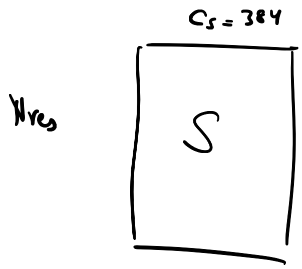
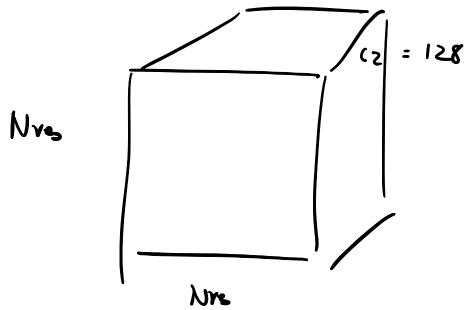
Structure Module

$S : \{s_i\}_{i=1..N_{res}}$ $\qquad s_i \in \mathbb{R}^{c_s}$

Single rep



$Z : \{z_{ij}\}$

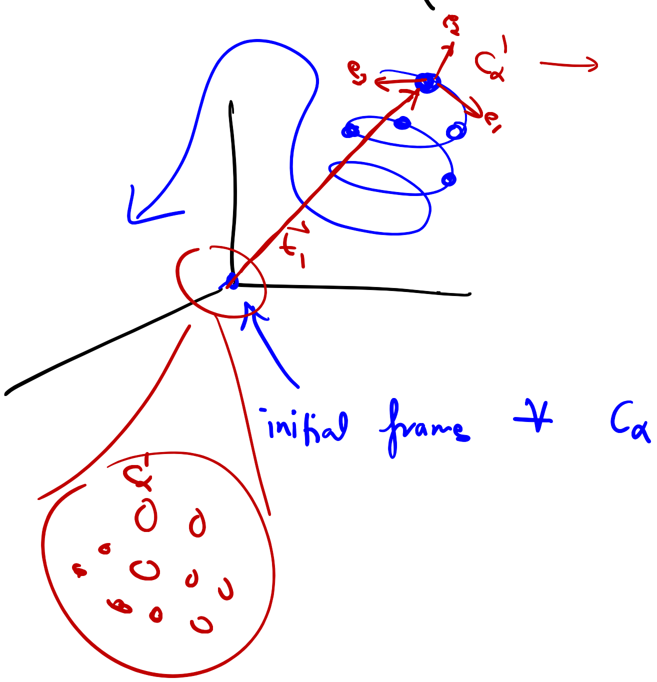


$T = \{T_i\}_{i=1..N_{res}}$

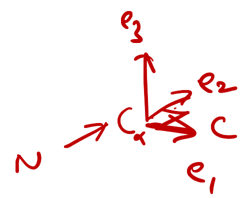
$(I, \vec{0})$

T_i : mapping from local to global ref frame

IPA: Invariant Point Attn (S, Z, T)



$\longrightarrow \quad T_1^{true} = \left(R_1 = \begin{bmatrix} | & | & | \\ e_1 & e_2 & e_3 \\ | & | & | \end{bmatrix}, \vec{t}_1\right)$



$C_\alpha - C$ is $\vec{e_1}$

$N - C_\alpha$ is $\vec{e_2}$ after orthogonalization

$\vec{e_3} = \vec{e_1} \times \vec{e_2}$

Regular attn + bias + 3D attn

K, Q, V

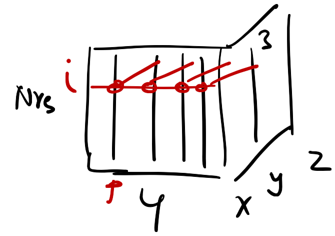
12 heads

c = 16 (dproj)

(Z)

$S \to Q$, $S \to K$, $S \to V$

$S \to Q^{3d}$, $S \to K^{3d}$, $S \to V^{3d}$ $(N_v=8)$



$N_q = 4$

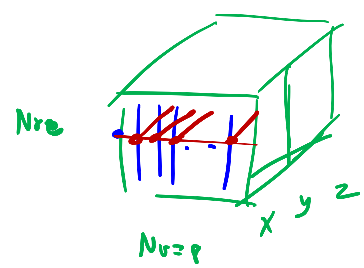
$$a_{ij} = w_L \left(\frac{q_i \cdot k_j}{\sqrt{c}} + b_{ij} + \frac{w_c}{2} \sum_{p=1}^{N_q} \left\| T_i \circ \vec{q}_p^{\,i} - T_j \circ \vec{k}_p^{\,j} \right\| \right)$$

Output Components

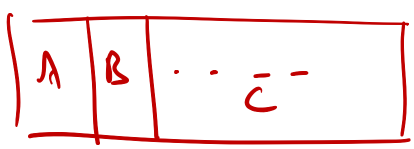
A) $\sum_j a_{ij} z_{ij}$ ← pair info

B) $\sum_j a_{ij} v_j$ ← regular value

C) $T_i^{-1} \circ \left(\sum_j a_{ij} \left(T_j \circ \vec{v}_p^{\,j} \right) \right)$ $\forall p = 1 \dots N_r$

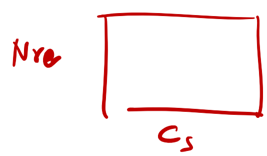


$\tilde{O}$ =



⇓

S =



Structure Module($S_i^{initial}$, z):

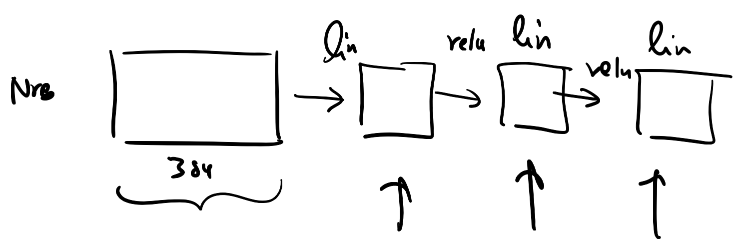
$T_i = (I, \emptyset)$ $\forall i = 1 \dots N_{res}$

for $l = 1 \dots 8$

$S_i = \text{IPA}(S_i) + S_i^{initial}$

$S_i = \text{Transition}(S_i) + S_i$ ← 3 linear with relu in-between

$T_i = T_i \circ \text{BackboneUpdate}(S_i)$

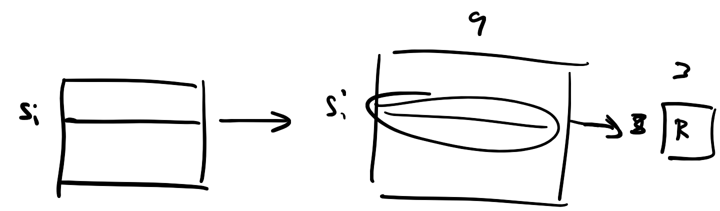


Backbone Update (s_i)

$$T_{new} = (R_{new}, \vec{t}_{new})$$

rotation matrix

- orthogonal $R^{-1} = R^{T}$ $R \cdot R^{T} = I$
- $\det(R) = 1$
- 3x3 matrix



Quarternions

Any rotation w.t can be represented by a rotation about an Euler axis with angle θ

Structure Module (s, z)

$T_i = (I, \vec{0}) \quad \forall\, i = 1 \dots N_{res}$

for $\ell = 1 \dots 8$

$$\left[\begin{array}{l}
s_i = \text{IPA}(s_i) + s_i \\
s_i = \text{Transition}(s_i) + s_i \\
T_i = T_i \circ \text{BackboneUpdate}(s_i) \\
\boxed{\alpha_i^{\phi,\psi} = \text{Torsion Angle Prediction}(s_i)} \\
x_i^{\alpha} = \vec{t}_i \quad \forall\, i = 1 \dots N_{res} \\
L_{aux}^{\ell} = \left[\text{FAPE}(T_i x_i \,\downarrow) + \text{torsion Angle Loss}(\alpha_i^{\phi,\psi})\right]
\end{array}\right.$$

$T_i = (R_i, \vec{t}_i)$

Predicted

true

$L_{aux} = \underset{\ell}{\text{mean}} \{ L_{aux}^{\ell} \}$

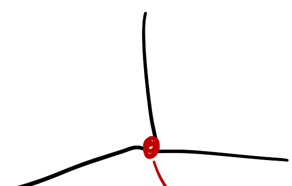
final atom coordinate

$L = \text{loss\_final}$.

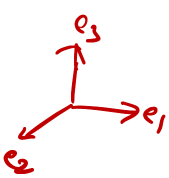
Backbone Update (s_i)

we need to update
rotation matrix $\underline{\underline{R_i}}$
and compute updated translation $\vec{t}_i$

$\underline{\underline{\vec{t}_i \in \mathbb{R}^3}}$
(x, y, z)



$T_1 \equiv C_{\alpha_1}$
$\swarrow$
$\underline{R_1}, \underline{\vec{t}_1}$



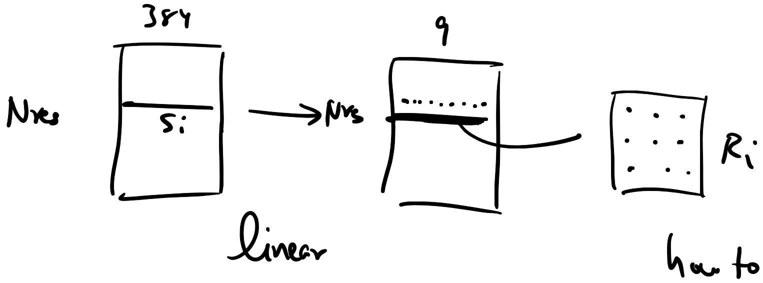
$R_i \in \mathbb{R}^{3\times3}$

$\hookrightarrow$ orthogonal $\qquad R_i^T R_i = I$

$\hookrightarrow$ $\det(R_i) = +1 \qquad R_i R_i^T = I$

$\equiv \underline{\underline{R_i^{-1} = R_i^T}}$

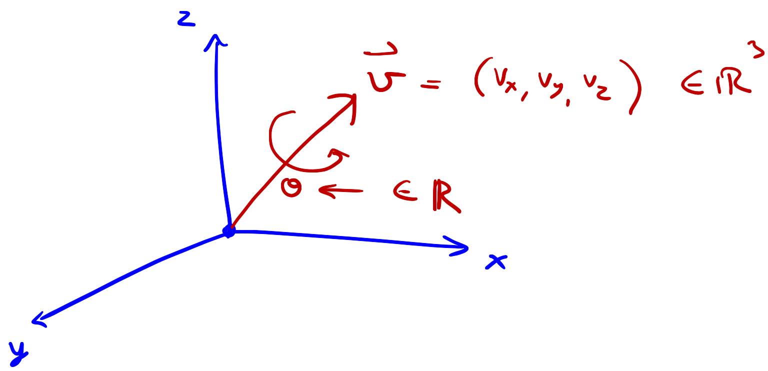
$\times$ not allowed $\left(\text{roto-}\underline{\underline{\text{reflection}}} \quad \det(R_i) = -1 \right)$



how to guarantee that R_i is orthogonal and $\det(R_i) = 1$?

Quarternions

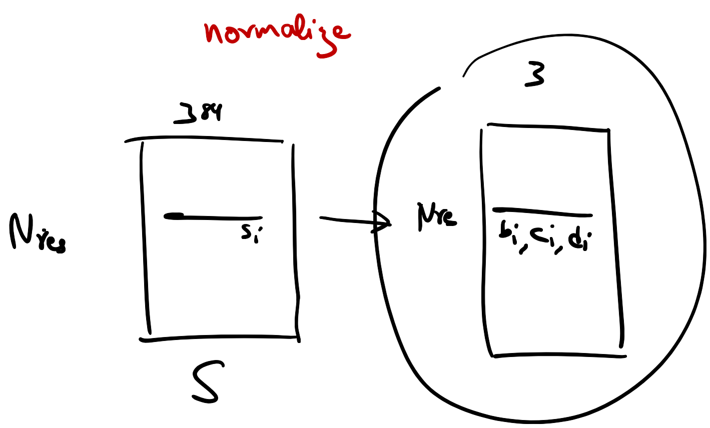
Any rotation in 3D space, about a fixed point, can be represented as a rotation about the Euler axis $\vec{v}$ by angle θ.



$$R_{\vec{v}}^{\theta} = \underbrace{\cos\left(\frac{\theta}{2}\right)}_{\text{real}} + \underbrace{\left(v_x i + v_y j + v_z k\right)}_{\text{Vector}} \cdot \sin\left(\frac{\theta}{2}\right)$$

$a \quad b \quad c \quad d$

normalize



$$q_i = 1 + b_i \cdot i + c_i \cdot j + d_i \cdot k$$

$$= (1, \underline{b_i, c_i, d_i})$$

Normalize

$$q_i / \|q_i\| = (1, b_i, c_i, d_i) \Big/ \sqrt{1 + b_i^2 + c_i^2 + d_i^2}$$

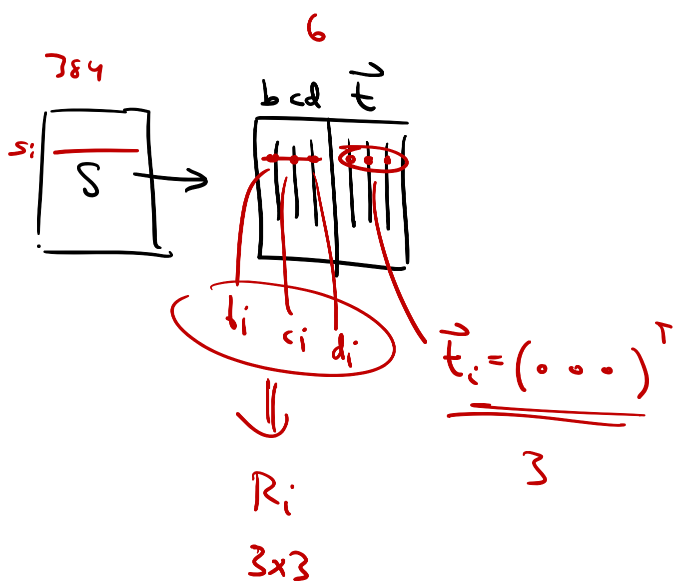
$$q_i \equiv (a_i, b_i, c_i, d_i)$$

$\Downarrow$ convert quaternion to a rotation matrix

R_i

$$R_i = \begin{pmatrix} a_i^2 + b_i^2 - c_i^2 - d_i^2 & 2b_ic_i - 2a_id_i & 2b_id_i + 2a_ic_i \\ \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots \end{pmatrix}$$

3x3 rotation matrix

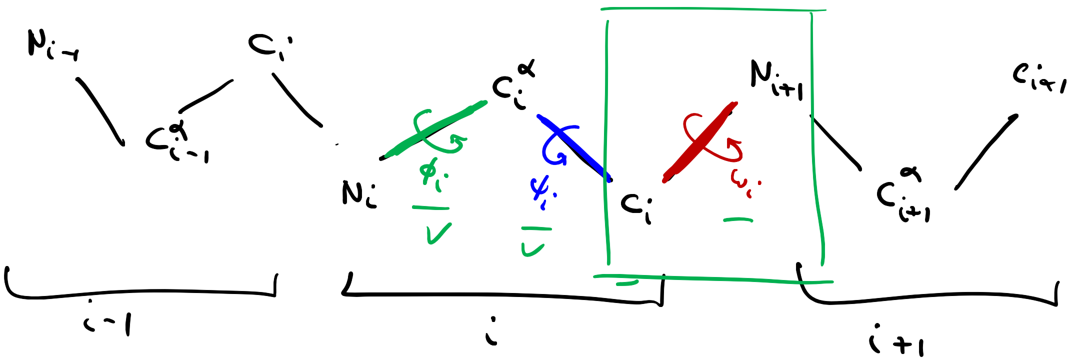


return $T_i = (R_i, \vec{t}_i)$ $\forall\, i = 1 \ldots N_{res}$

Torsion Angle Prediction : $\boxed{\phi, \psi,}\ \omega$, $\chi_1, \chi_2 \cdots \chi_5$

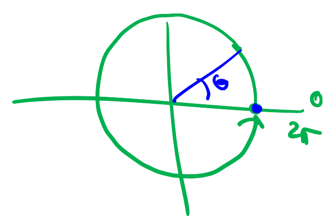
backbone — side chain torsions

i — ϕ_i, ψ_i $\qquad i = 1 \ldots N_{res}$

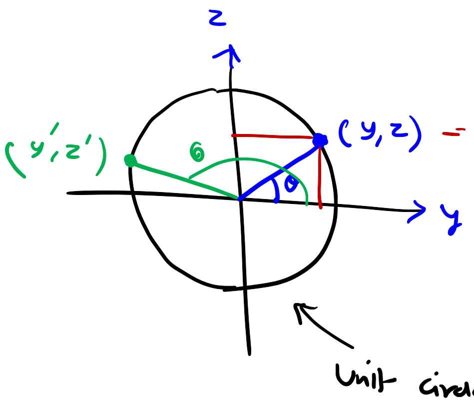


$\omega_i = 0,\ \underline{\underline{180}}$

$(-\pi, \pi)$ or $(0, 2\pi)$



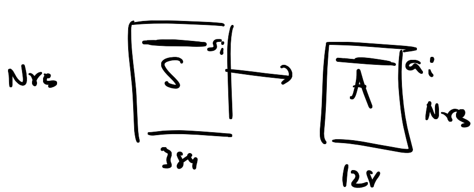
discontinuity @ $(0, 2\pi, \ldots)$



$= (\underline{\cos\theta}, \underline{\sin\theta})$

$\cos^2\theta + \sin^2\theta = 1$

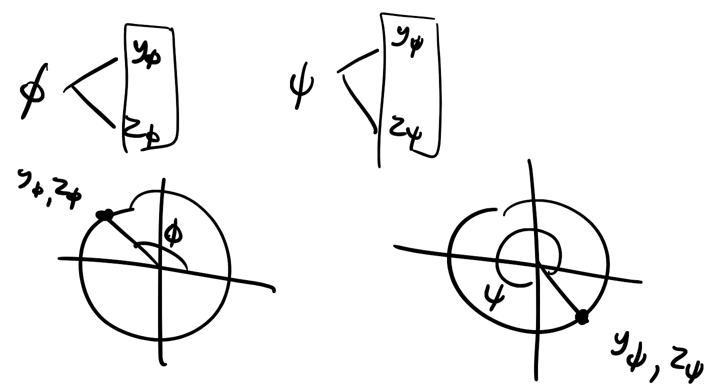
$$\vec{a}_i = \text{linear}(s_i) + \text{linear}(s_i^{\text{initial}}) \qquad a_i \in \mathbb{R}^{c=128}$$



$\vec{a}_i = \vec{a}_i + \text{lin}(\text{relu}(\text{lin}(\text{relu}(a_i))))$

$\vec{a}_i = \vec{a}_i +$ "

$\alpha_i^\phi, \alpha_i^\psi = \text{linear}(\text{relu}(a_i))$ $\qquad \alpha_i^{\phi,\psi} \in \mathbb{R}^2$



FAPE : Frame Aligned Point Error

$\underbrace{\{T_i, \vec{x}_i^\alpha\}}_{\text{Predicted}}{}_{i=1\ldots N_{res}}, \qquad \underbrace{\{T_i^{true}, \vec{x}_i^{\alpha,true}\}}_{\text{true}}$

$T_i = (\vec{R}_i, \vec{t}_i)$

$\vec{x}_i^\alpha \in \mathbb{R}^3$

1) align T_i and T_i^{true}

2) $\forall j = 1 \ldots N_{res}$

Compute the distance when frame @ i are aligned

$\forall i$, align T_i & T_i^{true}

$O(N_{res}^2)$

$\forall j$ compute dist between x_j^α, $x_j^{\alpha true}$

$d_{ij} = \sqrt{\| \underbrace{\underbrace{x_j^\alpha}_{\mathbb{R}^3} - \underbrace{x_j^{\alpha,true}}_{\mathbb{R}^3}}_{\text{global ref frame}} \|^2 + \varepsilon}$

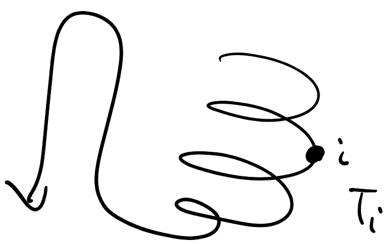
after rotation & translation

$T_i(\underbrace{x_j^\alpha}_{\text{local}}) = \underbrace{x_j^\alpha}_{\text{global}}$

$T_i^{true}(\underbrace{x_j^{\alpha,true}}_{\text{local}}) = \underbrace{x_j^{\alpha,true}}_{\text{global}}$

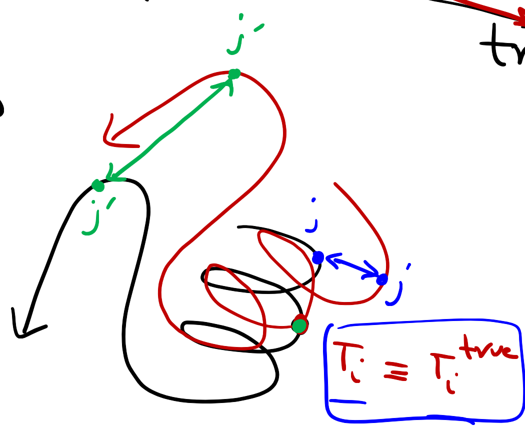
$$\mathcal{L}_{FAPE} = \frac{1}{10}\,\underset{i,j}{\text{mean}}\left\{ \underbrace{\min(d_{ij}, 10)}_{\text{don't penalize for } > 10\,\text{Å distance}} \right\}$$

T_i = map from local to global



Predicted

true structure



torsion angle Loss $\left(\underbrace{\alpha_i^{\phi}, \alpha_i^{\psi}}_{\text{predicted}}, \underbrace{\alpha_i^{\phi,true}, \alpha_i^{\psi,true}}_{\text{true}} \right)$

Angle as vectors $\mathbb{R}^2$

$$l_1^{\phi} = \|\alpha_i^{\phi}\|$$

Make unit vector $\longrightarrow$ $\alpha_i^{\phi} = \alpha_i^{\phi} / l_1^{\phi}$ | $\alpha_i^{\psi} = \alpha_i^{\psi} / \|\alpha_i^{\psi}\|$

$$\mathcal{L}_{torsion} = \underset{\substack{a_i \\ a_i = \phi_i \\ a_i = \psi_i \\ \forall i}}{\text{mean}} \left\{ \left\| \alpha_i^{a_i} - \alpha_i^{a_i,true} \right\|^2 \right\} + \text{angle norm} \; \underset{\substack{a_i = \\ \phi_i, \psi_i \\ \forall i}}{\text{mean}} \left\{ \left| l_1^{a_i} - 1 \right| \right\}$$

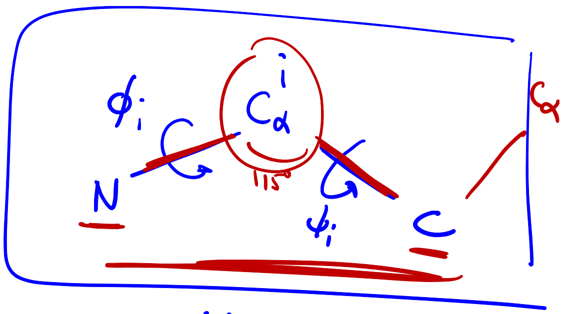
Final Coordinate prediction $(\alpha_i^{\phi}, \alpha_i^{\psi})$

$\cos \quad y_i^{\phi} \qquad\qquad y_i^{\psi} \quad \cos$

$\sin \quad z_i^{\phi} \qquad\qquad z_i^{\psi} \quad \sin$

$$R^{\phi} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi & \cos\phi \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & y_i & -z_i \\ 0 & z_i & y_i \end{pmatrix}$$

$\underline{T_i} \cdot$



$\cdot \; \underline{R^{\phi}} \begin{pmatrix} x_i^{\alpha} \end{pmatrix} = \boxed{x_i^{\alpha,\,final}}$

ideal config
coords

rotate into proper orientation : local ref frame

rotate into global ref frame