimination rules
  - WMG-based rules
- Criteria (axioms) for “good” rules
  - Fairness axioms
  - Other axioms
- Evaluation
  - Impossibility theorems
  - Axiomatic characterization