Linear regression

$$\dot{y} = \dot{\omega}^{\dagger} \dot{z}$$

$$= \omega_0 + \omega_1 x_1 + \cdots$$

$$+ \omega_d x_d$$

$$\omega$$
hyperplane

cugmental space $f = \sum_{i=1}^{N} (y_i - y_i)^2$

SSE Suared squared

$$P(y=(x))$$

$$P(y=0|x) = 1 - P(y=1|x)$$

Ginary Lyishe regression

y is categorical

 $y: \in \{0,1\}$ binary class

classification

$$P(y=1|\vec{x}) = \Theta(\vec{\omega}\vec{x})$$

(Lugmental Space)

$$\theta = \text{Sigmoid}$$

$$\theta(z) = \frac{1}{110^{2}} = \frac{2}{110^{2}}$$

$$\Theta(\vec{\omega}^{T}\vec{z}) = \frac{e^{\vec{\omega}\vec{x}}}{1 + e^{\vec{\omega}^{T}\vec{z}}}$$

$$P(y=0|\vec{x}) = 1 - \theta(\vec{\omega}^T \vec{x}) = \theta(-\vec{\omega}^T \vec{x})$$

$$\theta(z) = \frac{e}{1 + e^z} \quad \text{then} \quad \theta(z) = \frac{\partial \theta(z)}{\partial z} = \theta(z) \cdot (1 - \theta(z))$$

$$= \frac{\partial \theta(z)}{\partial z} = \frac{\partial \theta(z)}{\partial z} = \frac{\partial \theta(z)}{\partial z} \cdot \frac{\partial \theta(z)}{\partial z} = \frac{\partial \theta(z)$$

$$P(y|\vec{x}) = \begin{cases} 0(z) & \text{if } y=0 \\ 1-0(z) & \text{if } y=0 \end{cases}$$

$$D = \begin{cases} \vec{x}_{c}, y_{c} \end{cases}^{n} \text{ classer}$$

$$P(y|\vec{x}) = \begin{cases} 0(z) & (1-0(z))^{1-y} \\ 1-y & \text{model} \end{cases}$$
Alternative expression

alternative expression

$$P(y_i | \vec{x}_i) = O(\vec{\omega} \vec{x}_i) (1 - O(\vec{\omega} \vec{x}_i)) + i = 1...n$$

$$\vec{\omega} \in \mathbb{R}^{d+1} \text{ is the unknown}$$

Q: how to find to? that's the objective?

Objective: Maximize the likelihood = maximum likelihood Estimation (ME)

Ditelinood

Ditelinood

Note P(D|
$$\vec{\omega}$$
)

The $\vec{\omega}$ that gives

the highest prob to

the observed sample

is the optimal one of

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 $\vec{\omega}$

Directly all

product $\vec{\omega}$
 $\vec{\omega}$

$$\max_{\overrightarrow{\omega}} \log L(\overrightarrow{\omega}) = \sum_{i=1}^{n} \log \left(O(z_i)^{S_i} O(-z_i)^{S_i} \right) ; z_i' = \omega^T x_i'$$

$$= \sum_{i=1}^{n} \log \left(\Theta(z_i)^{y_i} \right) + \log \left(\Theta(-z_i)^{1-y_i} \right)$$

$$m_{cx}$$
 log $L(\vec{x}) = \sum_{i=1}^{n} y_i \log \Theta(z_i) + (1-y_i) \log (1-\Theta(z_i))$

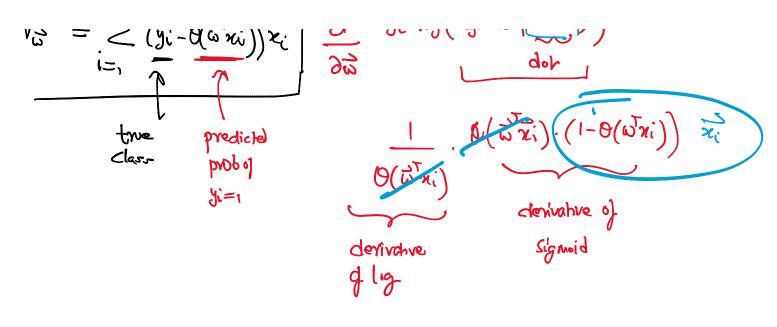
Log-likelihood (LL)

Negative

$$\text{min BCE}(\vec{\omega}) \equiv \min_{\vec{\omega}} \text{NLL}(\vec{\omega}) \equiv - \log L(\vec{\omega})$$

$$M_{c_{\times}} \log L(\vec{a}) = M_{c_{\times}} \sum_{i=1}^{N} y_{i} \log Q(z_{i}) + (1-y_{i}) \log (1-Q(z_{i}))$$

$$\nabla_{\omega} = \sum_{i=1}^{N} (y_i - \theta(\tilde{\omega}^{\frac{7}{N}})) \tilde{x}_i$$
 $\frac{\partial}{\partial \tilde{\omega}}$ $y_i \log(\tilde{y}_i - \tilde{y}_i)$ $\frac{\partial}{\partial \omega}$ $\frac{\partial}{\partial \omega}$ $\frac{\partial}{\partial \omega}$ $\frac{\partial}{\partial \omega}$ $\frac{\partial}{\partial \omega}$



$$\sum (y_i - \Theta(i x_{n_i})) \cdot x_i = 0$$

$$\geq 6(\vec{k}^T x_i) x_i = \geq y_i x_i$$

No closed form solution 1

$$\nabla \vec{b} = \sum_{i=1}^{N} (y_i - \theta(\vec{b}_{x_i})) \cdot \vec{x}_i$$
Scalar Vector

$$\vec{\omega}^{(t)} = \vec{\omega}^{(t-1)} + \vec{\eta} \cdot (\vec{\nabla} \vec{\omega})$$

$$\leq \text{tep size (eta)}$$

batch gradient ascent 1

Stochashe Gradian Ascent

based on a single point

Practical

$$\nabla_{\vec{\omega}}(\mathbf{x}_i) = (y_i - \theta(\vec{\omega}^T \mathbf{x}_i))^{\frac{1}{\mathbf{x}_i}}$$

$$\mathcal{L}_{(t)} = \mathcal{L}_{(t-1)} + \mathcal{A} \cdot \mathcal{L}_{\mathcal{L}}(x;)$$

testing / Inference ?

given an unknown
$$\approx_{n+1}$$

$$f(\tilde{z}_{nn}) = \begin{cases} \tilde{y} = 0 \\ \tilde{y} = 0 \end{cases}$$

Otherwise

Multiclass logistic Regression

$$\{\vec{z}_i, y_i\}_{i=1}^n$$

$$y_i \in \{c_1, c_2, \dots, c_k\}$$

K - Symbolic classes

given yi, xi

predict k probabilités

$$P(y_i = C_i \mid \widehat{x_i}) = \underline{T_i}(\widehat{x_i})$$

$$P(y_{i} = C_{1} \mid x_{i}) = \frac{T_{2}(x_{i})}{T_{2}(x_{i})}$$

$$P(y_{i} = C_{k} \mid x_{i}) = \frac{T_{k}(x_{i})}{T_{k}(x_{i})}$$

$$\frac{k}{j=1} T_{j}(x_{i}) = 1$$

we need to different weight vectors

$$\vec{w}_1, \vec{w}_2, \dots, \vec{w}_k \in \mathbb{R}^{d+1}$$

$$T_1(\vec{x}_i) = f(\vec{\omega}_i^T x_i)$$

$$\pi_{2}(\vec{x}_{i}) = \left(\vec{\omega}_{2}^{T}\vec{x}_{i}\right)$$

$$\overline{q}_{k}(\overline{z}_{i}) = \left(\overline{w}_{k}^{T}x_{i}\right)$$

f has to be some
function that sum
to 1 with all values
in [0,1]

f= Signoib

$$f(\vec{\omega}_{j}^{T}\vec{x}_{i}) = \frac{e^{i\vec{\omega}_{j}^{T}}\vec{x}_{i}}{e^{i\vec{\omega}_{j}^{T}}\vec{x}_{i}}$$
over all classes
$$a=1$$

Signaid
$$O(\omega^{T} x^{2})$$

$$= O(\omega^{T} x^{2})$$

$$W = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$
 (d+1) × K

weight matrix

$$\frac{MLE: \text{ wer log } L(W) = \sum_{i=1}^{N} \log P(y_i | W)$$

Under Independence acromption

$$P(y_i|W) = \begin{cases} II_1(\vec{v}_i^T x_i^2) & \text{if } y_i = C_1 \\ II_2(\vec{w}_i^T x_i^2) & \text{if } y_i = C_2 \end{cases}$$

$$\vdots$$

$$T_k(\vec{w}_k^T x_i^2) & \text{if } y_i = C_k$$

$$E_s.$$

$$K = Y$$

$$\begin{pmatrix} \lambda_{i}, y_{i} \end{pmatrix} \longrightarrow \begin{pmatrix} \lambda_{i}, y_{i} \end{pmatrix}$$

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$$\begin{pmatrix} \lambda_{i}, y_{i} \end{pmatrix} \longrightarrow \begin{pmatrix} \lambda_{i}, y_{i} \end{pmatrix}$$

yi 6 de(e2,00,00)

encode the k classe into one-hot vectors for the the class

)
$$\overrightarrow{x}_{i} = (1, 2)$$
 $\overrightarrow{y}_{i} = \overrightarrow{e}_{3} = (0010)^{T}$

$$y_i \in \mathbb{R}^k$$
One-hor

$$P(\vec{y}_i \mid z_i) = \begin{bmatrix} T_1(\omega_1^T z_i) & T_2(\omega_2^T z_i^T) \\ T_3(\omega_2^T z_i^T) & T_4(\omega_3^T z_i^T) \end{bmatrix}$$

e-9.
$$y_i = e_3 = (0010) = (y_{i1}, y_{i2}, y_{i3}, y_{in})$$
 $k = 4$

$$w_{x}$$
 $\log L(w) = \sum_{i=1}^{N} y_{i1} \log T_{i}(w_{i}^{T} y_{i}) + y_{i2} \log T_{2}(w_{2}^{T} y_{i}) - 1$

$$\omega_{\alpha}^{(0)} = \text{Yandom (d+1) dim Vector} \qquad \forall \alpha = 1 \dots k$$

Uniform (2,1) or random normal (0, ==0.1)

For
$$i = 1 - N$$
 in Yandom Order \leftarrow Stochastic

for $j = 1 - k$ \leftarrow GII Classes

Tij $(\vec{k} \cdot \vec{x}_{i}) \leftarrow$ Softmax values

Epoch

 $V_{\omega_{j}}(x_{i}) = (y_{ij} - T_{j}(\vec{\omega}_{j} \cdot \vec{x}_{i})) \cdot x_{i}$
 $(tri) = \omega_{j} + \eta \cdot V_{\omega_{j}}(\eta_{i})$

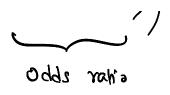
Check for Convergence
$$S = \sum_{j=1}^{k} ||\widetilde{w}_{j}^{t+1} - \widetilde{w}_{j}^{t}||$$
Stop is $S < E$
Hereshold, e.s. $E = 0.0001$

legistic model

> Computer log-odd ratios

tog odds vatio
$$\equiv \log \left(\frac{P(y_{i=1} | n_{i})}{P(y_{i=0} | x_{i})} \right) = \overline{\omega}_{x_{i}}^{x_{i}}$$

think of this as



also tre los k- class situation (softmen)

$$\log sdds rahin = \log \left(\frac{P(y_i = C_1 \mid m_i)}{P(y_i = C_k \mid m_i)} \right) = \frac{1}{m_i}$$

$$lor_2 = log \left(\frac{l(y_i' = c_2 \mid x_i')}{l(y_i' = c_k \mid x_i)} \right) = \frac{lor_2}{l(x_i' = c_k \mid x_i')}$$
wishake

ice log odds ratio is related to the difference of the weight vectors