Economics and Computation
Homework 1: Game Theory

Please do not hand in. Just do it by yourself to prepare for the exam.

1 Highly Recommended

Problem 1. (ILP) You are playing a card game called Hearthstone. The enemy hero has K health and has just one minion with attack A and health H. You have n minions whose attack and health are $(a_1, h_1), \ldots, (a_n, h_n)$. Each of your minions can attack once, either at the enemy minion or at the enemy hero. If your minion attacks the enemy minion, then the enemy minion's health is reduced by the attack of your minion, and once the health becomes 0 or below, the enemy minion is eliminated (whether your minion gets hurt is irrelevant in this problem). If your minion attacks enemy hero, then the enemy hero's health is reduced by the attack of your minion.

Your goal is to design an ILP to compute an attack plan such that (1) the enemy minion is eliminated (health reduced to 0 or less), and (2) enemy hero's health is reduced to 0 or less. Your algorithm should run in time $O(Hn)$.

For example, suppose $K = 12$ and the enemy minion is $(4, 7)$, and your minions are $(3, 1), (3, 1), (1, 1)$, and $(12, 12)$. Then you can win the game by letting $(3, 1), (3, 1), (1, 1)$ attack the enemy minion, and let $(12, 12)$ attack enemy hero. If you let $(12, 12)$ attacks the enemy minion and let the rest attack enemy hero, then the total damage done to enemy hero is $7 < 12$, and you are not winning the game.

Hint: You can think about how to divide your minions into two groups: the first group attacks the enemy minion and the second group that attacks the enemy hero. Because the total damage is fixed, to maximize damage done to the enemy hero by the second group, equivalently, you can compute an optimal first group that deals as little total damage to the enemy minion as possible, as long as the total is $\geq H$.

Problem 2. (Normal-form games 4pts). Consider the following game.

	L	M	R
U	5, 0	1, 3	4, 0
C	2, 4	2, 4	3, 5
D	0, 1	4, 0	4, 0

1. (2pt) Remove dominated strategies iteratively. Every time a strategy is removed, you must show which strategy (which can be a mixed strategy) dominates it.

2. (2pt) Compute a mix-strategy NE.

3. (2pt) Write down the linear constraints for computing a correlated equilibrium (no need to solve it).

Problem 3. (Game modeling 4pts). Suppose you are the only student in Computational Social Processes. You can choose to either pay attention or not, and the instructor can choose to prepare the class or not.

If the instructor prepares and you pay attention, then both of you will get utilities 4. If the instructor does not prepare and you do not pay attention, then both of you get utilities 0. If the instructor does not prepare but you pay attention, then you will be disappointed (-16 utility), and the instructor will be penalized by a low course evaluation, leading to a utility of -14 (btw, this is not true in my case—giving low evaluation score does not really penalize me...) Finally, if the instructor prepares for the class but you pay no attention, then you will learn nothing (utility 0) and the instructor will be disappointed (utility -2).

1. (2pt) Model the situation as a game (identify the players, actions, and draw the game matrix).

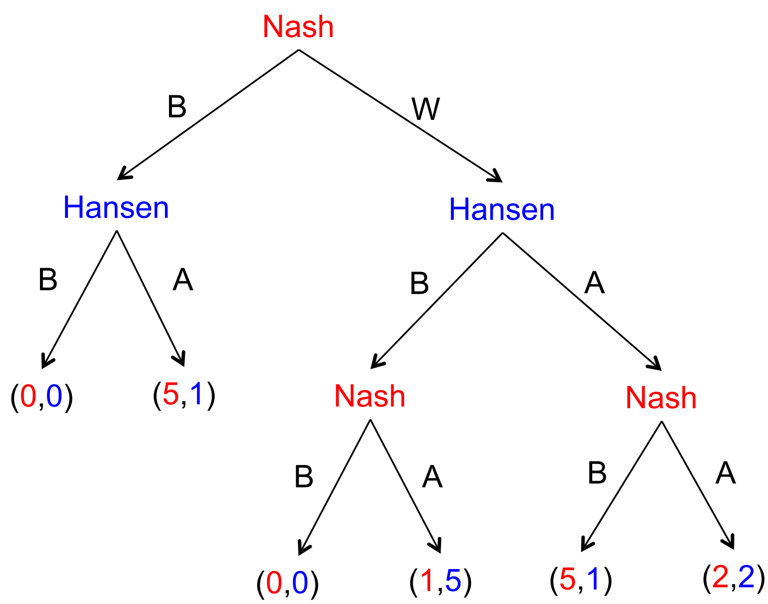

Figure 1: An extensive-form game.

2. (2pt) Compute all mixed strategy NE of the game.

Problem 4. (Extensive-form game 6pts) Consider the extensive-form game illustrated in Figure 1. In the first round Nash can either choose to go for the Blond (B) or wait for Hanson's move (W).

- **(2pt)** Find the SPNE using backward induction.

2 Food for Thought

Problem 5. Utility Theory (2pts) The **Allais paradox** arises when people are tested in the two experiments illustrated in Table 1. In each experiment, a person can choose one of the two options (A or B).

Suppose a person chooses Lottery A for Experiment 1 and Lottery B for Experiment 2 (in fact many people do so). Show that this is a failure of utility theory.

Problem 6. We know that no strictly dominated strategy can appear in any Nash Equilibrium. Does this hold for correlated equilibrium? In other

Experiment 1		Experiment 2	
Lottery A	Lottery B	Lottery A	Lottery B
\$1M@100%	\$1M@89% + \$5M@10% + 0@1%	\$1M@11%+0@89%	\$0M@90% + \$5M@10%

Table 1: Allais paradox.

words, is it true that in each support of any correlated equilibrium (which is a pure strategy profile), no player can play a strictly dominated strategy?

Problem 7. Prove that any mixed-strategy Nash equilibrium is a correlated equilibrium. Prove that any probability mixture of pure-strategy Nash equilibrium is a correlated equilibrium. Is it true that any probability mixture of mixed-strategy Nash equilibrium must be a correlated equilibrium?

Problem 8. Consider the game of Chicken with the following utility functions where u is a real number

	D	C
D	0,0	7,2
C	2,7	u,u

What is the range of u so that the only correlated equilibria are probability mixture of pure NE?