Informed search

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Last class

- Search problems
  - state space graph: modeling the problem
  - search tree: scratch paper for solving the problem
- Uninformed search
  - BFS
  - DFS

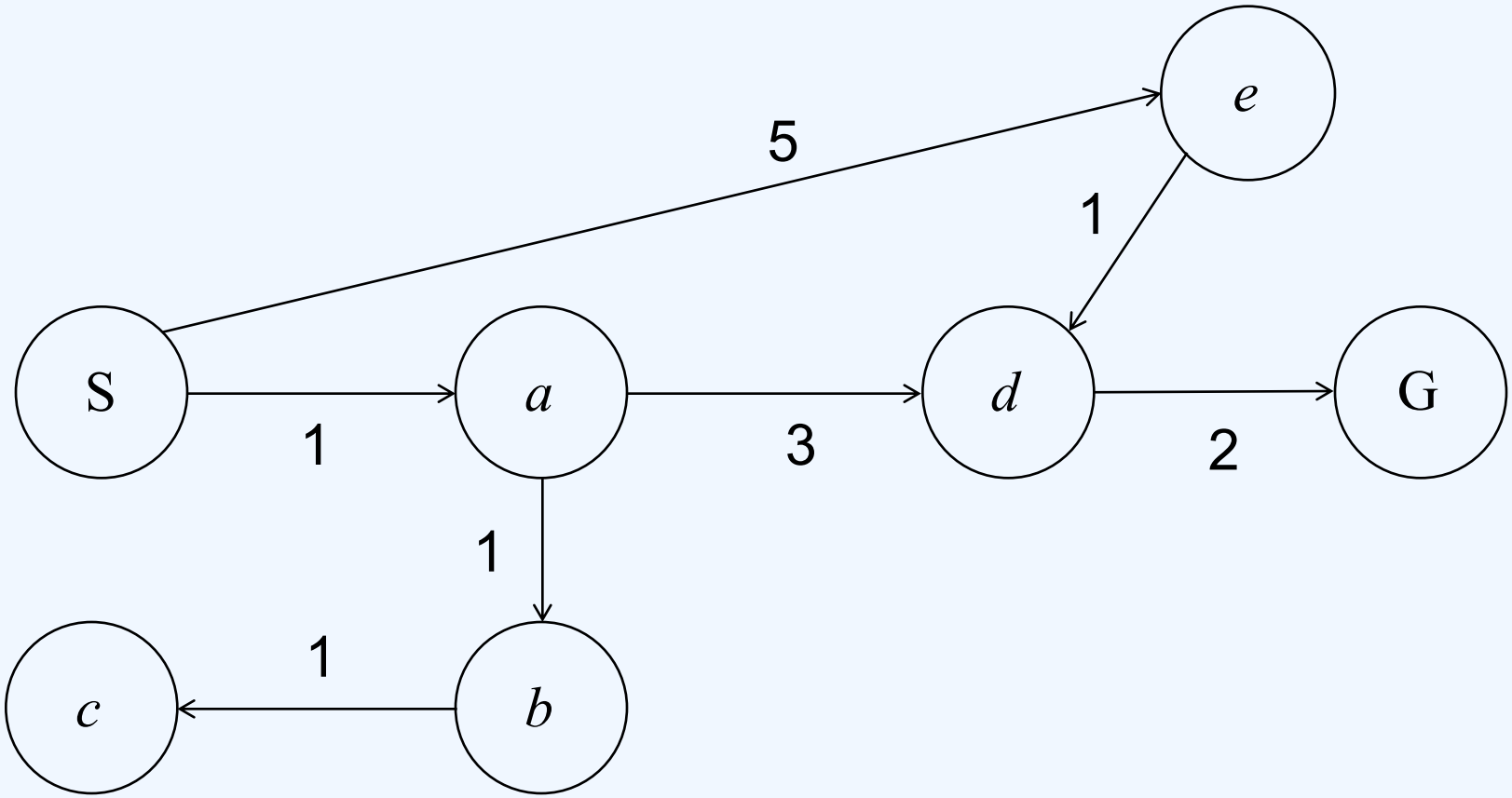
Today's schedule

- More on uninformed search
  - iterative deepening: BFS + DFS
  - uniform cost search (UCS)
  - searching backwards from the goal
- Informed search
  - best first (greedy)
  - A*

Combining good properties of BFS and DFS

- Iterative deepening DFS:
  - Call limited depth DFS with depth 0;
  - If unsuccessful, call with depth 1;
  - If unsuccessful, call with depth 2;
  - Etc.
- Complete, finds shallowest solution
- Flexible time-space tradeoff
- May seem wasteful timewise because replicating effort

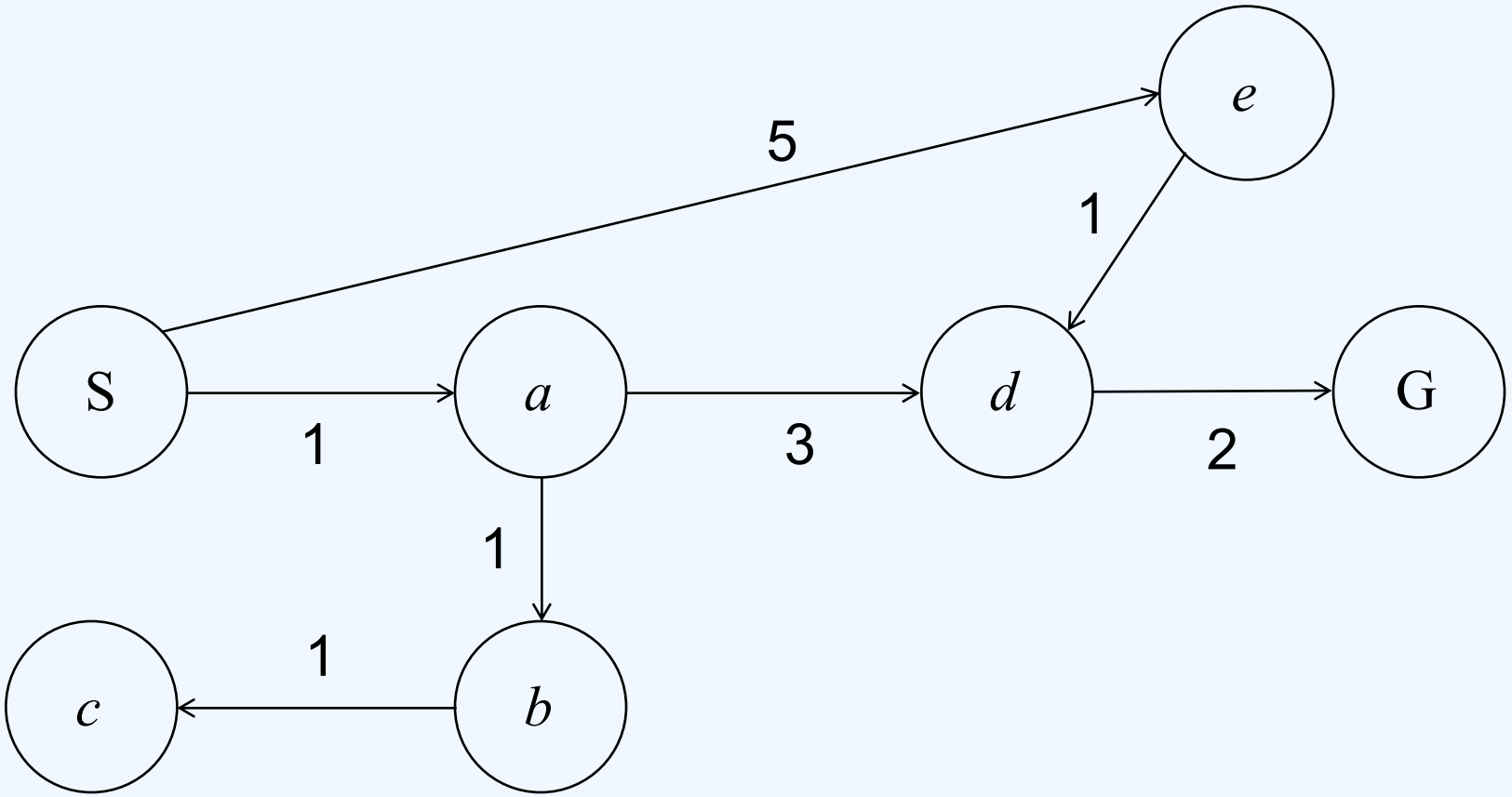
Costs on Actions



Uniform-Cost Search (UCS)

- BFS: finds shallowest solution
- Uniform-cost search (UCS): work on the lowest-cost node first
  - so that all states in the fringe have more or less the same cost, therefore “uniform cost”
- If it finds a solution, it will be an optimal one
- Will often pursue lots of short steps first
- Project 1 Q3

Example



Priority Queue Refresher

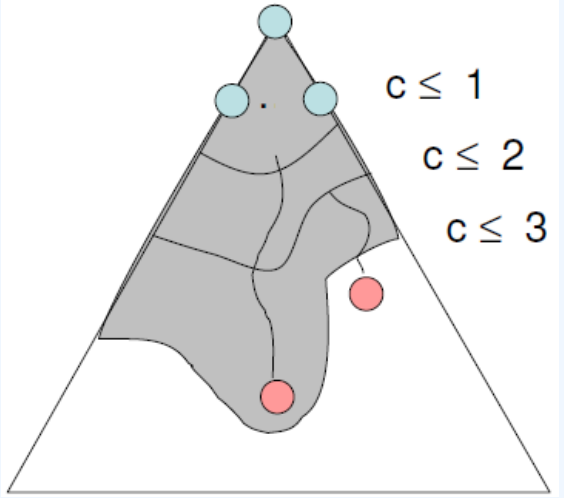
•A priority queue is a data structure in which you can insert and retrieve (key, value) pairs with the following operations:

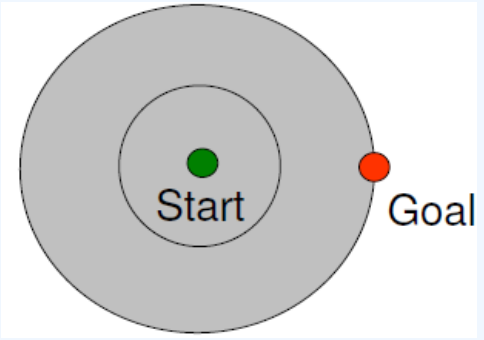
pq.push(key, value)	Inserts (key, value) into the queue.
pq.pop()	returns the key with the lowest value, and removes it from the queue

- Insertions aren't constant time, usually
- We'll need priority queues for cost-sensitive search methods

Uniform-Cost Search

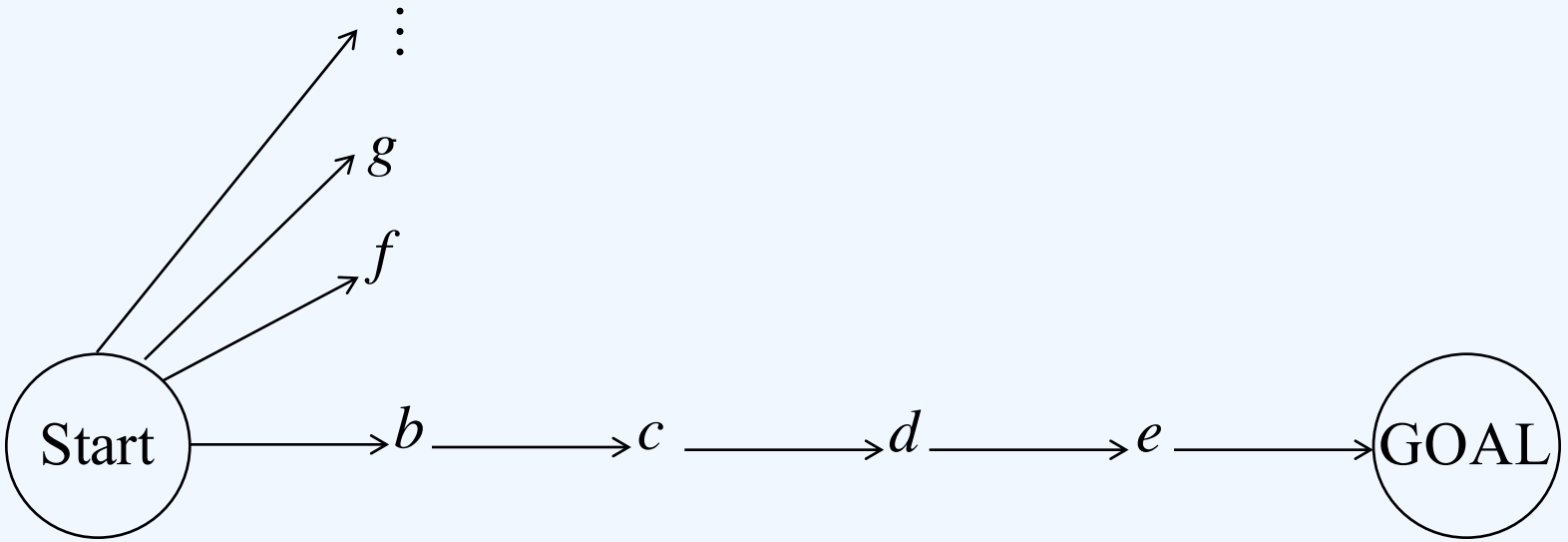
- The good: UCS is
  - complete: if a solution exists, one will be found
  - optimal: always find the solution with the lowest cost
  - really?
    - yes, for cases where all costs are positive
- The bad
  - explores every direction
  - no information about the goal location





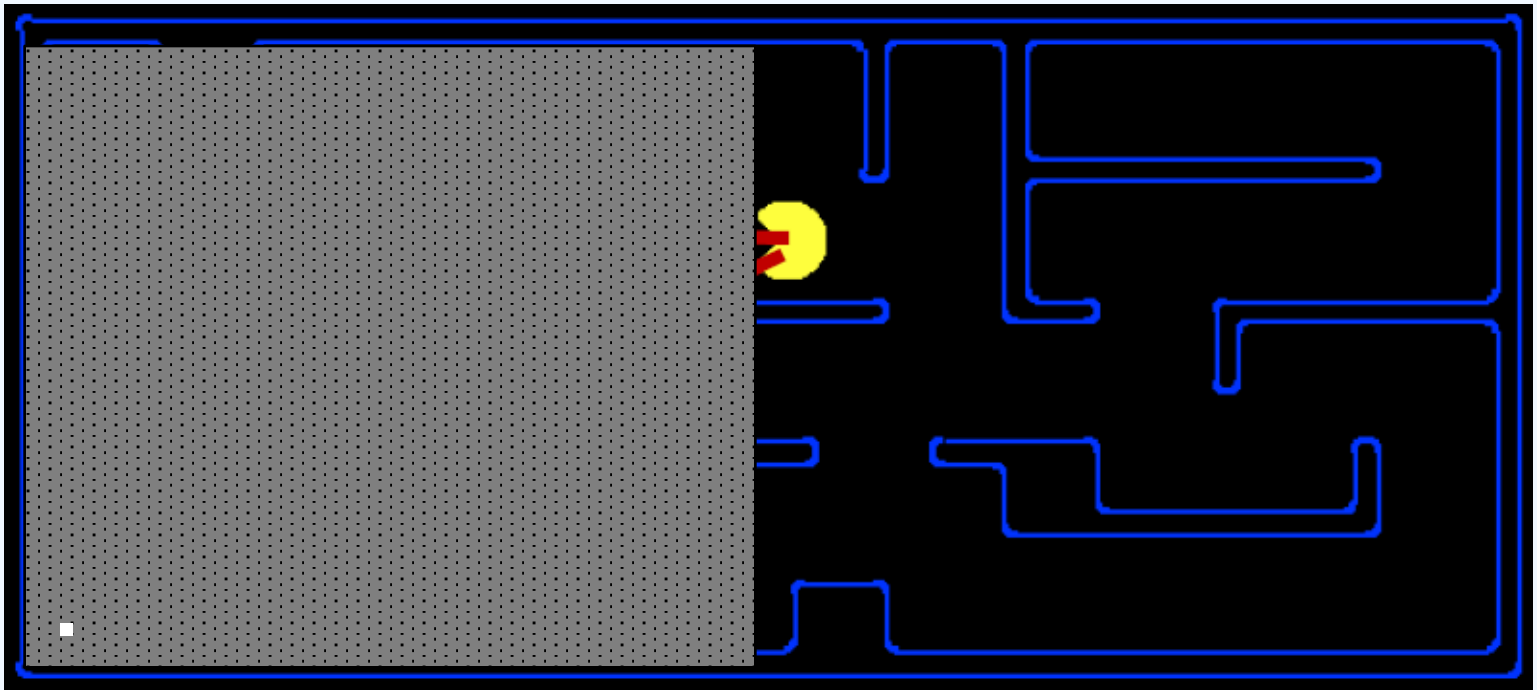
Searching backwards from the goal

- Switch the role of START and GOAL
- Reverse the edges in the problem graph
- Need to be able to compute predecessors instead of successors



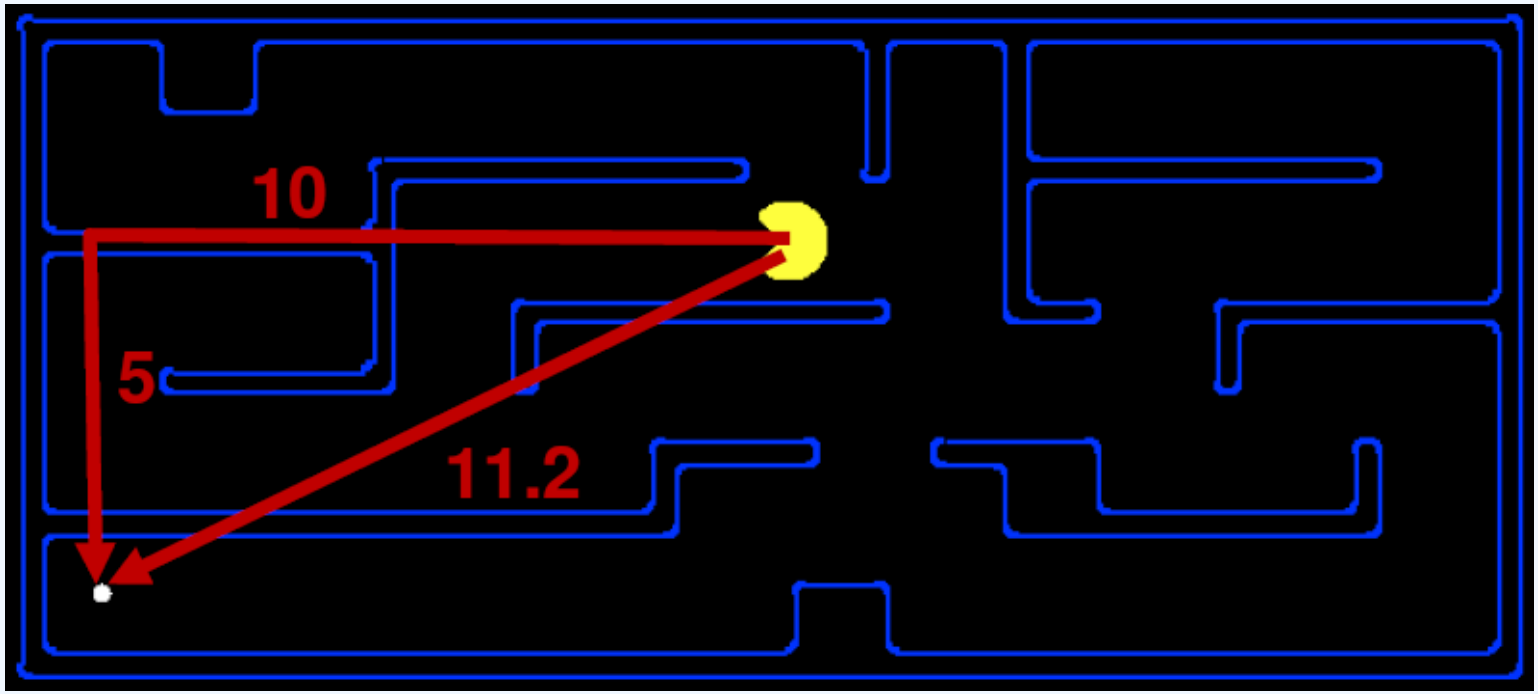
Informed search

- So far, have assumed that no non-goal state looks better than another
- Unrealistic
  - Even without knowing the road structure, some locations seem closer to the goal than others
- Makes sense to expand seemingly closer nodes first



Search Heuristics

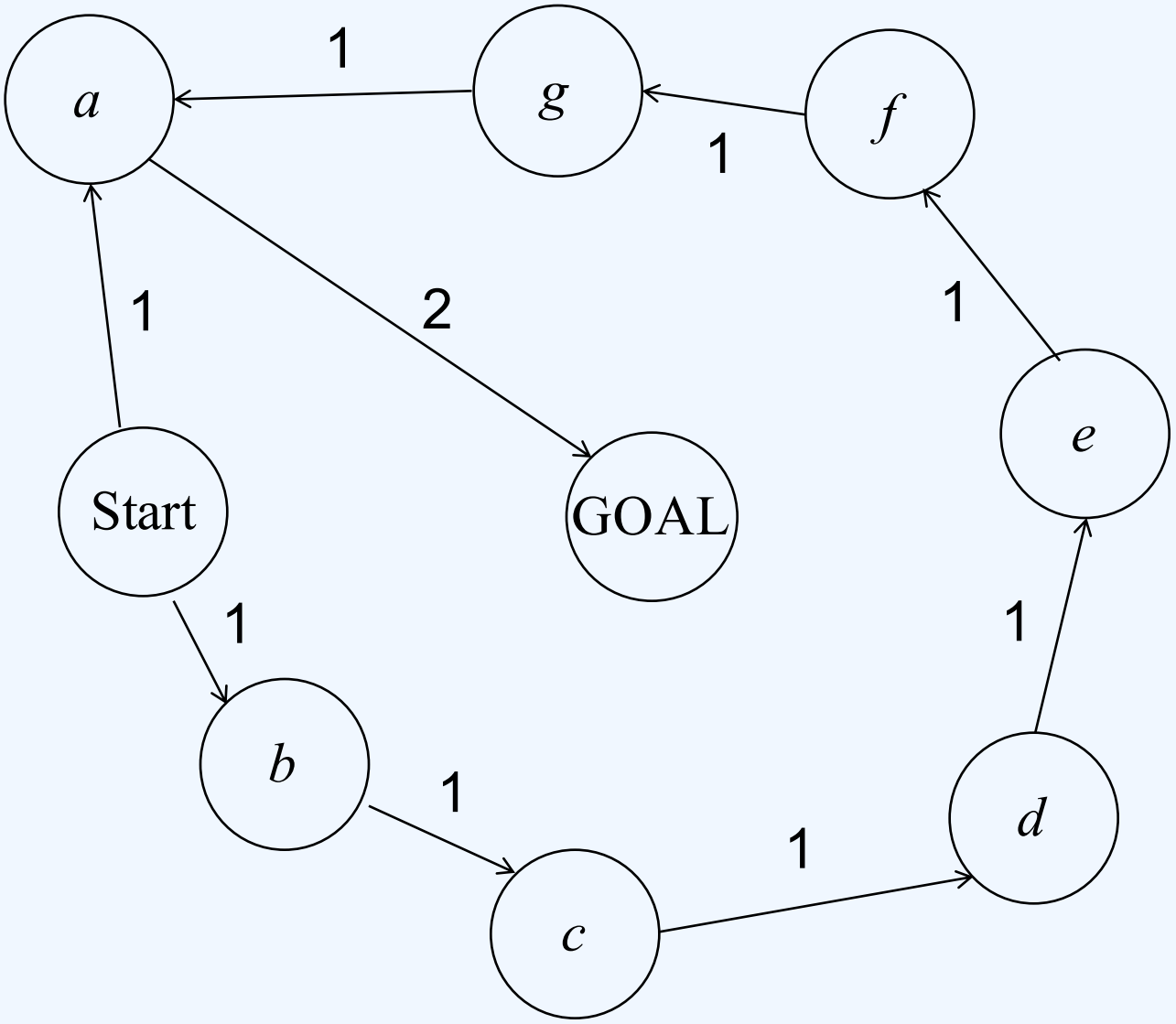
- Any estimate of how close a state is to a goal
- Designed for a particular search problem
- Examples: Manhattan distance, Euclidean distance



Best First (Greedy)

- Strategy: expand a node that you think is closest to a goal state
  - Heuristic function: estimate of distance to nearest goal for each state
  - h(n)
  - Often easy to compute
- A common case:
  - Best-first takes you straight to the (wrong) goal
- Worst-case: like a badly-guided DFS

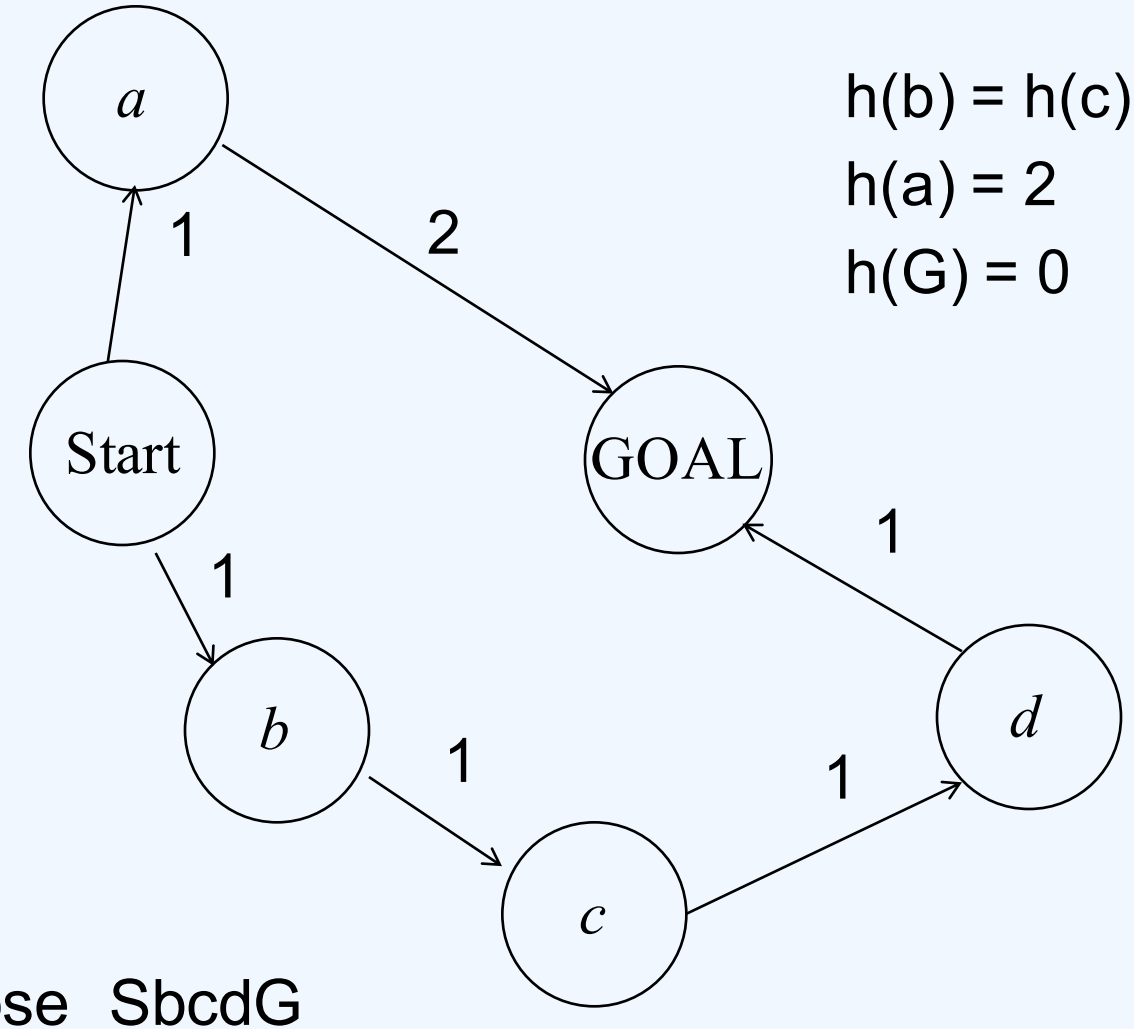
Example



h(b) = h(c) = h(d) = h(e) = h(f) = h(g) =1

h(a) = 2

A more problematic example



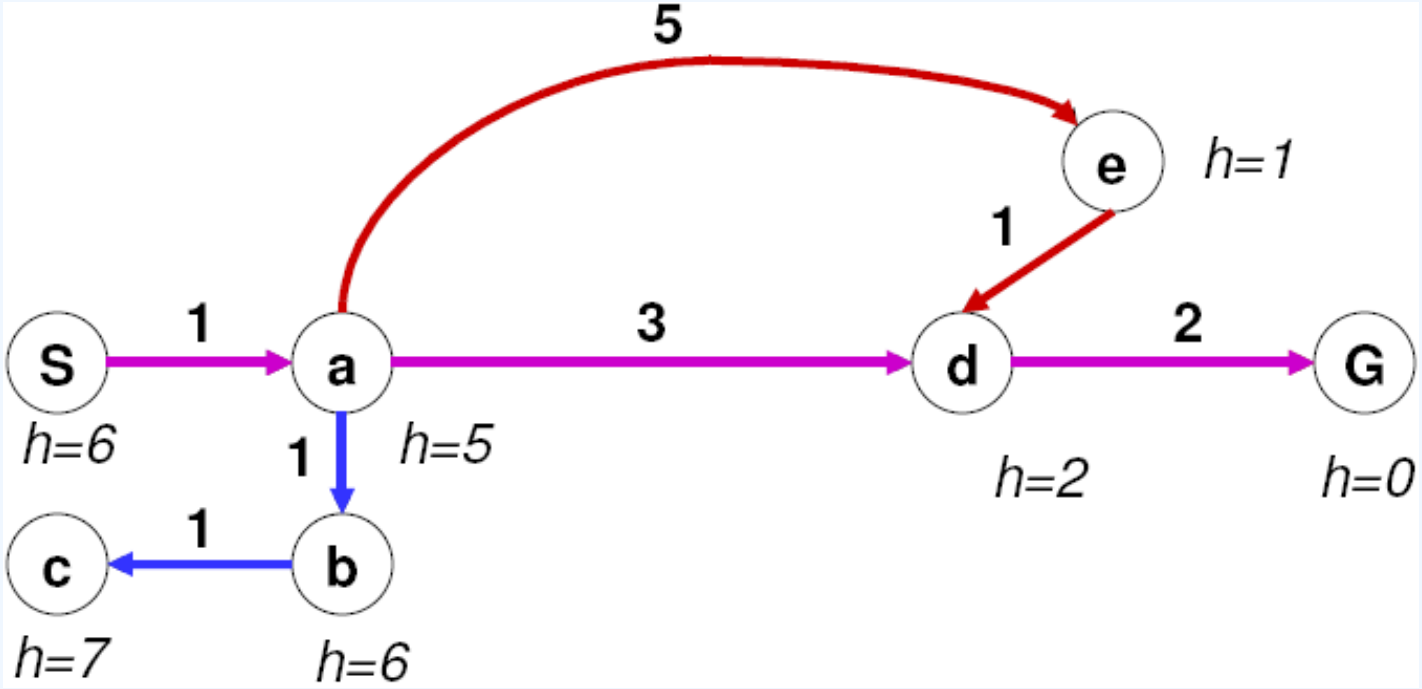
h(b) = h(c) = h(d)=1
h(a) = 2
h(G) = 0

- Greedy will choose SbcdG
- Optimal: SaG
- Should we stop when the fringe is empty?

A*

A*: Combining UCS and Greedy

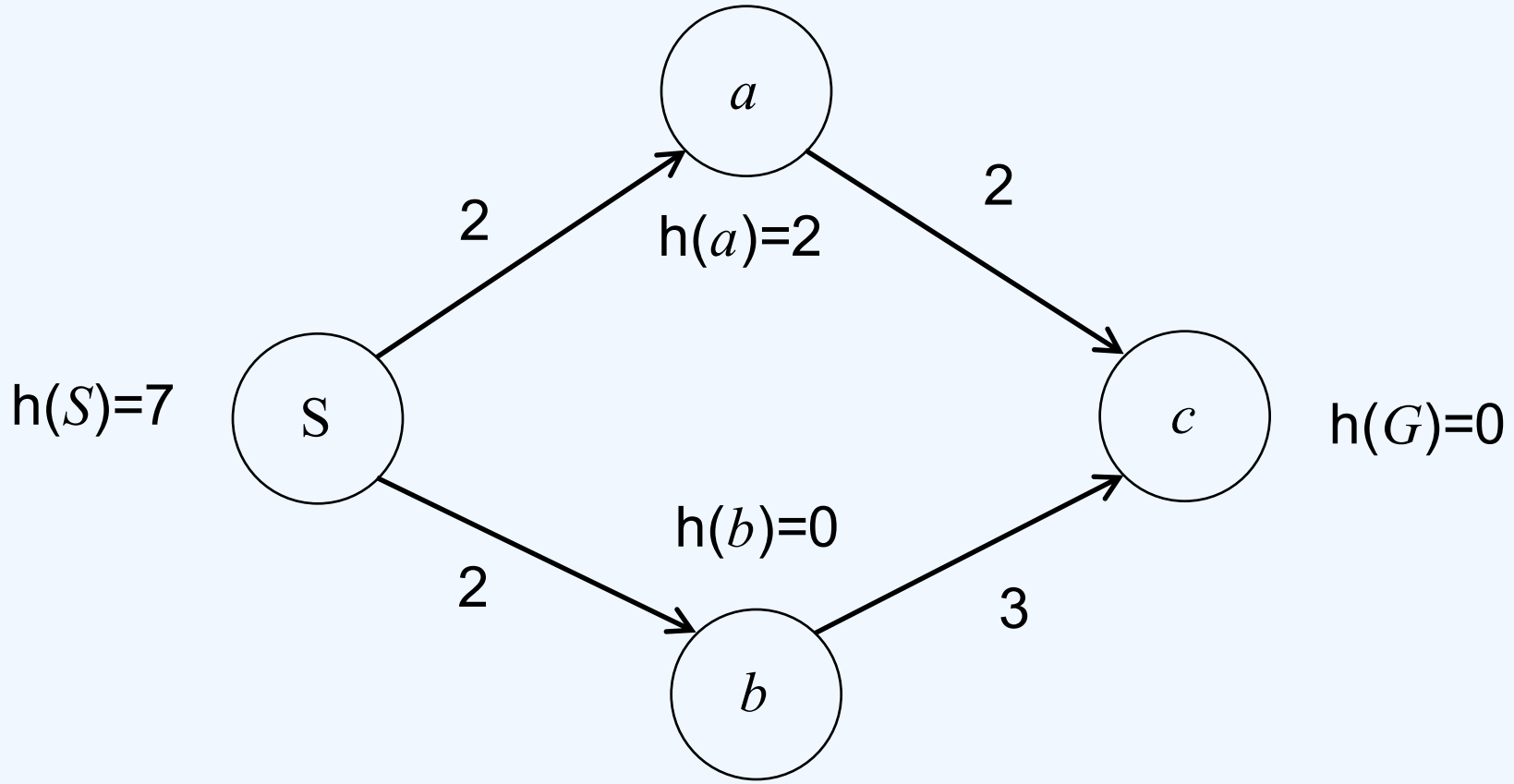
- Uniform-cost orders by path cost $g(n)$
- Greedy orders by goal proximity, or forward cost $h(n)$



- A* search orders by the sum: $f(n) = g(n) + h(n)$

When should A* terminate?

- Should we stop when we add a goal to the fringe?

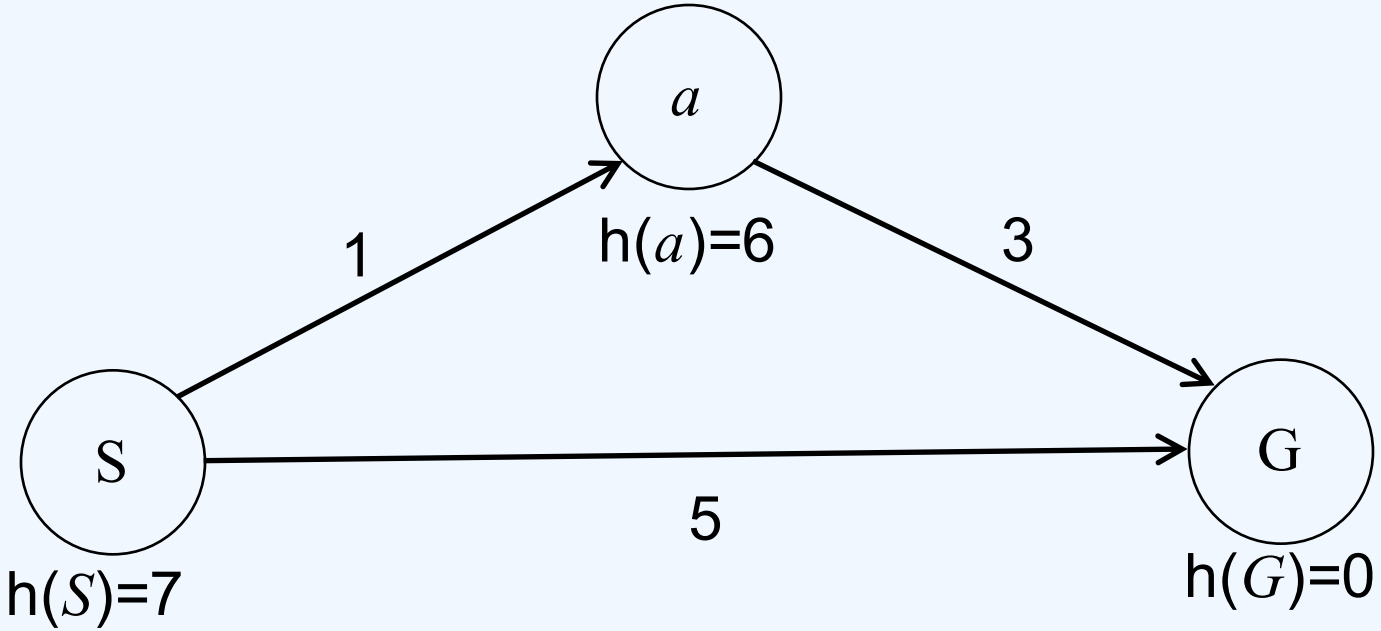


- No: only stop when a goal pops from the fringe

A*

- Never expand a node whose state has been visited
- Fringe can be maintained as a priority queue
- Maintain a set of visited states
- *fringe := {node corresponding to initial state}*
- *loop:*
  - *if fringe empty, declare failure*
  - *choose and remove the top node v from fringe*
  - *check if v's state s is a goal state; if so, declare success*
  - *if v's state has been visited before, skip*
  - *if not, expand v, insert resulting nodes with f(v)=g(v)+h(v) to fringe*

Is A* Optimal?



- Problem: overestimate the true cost to a goal
  - so that the algorithm terminates without expanding the actual optimal path

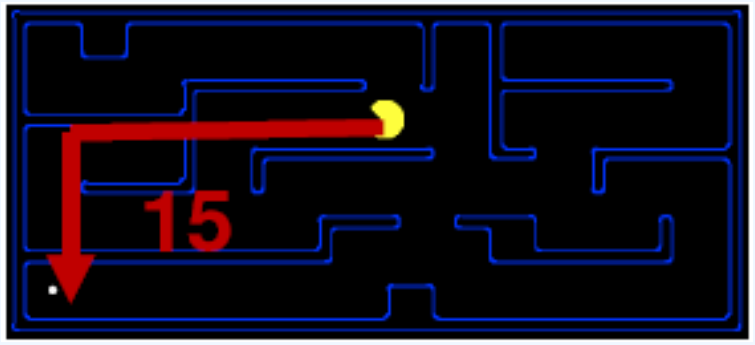
Admissible Heuristics

- A heuristic h is admissible if:

$$h(n) \leq h^*(n)$$

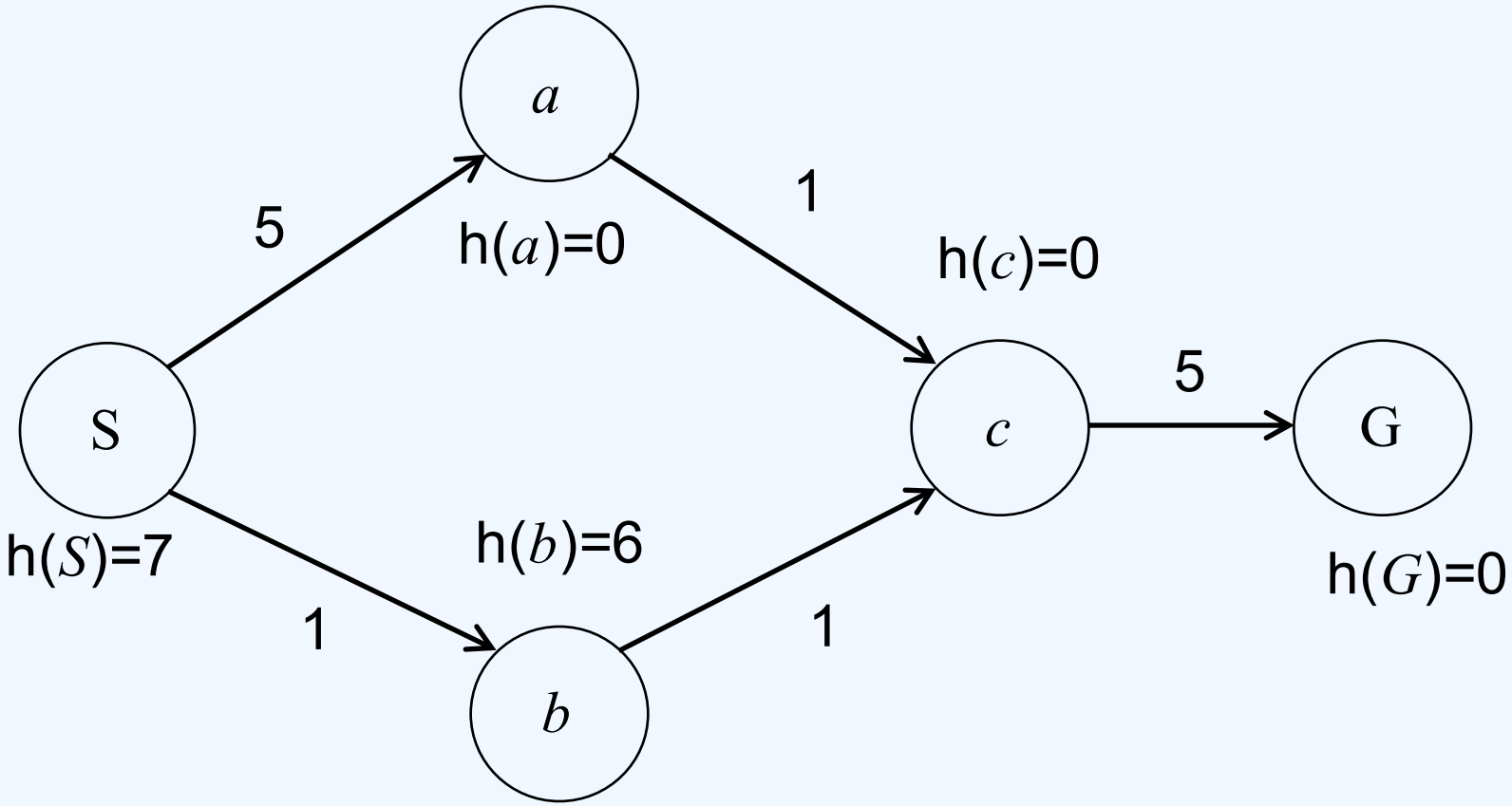
where $h^*(n)$ is the true cost to a nearest goal

- Examples:
  - $h\,(n)=0$
  - $h(n) = h^*(n)$
  - another example



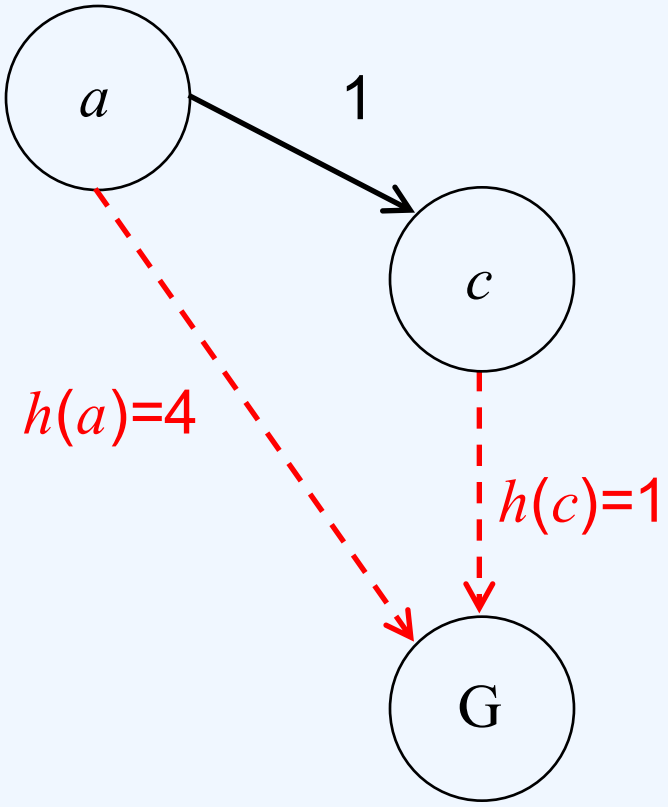
- Coming up with admissible heuristics is most challenging part in using A* in practice

Is Admissibility enough?



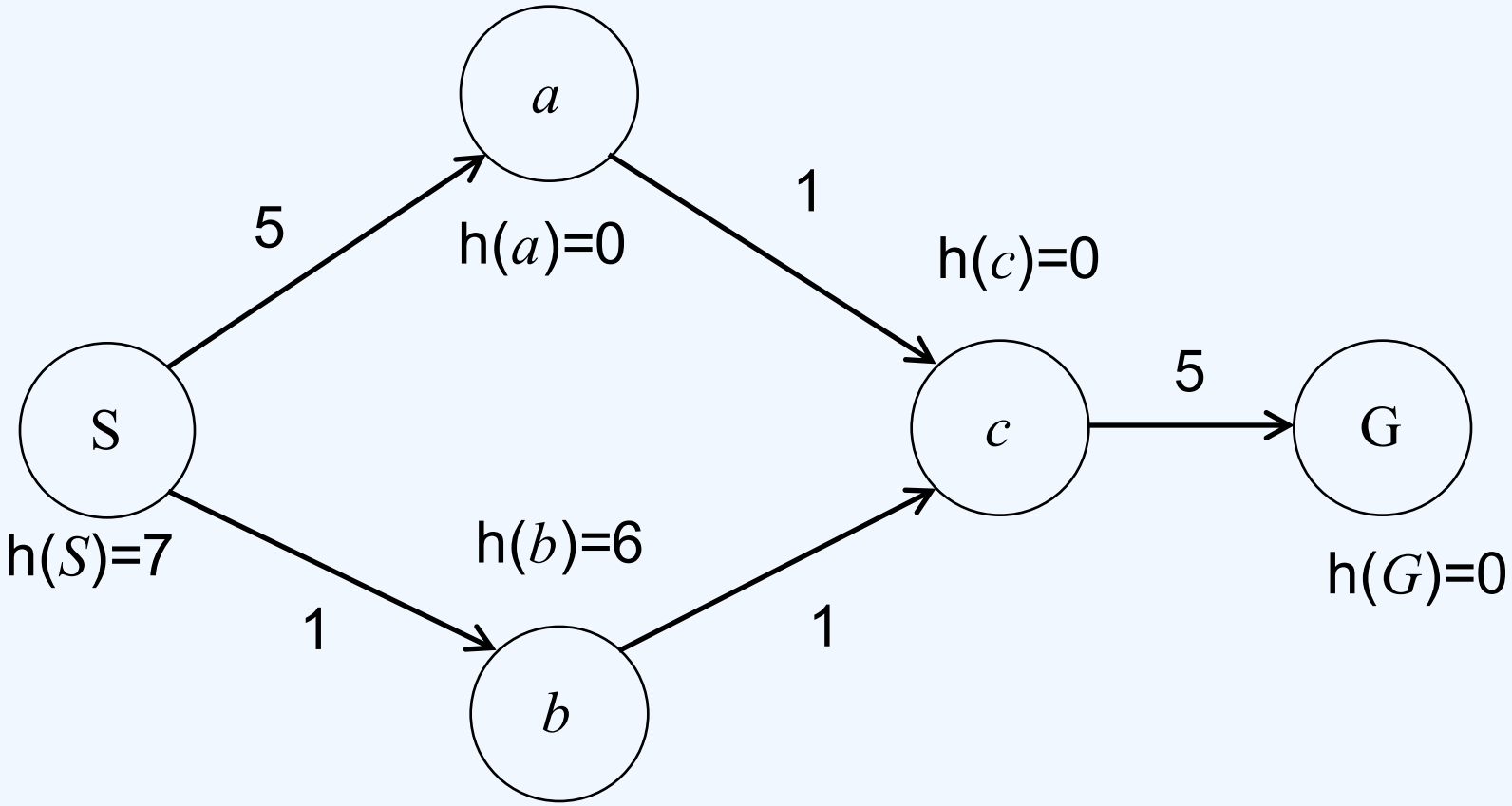
- Problem: the first time to visit a state may not via the shortest path to it

Consistency of Heuristics



- Stronger than admissibility
- Definition:
- cost(a to c) + $h(c) \geq h(a)$
  - Triangle inequality
  - Be careful about the edges
- Consequences:
  - The f value along a path never decreases
  - so that the first time to visit a state corresponds to the shortest path

Violation of consistency



- b→ c

Correctness of A*

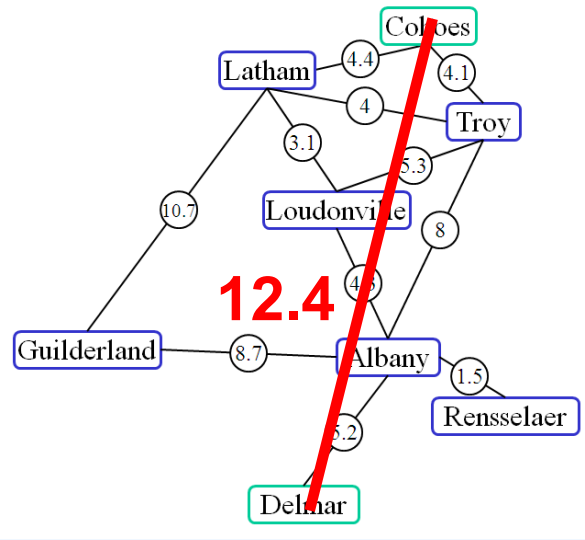
- A* finds the optimal solution for consistent heuristics
  - but not for the admissible heuristics (as in the previous example)
- However, if we do not maintain a set of "visited" states, then admissible heuristics are good enough

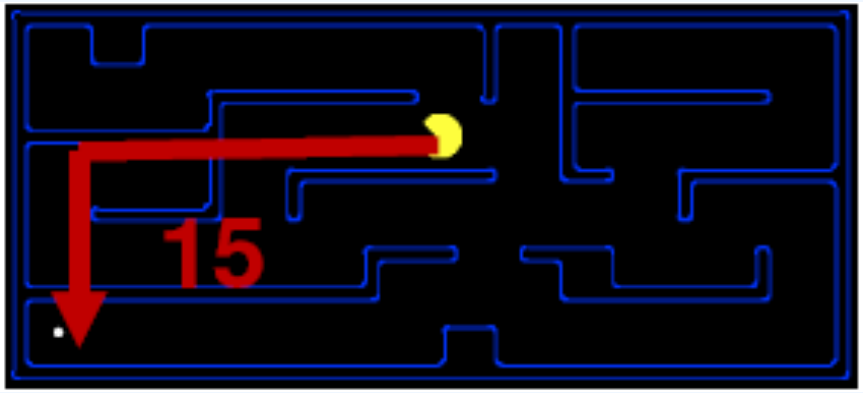
Proof

- Proof by contradiction: suppose the output is incorrect.
  - Let the optimal path be S→ a→ … → G
    - for simplicity, assume it is unique
  - When G pops from the fringe, f(G) is no more than f(v) for all v in the fringe.
  - Let v denote the state along the optimal path in the fringe
    - guaranteed by consistency
  - Contradiction
    - addimisbility: f(G) <= f(v) <= g(v) + h*(v)

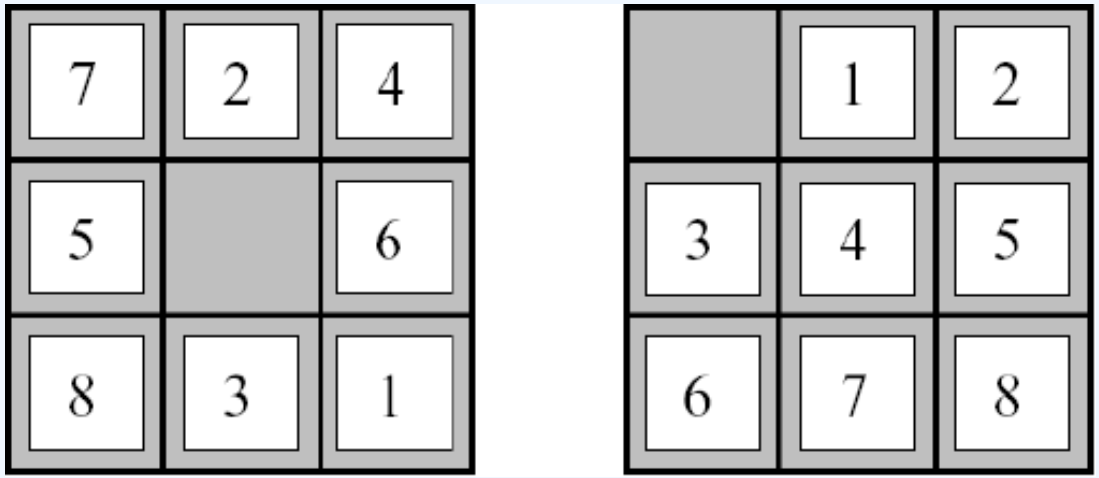
Creating Admissible Heuristics

- Most of the work in solving hard search problems optimally is in coming up with admissible heuristics





Example: 8 Puzzle

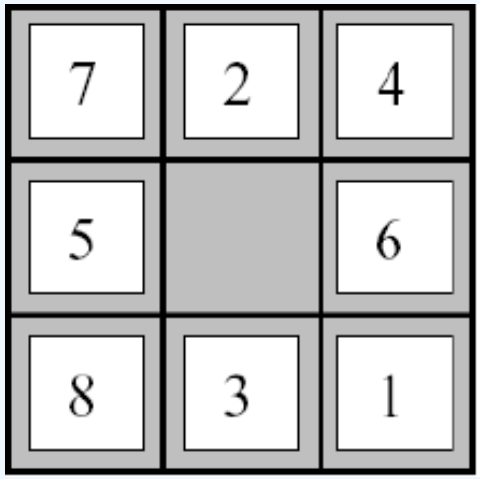


Start State **Goal State**

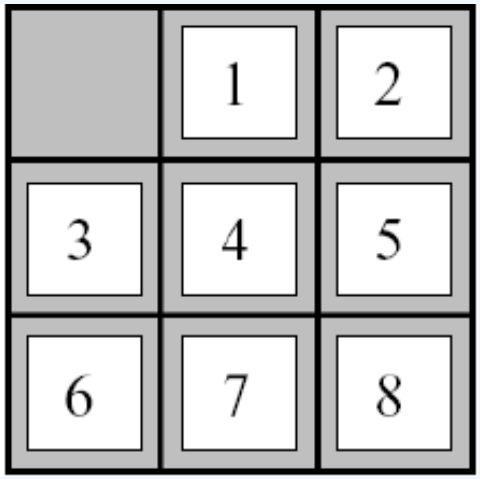
- What are the states?
- How many states?
- What are the actions?
- What should the costs be?

8 Puzzle I

- Heuristic: Number of tiles misplaced
- Why is it admissible?
- h(START) = 8



Start State

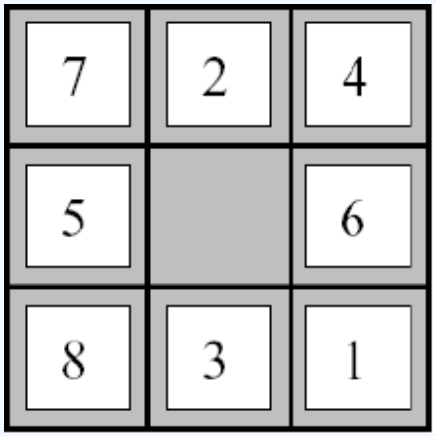


Goal State

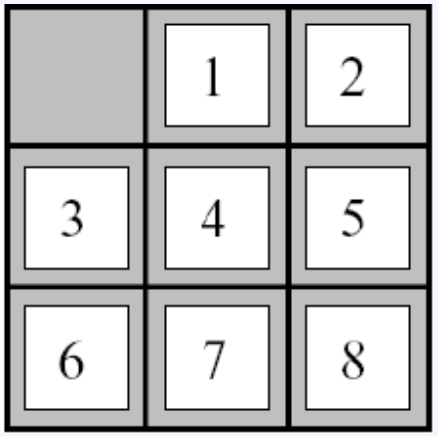
	Average nodes expanded when optimal path has length….		
	…4 steps	…8 steps	…12 steps
UCS	112	6,300	3,600,000
TILES	13	39	227

8 Puzzle II

- What if we had an easier 8-puzzle where any tile could slide any direction at any time ignoring other tiles?
- Total Manhattan distance
- Why admissible?
- h(START) = 3+2+…+18



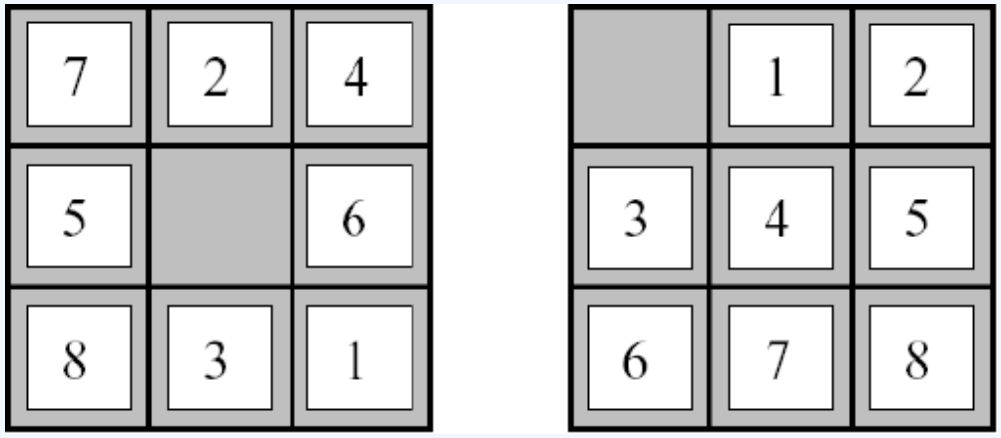
Start State



Goal State

	Average nodes expanded when optimal path has length….		
	…4 steps	…8 steps	…12 steps
TILES	13	39	227
Manhattan	12	25	73

8 Puzzle III



Start State **Goal State**

- Total Manhattan distance for grid 1 and 2
- Why admissible?
- Is it consistent?
- h(START) = 3+1=4

8 Puzzle IV

- How about using the actual cost as a heuristic?
  - Would it be admissible?
  - Would we save on nodes expanded?
  - What's wrong with it?
- With A*: a trade-off between quality of estimate and work per node!

Other A* Applications

- Pathing / routing problems
- Resource planning problems
- Robot motion planning
- Language analysis
- Machine translation
- Speech recognition
- Voting!

Features of A*

- A* uses both backward costs and (estimates of) forward costs
- A* computes an optimal solution with consistent heuristics
- Heuristic design is key

Search algorithms

- Maintain a fringe and a set of visited states
- *fringe := {node corresponding to initial state}*
- *loop:*
  - *if fringe empty, declare failure*
  - *choose and remove the top node v from fringe*
  - *check if v's state s is a goal state; if so, declare success*
  - *if v's state has been visited before, skip*
  - *if not, expand v, insert resulting nodes to fringe*
- Data structure for fringe
  - FIFO: BFS
  - Stack: DFS
  - Priority Queue: UCS (value = h(n)) and A* (value = g(n)+h(n))

Take-home message

- No information
  - BFS, DFS, UCS, iterative deepening, backward search
- With some information
  - Design heuristics h(n)
  - A*, make sure that h is consistent
  - Never use Greedy!

Project 1

- You can start to work on UCS and A* parts
- Start early!
- Ask questions on Piazza