

Introduction to Mechanism Design

Lirong Xia



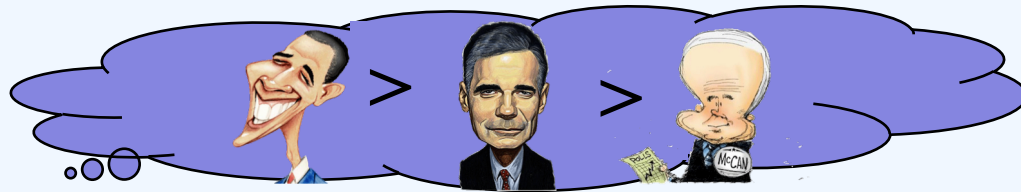
Rensselaer

Voting game of strategic voters



Alice

Strategic vote



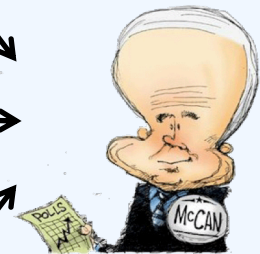
Bob

Strategic vote



Carol

Strategic vote



Game theory is predictive

- How to design the “rule of the game”?
 - so that when agents are strategic, we can achieve a given outcome w.r.t. their **true** preferences?
 - “reverse” game theory
- Example
 - Lirong’s goal of this course: students learned economics and computation
 - Lirong can change the rule of the course
 - grade calculation, curving, homework and exam difficulty, free food, etc.
 - Students’ incentives (you tell me)

Today's schedule: mechanism design

- Mechanism design: Nobel prize in economics 2007



Leonid Hurwicz
1917-2008



Eric Maskin



Roger Myerson

- VCG Mechanism: Vickrey won Nobel prize in economics 1996

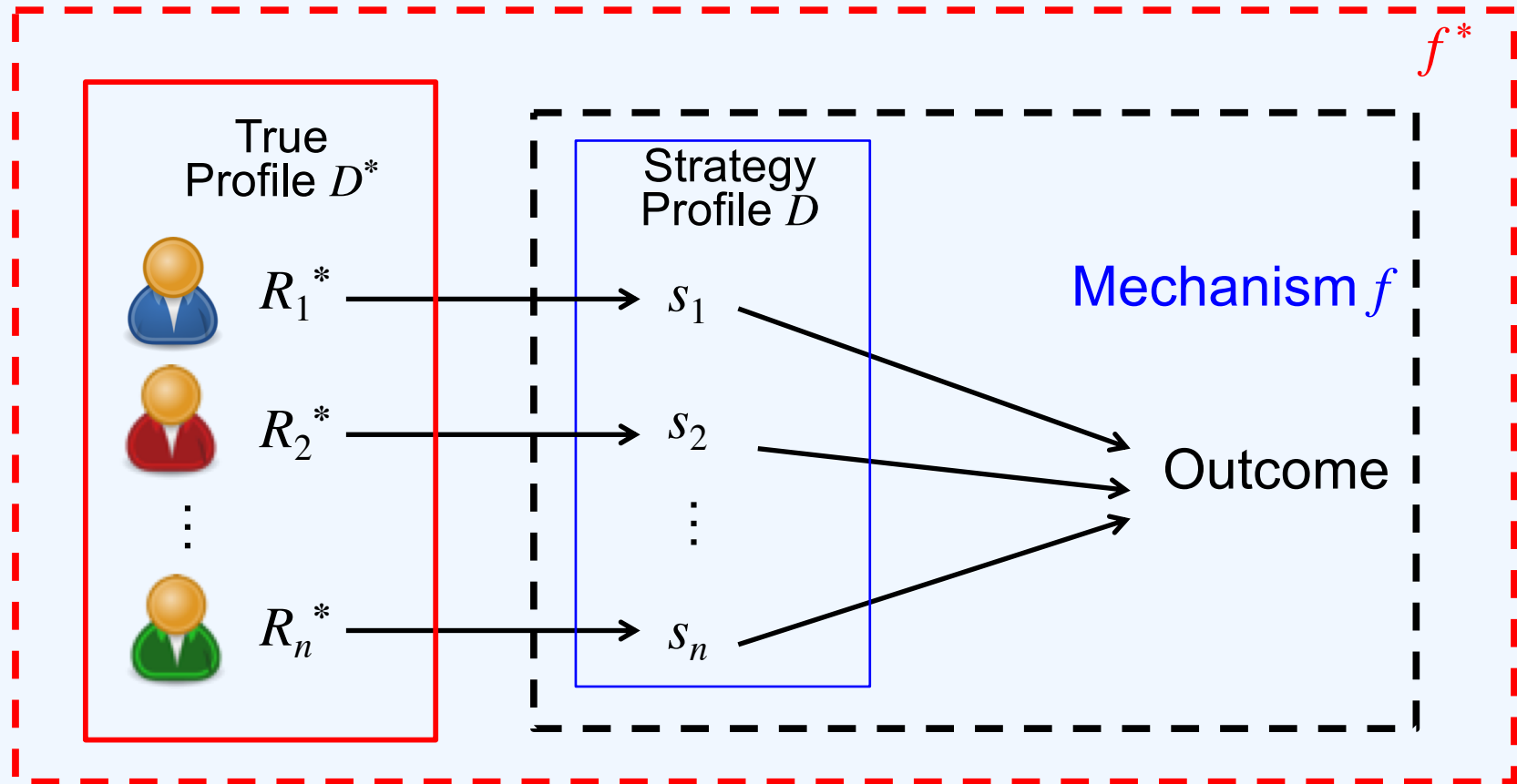


William Vickrey
1914-1996

Mechanism design with money

- With monetary transfers
- Set of **alternatives**: A
 - e.g. allocations of goods
- Outcomes: $\{ (\text{alternative}, \text{payments}) \}$
- Preferences: represented by a **quasi-linear** utility function
 - every agent j has a private value $v_j^*(a)$ for every $a \in A$. Her utility is
$$u_j^*(a, p) = v_j^*(a) - p_j$$
 - It suffices to report a value function v_j

Implementation



- A game and a solution concept **implement** a function f^* , if
 - for every **true** preference profile D^*
 - $f^*(D^*) = \text{OutcomeOfGame}(f, D^*)$
- f^* is defined w.r.t. the true preferences
- f is defined w.r.t. the reported preferences

Can we adjust the payments to maximize social welfare?

➤ Social welfare of a

- $SW(a) = \sum_j v_j^*(a)$

➤ Can any ($\operatorname{argmax}_a SW(a)$, payments) be implemented w.r.t. dominant strategy NE?

The Vickrey-Clarke-Groves mechanism (VCG)

➤ The Vickrey-Clarke-Groves mechanism (VCG) is defined by

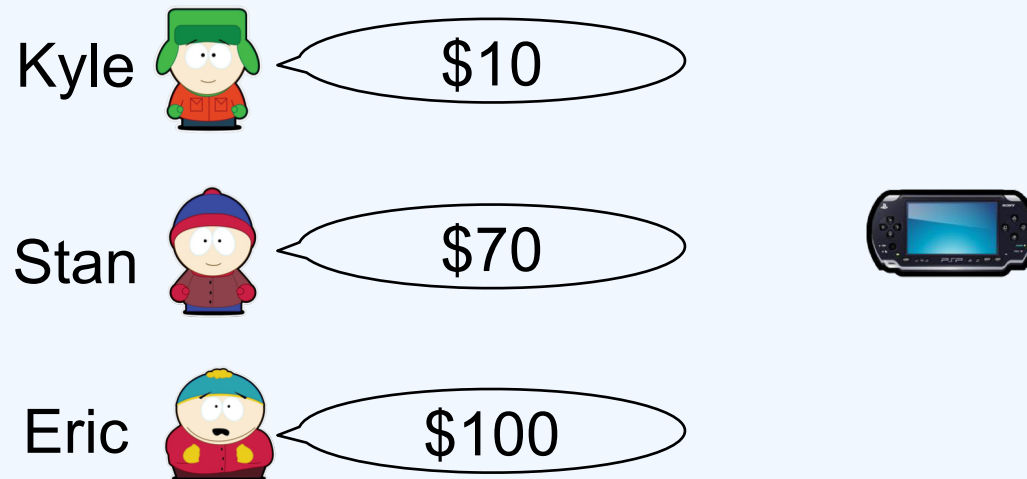
- Alternative in outcome: $a^* = \operatorname{argmax}_a SW(a)$
- Payments in outcome: for agent j

$$p_j = \max_a \sum_{i \neq j} v_i(a) - \sum_{i \neq j} v_i(a^*)$$


- **negative externality** of agent j of its presence on other agents

➤ Truthful, efficient

Example: auction of one item



➤ Alternatives = (give to K, give to S, give to E)

➤ $a^* =$ 

➤ $p_1 = 100 - 100 = 0$

➤ $p_2 = 100 - 100 = 0$

➤ $p_3 = 70 - 0 = 70$

Example: Ad Auction

macbook keyword

All Shopping News Images Videos More Settings Tools

About 222,000,000 results (0.61 seconds)

Slot 1	Slot 2	Slot 3	Slot 4	Slot 5
See MacBook				Sponsored
13-inch MacBook Pro - Space Gray \$1,299.00 Apple Free shipping	15-inch MacBook Pro - Space Gray \$2,399.00 Apple Free shipping	13-inch MacBook Air \$999.00 Apple Free shipping	Apple Air, Silver \$374.99 Walmart ★★★★★ (446)	Refurbished Apple MacBook... \$349.20 Refurbished Walmart
winner 1	winner 2	winner 3	winner 4	winner 5

[MacBook - Behind the Mac | Apple](#)
Ad www.apple.com/ ▼
Behind the Mac people are making wonderful things and so could you. Shop now. More powerful than ever. Free two-day delivery. Apple Store pickup. Built-in Apps.
[Compare Mac models](#) · [Buy now](#) · [Apple GiveBack](#) · [Accessories for Mac](#)

Ad Auctions: Setup

➤ m slots

- slot i gets s_i clicks

➤ n bidders

- v_j : value for each user click
- b_j : pay (to service provider) per click
- utility of getting slot i : $(v_j - b_j) \times s_i$

➤ Outcomes: { (allocation, payment) }

Ad Auctions: VCG Payment

➤ 3 slots

- $s_1 = 100, s_2 = 60, s_3 = 40$

➤ 4 bidders

- true values $v_1^* = 10, v_2^* = 9, v_3^* = 7, v_4^* = 1,$

➤ VCG allocation: OPT = (1, 2, 3)

- slot 1->bidder 1; slot 2->bidder 2; slot 3->bidder 3;

➤ VCG Payment

- Bidder 1
 - not in the game, utility of others = $100 \cdot 9 + 60 \cdot 7 + 40 \cdot 1$
 - in the game, utility of others = $60 \cdot 9 + 40 \cdot 7$
 - negative externality = 540, pay per click = 5.4
- Bidder 2: 3 per click, Bidder 3: 1 per click

VCG is DSIC

- proof. Suppose for the sake of contradiction that VCG is not DSIC, then there exist j , v_j , v_{-j} , and v'_j such that

$$u_j(v_j, v_{-j}) < u_j(v'_j, v_{-j})$$

- Let a' denote the alternative when agent j reports v'_j

$$\begin{aligned} \Leftrightarrow v_j(a^*) - (\max_a \sum_{k \neq j} v_j(a) - \sum_{k \neq j} v_j(a^*)) \\ < v_j(a') - (\max_a \sum_{k \neq j} v_j(a) - \sum_{k \neq j} v_j(a')) \end{aligned}$$

$$\Leftrightarrow v_j(a^*) + \sum_{k \neq j} v_j(a^*) < v_j(a') + \sum_{k \neq j} v_j(a')$$

Contradiction to the maximality of a^*