Introduction to Game Theory

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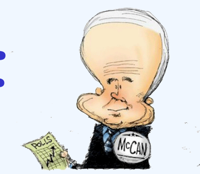
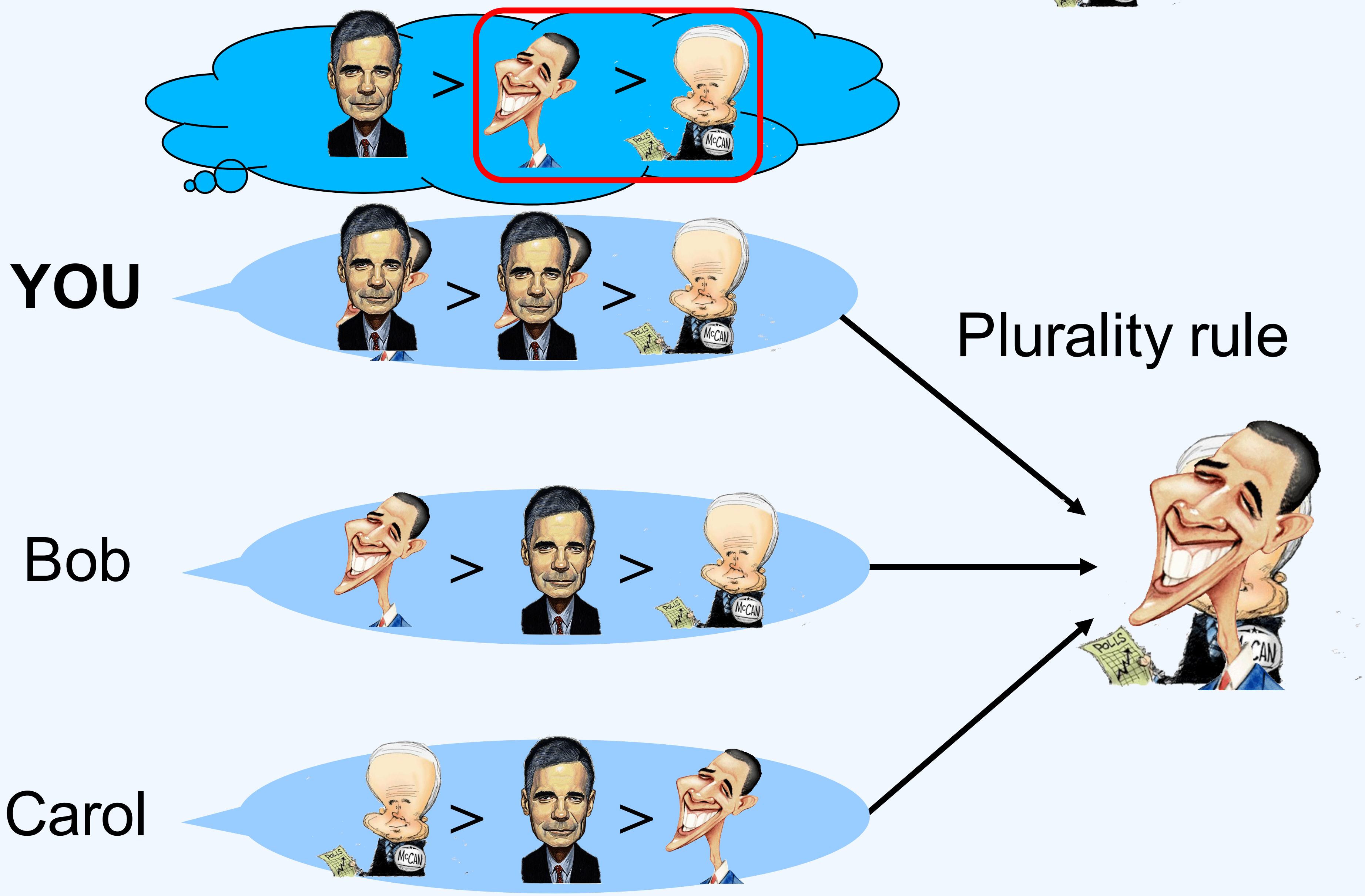
Rensselaer

Today: Game theory

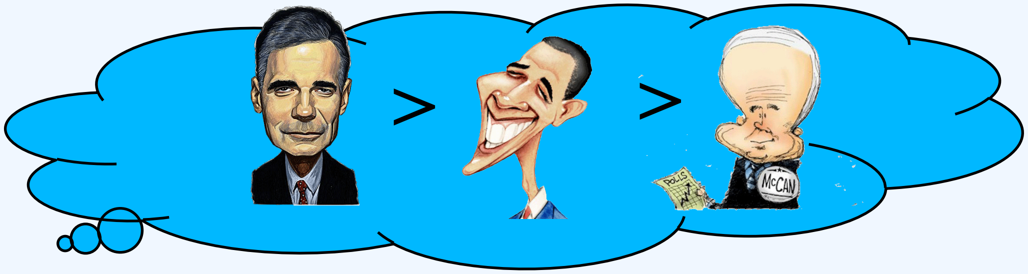
- Related grad-level courses at RPI
  - Algorithmic Game Theory (Prof. Elliot Anshelevich)
  - Computational Social Choice (me, Spring 2019)

plurality rule

(ties are broken in favor of )

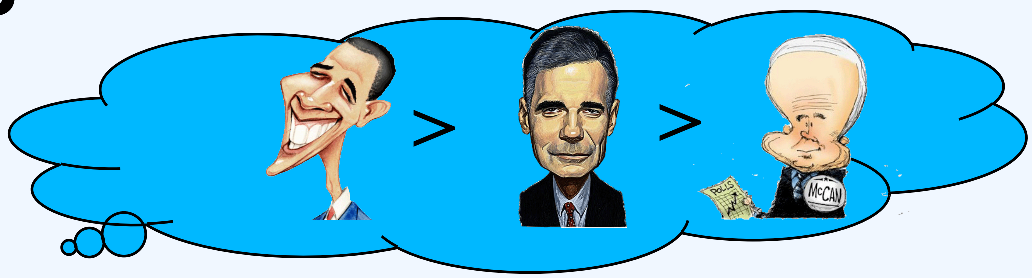


What if everyone is incentivized to lie?

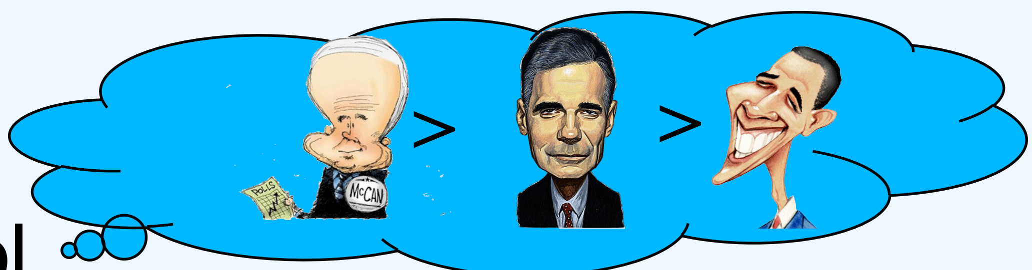


YOU

Plurality rule



Bob

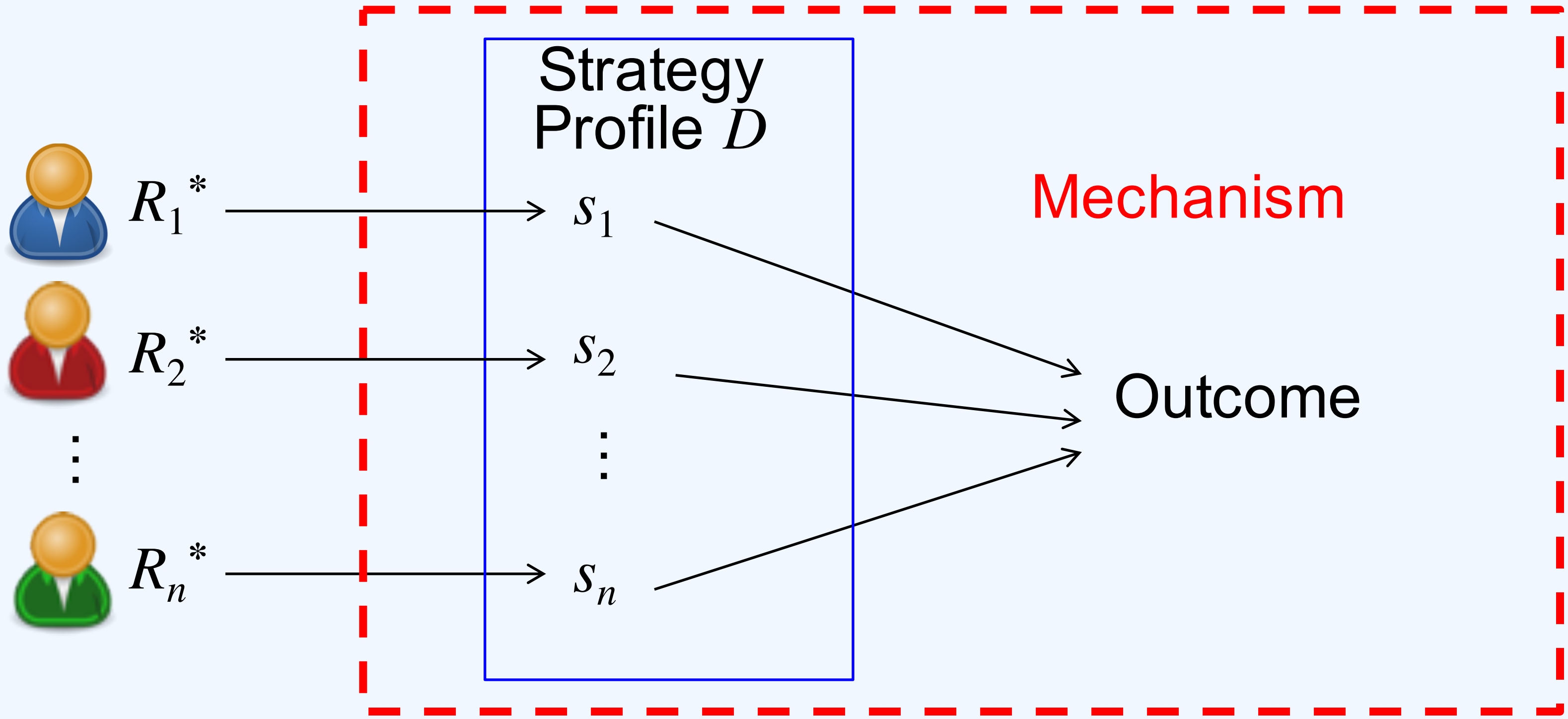


Carol

High-level questions

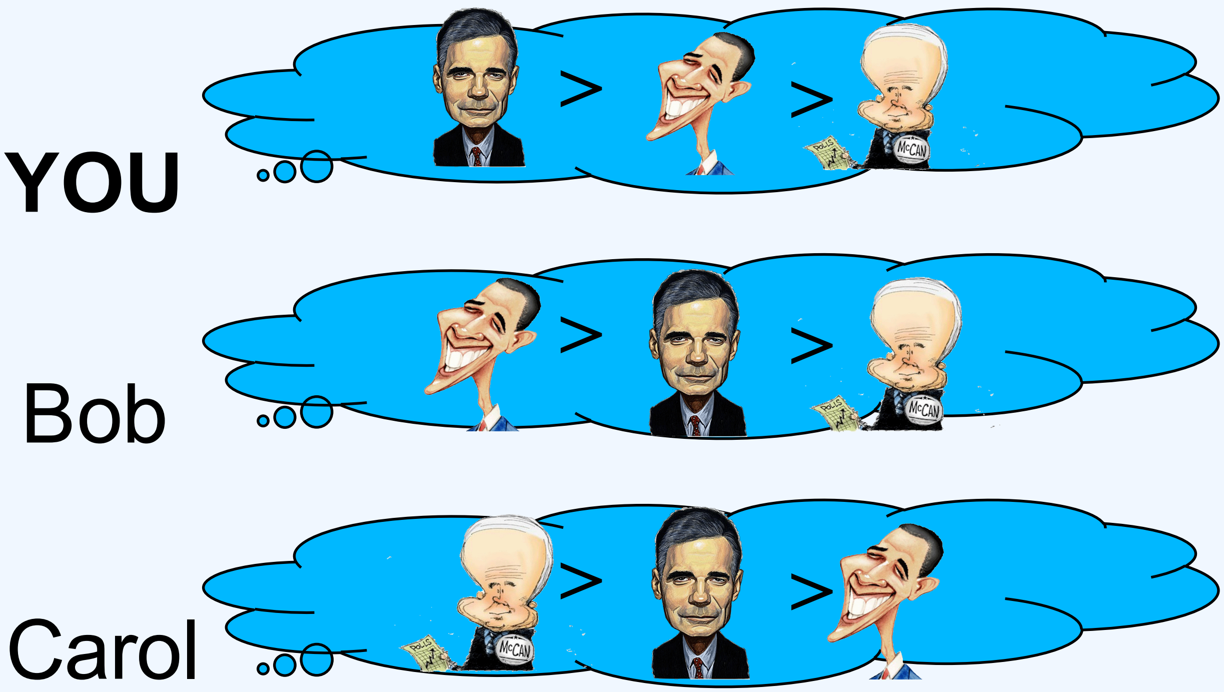
- What?
  - Agents may have incentives to lie
- Why?
  - Hard to predict the outcome when agents lie
- How?
  - A general framework for games
    - Solution concept: Nash equilibrium
  - Modeling preferences and behavior: utility theory
  - Special games
    - Normal form games: mixed Nash equilibrium
    - Extensive form games: subgame-perfect equilibrium

A game

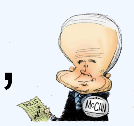


- Players: $N=\{1,\ldots,n\}$
- Strategies (actions):
  - S_j for agent j, $s_j \in S_j$
  - $(s_1,\ldots,s_n)$ is called a strategy profile.
- Outcomes: O
- Preferences: total preorders (full rankings with ties) over O
- Mechanism $f: \Pi_j S_j \rightarrow O$

A game of plurality elections



Plurality rule

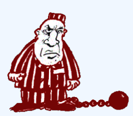
- Players: { YOU, Bob, Carol }
- Outcomes: O = {  ,  ,  }
- Strategies: S_j = Rankings(O)
- Preferences: See above
- Mechanism: the plurality rule

A game of two prisoners

Column player

Row player

	Cooperate	Defect
Cooperate	(-1 , -1)	(-3 , 0)
Defect	(0 , -3)	(-2 , -2)

- Players:  
- Strategies: { Cooperate, Defect }
- Outcomes: {(-2 , -2), (-3 , 0), (0 , -3), (-1 , -1)}
- Preferences: self-interested 0 > -1 > -2 > -3
  -  : (0 , -3) > (-1 , -1) > (-2 , -2) > (-3 , 0)
  -  : (-3 , 0) > (-1 , -1) > (-2 , -2) > (0 , -3)
- Mechanism: the table

Solving the game

- Suppose
  - every player wants to make the outcome as preferable (to her) as possible by controlling her own strategic (but not the other players')
- What is the outcome?
  - No one knows for sure
  - A "stable" situation seems reasonable
- A Nash Equilibrium (NE) is a strategy profile $(s_1,\ldots,s_n)$ such that
  - For every player j and every $s_j' \in S_j$,

$$f(s_j, s_{-j}) \geq_j f(s_j', s_{-j})$$

  - $s_{-j} = (s_1,\ldots,s_{j-1}, s_{j+1},\ldots,s_n)$
  - no single player can be better off by deviating

Prisoner's dilemma

Column player

Row player

	Cooperate	Defect
Cooperate	(-1 , -1)	(-3 , 0)
Defect	(0 , -3)	(-2 , -2)

A beautiful mind

- “If everyone competes for the blond, we block each other and no one gets her. So then we all go for her friends. But they give us the cold shoulder, because no one likes to be second choice. Again, no winner. But what if none of us go for the blond. We don’t get in each other’s way, we don’t insult the other girls. That’s the only way we win. That’s the only way we all get [a girl.]”

A beautiful mind: the bar game

Hansen Column player

Nash Row player

	Blond	Another girl
Blond	(0 , 0)	(5 , 1)
Another girl	(1 , 5)	(2 , 2)

- Players: { Nash, Hansen }
- Strategies: { Blond, another girl }
- Outcomes: {(0 , 0), (5 , 1), (1 , 5), (2 , 2)}
- Preferences: high value for self only
- Mechanism: the table

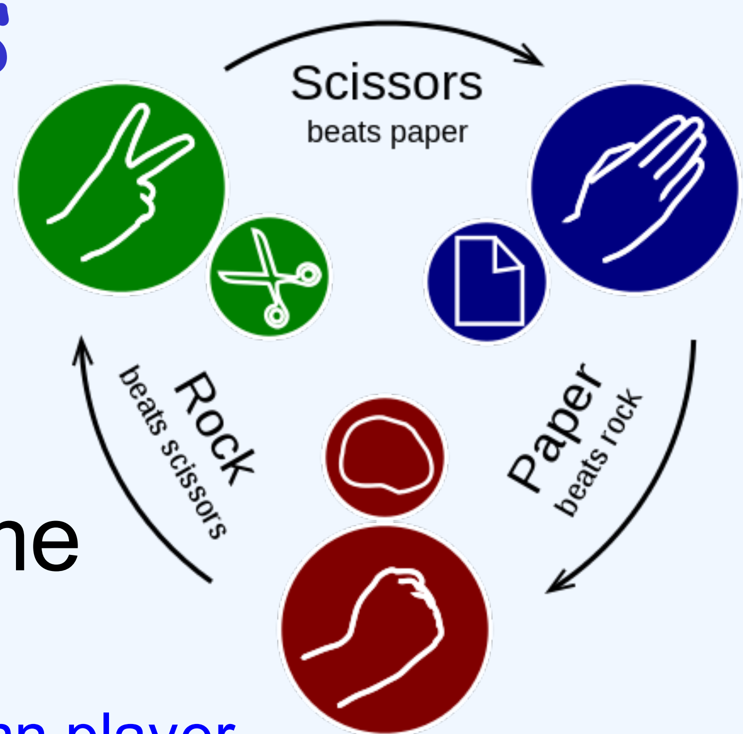
Does an NE always exists?

- Not always

	Column player L	Column player R
Row player U	(0 , 1)	(1 , 0)
Row player D	(1 , 0)	(0 , 1)

- But an NE exists when every player has a dominant strategy
  - s_j is a dominant strategy for player j, if for every $s_j' \in S_j$,
    - for every s_{-j} , $f(s_j, s_{-j}) \geq_j f(s_j', s_{-j})$
    - the preference is strict for some s_{-j}

Rock Paper Scissors



- Actions: {R, P, S}
- Two-player zero sum game

No pure NE

Column player

Row player	R	P	S
R	(0 , 0)	(-1 , 1)	(1 , -1)
P	(1 , -1)	(0 , 0)	(-1 , 1)
S	(1 , -1)	(1 , -1)	(0 , 0)

Dominant-strategy NE

- For player j, strategy s_j dominates strategy s_j', if
  1. for every s_{-j} , $u_j(s_j, s_{-j}) \geq u_j (s_j', s_{-j})$
  2. the preference is strict for some s_{-j}
- Recall that an NE exists when every player has a dominant strategy s_j, if
  – s_j dominates other strategies of the same agent
- A dominant-strategy NE (DSNE) is an NE where
  – every player takes a dominant strategy

Computing NE: Iterated Elimination

- Eliminate dominated strategies sequentially

Column player

Row player

	L	M	R
U	(1 , 0)	(1 , 2)	(0 , 1)
D	(0 , 3)	(0 , 1)	(2 , 0)