# Introduction to Mechanism Design

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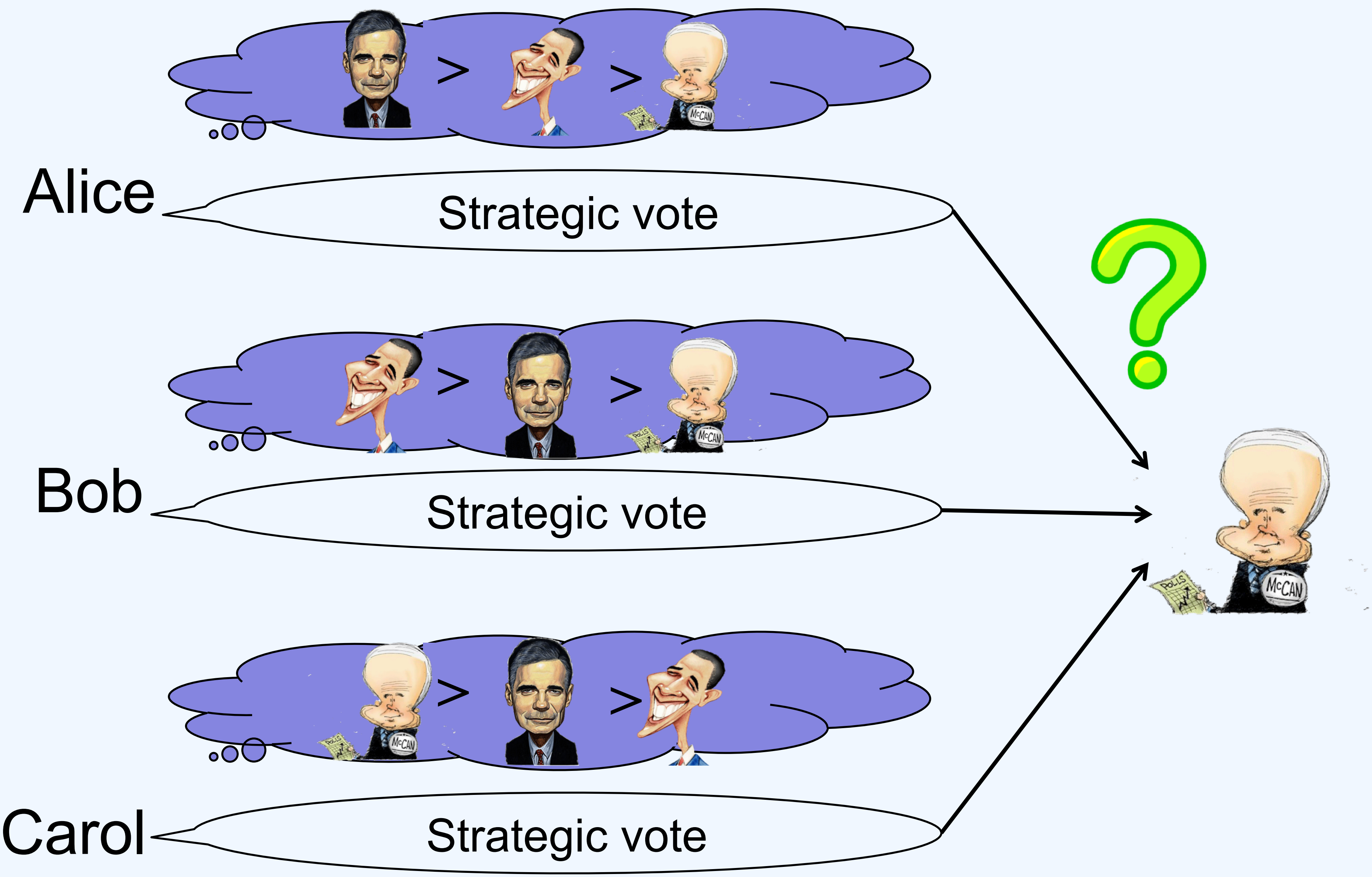
Rensselaer

# Voting game of strategic voters



# Game theory is predictive

- How to design the "rule of the game"?
  - so that when agents are strategic, we can achieve a given outcome w.r.t. their true preferences?
  - "reverse" game theory
- Example
  - Lirong's goal of this course: students learned economics and computation
  - Lirong can change the rule of the course
    - grade calculation, curving, homework and exam difficulty, free food, etc.
  - Students' incentives (you tell me)

# Today’s schedule: mechanism design

- Mechanism design: Nobel prize in economics 2007

Leonid Hurwicz
1917-2008

Eric Maskin

Roger Myerson

- VCG Mechanism: Vickrey won Nobel prize in economics 1996

William Vickrey
1914-1996

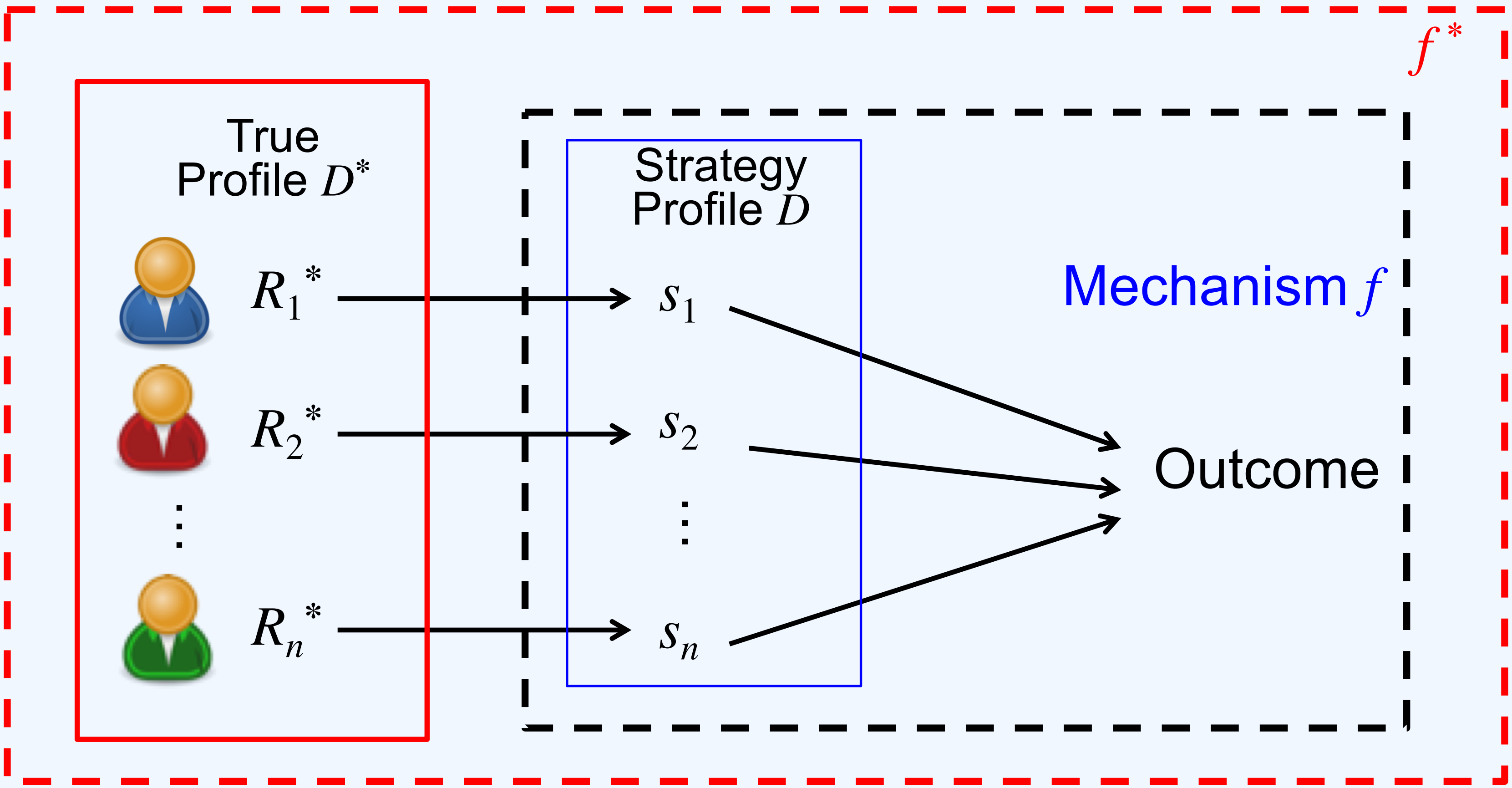
# Mechanism design with money

- With monetary transfers
- Set of alternatives: $A$
  - e.g. allocations of goods
- Outcomes: { (alternative, payments) }
- Preferences: represented by a quasi-linear utility function
  - every agent $j$ has a private value $v_j^*(a)$ for every $a \in A$. Her utility is

$$u_j^*(a, p) = v_j^*(a) - p_j$$

  - It suffices to report a value function $v_j$

# Implementation




- A game and a solution concept implement a function $f^*$, if
  - for every true preference profile $D^*$
  - $f^*(D^*)$ =OutcomeOfGame($f$, $D^*$)
- $f^*$ is defined w.r.t. the true preferences
- $f$ is defined w.r.t. the reported preferences

# Can we adjust the payments to maximize social welfare?

- Social welfare of $a$
  - $\text{SW}(a)=\Sigma_j v_j^*(a)$
- Can any ($\text{argmax}_a$ $\text{SW}(a)$, payments) be implemented w.r.t. dominant strategy NE?

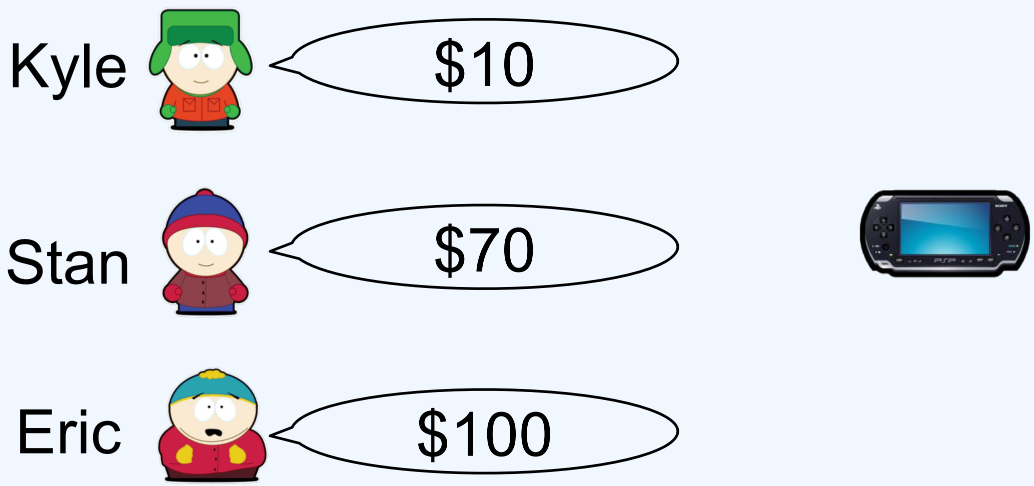
# The Vickrey-Clarke-Groves mechanism (VCG)

- The Vickrey-Clarke-Groves mechanism (VCG) is defined by
  - Alterative in outcome: $a^*$=argmax$_a$ SW($a$)
  - Payments in outcome: for agent $j$

$$p_j = \max_a \Sigma_{i \neq j} v_i(a) - \Sigma_{i \neq j} v_i(a^*)$$

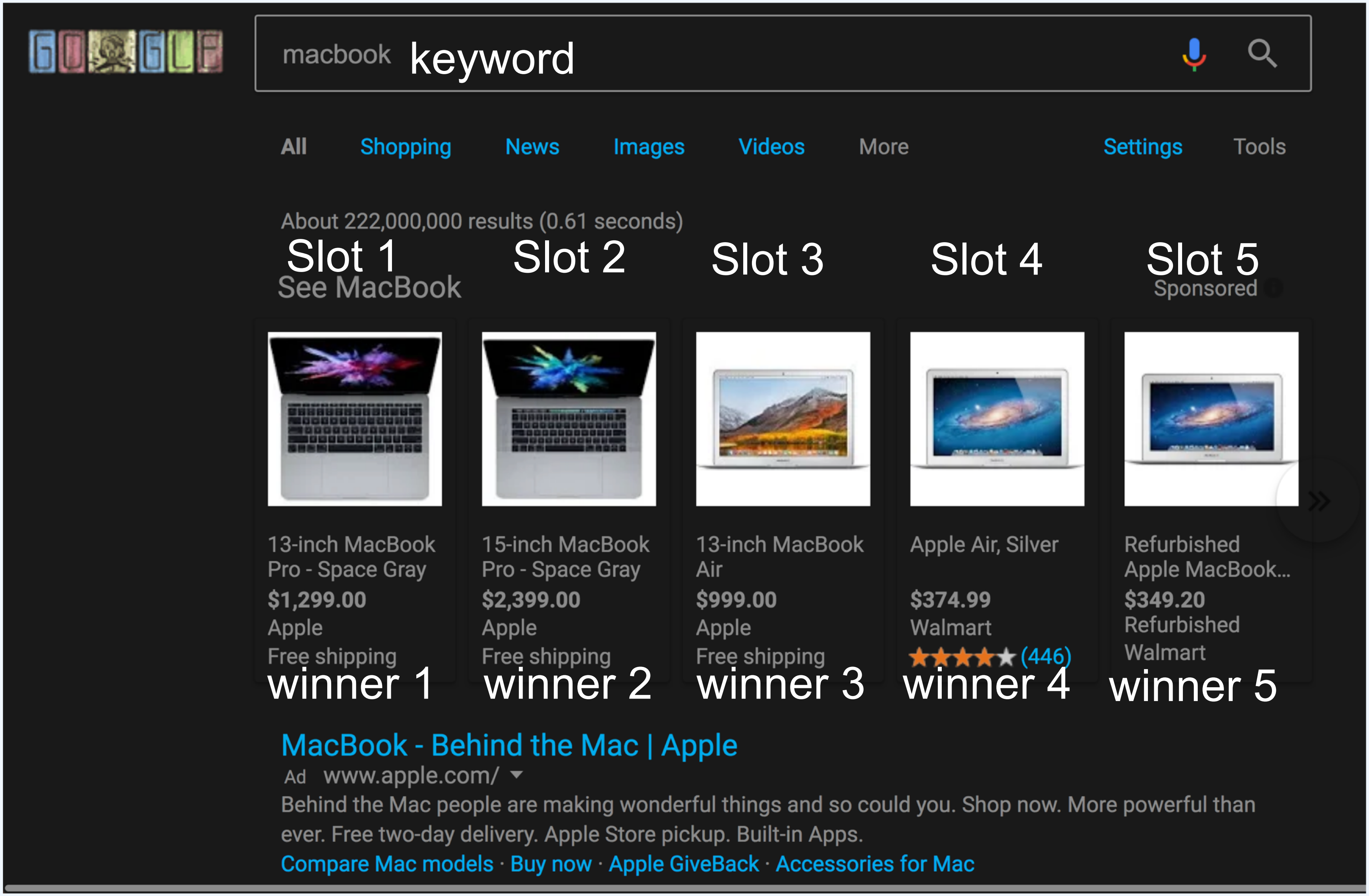
    - negative externality of agent $j$ of its presence on other agents
- Truthful, efficient

# Example: auction of one item




- Alternatives = (give to K, give to S, give to E)
- $a^*$ = 
- $p_1$ = 100 – 100 = 0
- $p_2$ = 100 – 100 = 0
- $p_3$ = 70 – 0 = 70

# Example: Ad Auction



# Ad Auctions: Setup

- $m$ slots
  - slot $i$ gets $s_i$ clicks
- $n$ bidders
  - $v_j$ : value for each user click
  - $b_j$ : pay (to service provider) per click
  - utility of getting slot $i$ : $(v_j - b_j) \times s_i$
- Outcomes: { (allocation, payment) }

# Ad Auctions: VCG Payment

- 3 slots
  - $s_1$ = 100, $s_2$ =60, $s_3$ =40
- 4 bidders
  - true values $v_1^*$ = 10, $v_2^*$ = 9, $v_3^*$ = 7, $v_4^*$ = 1,
- VCG allocation: OPT = (1, 2, 3)
  - slot 1->bidder 1; slot 2->bidder 2; slot 3->bidder 3;
- VCG Payment
  - Bidder 1
    - not in the game, utility of others = 100*9 + 60*7 + 40*1
    - in the game, utility of others = 60*9 + 40*7
    - negative externality = 540, pay per click = 5.4
  - Bidder 2: 3 per click, Bidder 3: 1 per click

# VCG is DSIC

- proof. Suppose for the sake of contradiction that VCG is not DSIC, then there exist $j$, $v_j$, $v_{-j}$, and $v'_j$ such that

$$u_j(v_j , v_{-j}) < u_j(v'_j , v_{-j})$$

- Let $a'$ denote the alternative when agent $j$ reports $v'_j$

$\Leftrightarrow$ $v_j(a^*) – (\max_a \sum_{k \neq j} v_j (a) - \sum_{k \neq j} v_j (a^*))$

$< v_j(a') – (\max_a \sum_{k \neq j} v_j (a) - \sum_{k \neq j} v_j (a'))$

$\Leftrightarrow$ $v_j(a^*) + \sum_{k \neq j} v_j (a^*) < v_j(a') + \sum_{k \neq j} v_j (a')$

Contradiction to the maximality of $a^*$