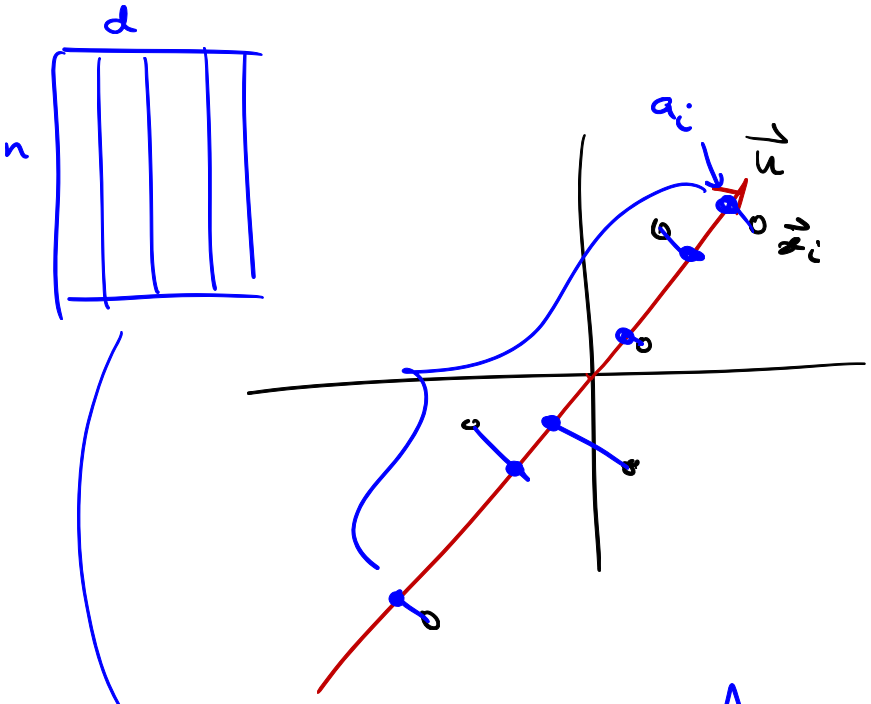


Center the data

$\overline{D}$

$$a_i = \underbrace{\text{proj}_{\vec{u}}(\vec{x}_i)}_{\text{scalar}}$$

$$A = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix} \longrightarrow \sigma_u^2 = \vec{u}^T \Sigma \vec{u}$$

Σ: cov matrix for D

$$J = \max_{\vec{u}} \vec{u}^T \Sigma \vec{u}, \quad \text{s.t.} \quad \underbrace{\vec{u}^T \vec{u} = 1}$$

$$J = \max_{\vec{u}} \vec{u}^T \Sigma \vec{u} - \underbrace{\alpha(\vec{u}^T \vec{u} - 1)}$$

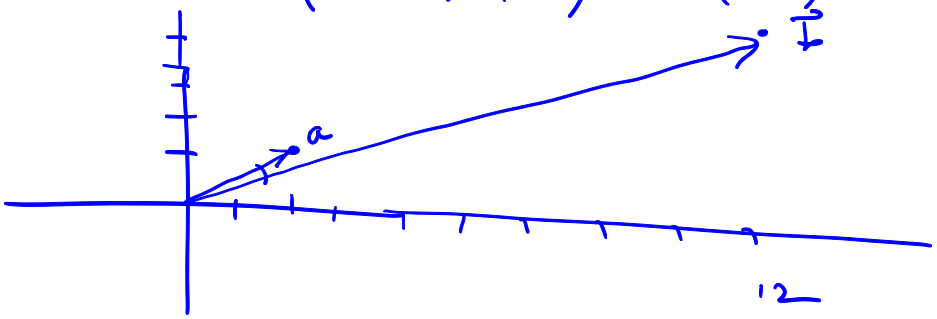
$$\frac{\partial J}{\partial \vec{u}} = 2\Sigma\vec{u} - 2\alpha\vec{u} = 0$$

$$\Rightarrow \boxed{\Sigma \vec{u} = \alpha \vec{u}}$$ eigen-equation

Σ: $d \times d$, $\vec{u}$: $d \times 1$, α: scalar 1×1

$$\Sigma = \begin{pmatrix} 5 & 2 \\ 2 & 1 \end{pmatrix} \qquad \vec{a} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

$$\Sigma \cdot \vec{a} = \begin{pmatrix} 5 & 2 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 12 \\ 5 \end{pmatrix} = \vec{b}$$



$$D = n \begin{bmatrix} \quad \end{bmatrix}_{n \times d} \begin{pmatrix} \vdots \\ \vdots \end{pmatrix}_{d \times 1} = \begin{pmatrix}$$